PROBLEM SET 2

NE 290H BARNARD & LUND — DUE 2/2/2009

(1.) PROBLEM: IN CLASS IT WAS STATED THAT IF:

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$$\chi \equiv \frac{x^2}{r_x^2+s} + \frac{y^2}{r_y^2+s}$$

AND IF

$$\rho(x,y) = \hat{\rho}\left(\frac{x^2}{r_x^2} + \frac{y^2}{r_y^2}\right) = \hat{\rho}(\chi)\Big|_{s=0} \equiv \frac{d\eta(\chi)}{d\chi}\Big|_{s=0}$$

AND IF

$$\Psi(x,y) = \frac{-r_x r_y}{4\varepsilon_0}\int_0^\infty \frac{\eta(\chi)\,ds}{\sqrt{r_x^2+s}\,\sqrt{r_y^2+s}}$$

THEN IT FOLLOWS THAT

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} = -\frac{\rho(x,y)}{\varepsilon_0}$$

and so $\Psi(x,y)$ is a solution of Poisson's equation.

a). Calculate $\frac{d\chi}{ds}$. (Note that x, y, r_x, and r_y are held fixed).

b). SHOW THAT

$$\nabla^2\psi = \frac{-r_x r_y}{4\varepsilon_0}\int_0^\infty \frac{-4\eta''(\chi)\frac{d\chi}{ds} + 2\eta'\left[\frac{1}{r_x^2+s}\right] + 2\eta'\left[\frac{1}{r_y^2+s}\right]}{\sqrt{r_x^2+s}\,\sqrt{r_y^2+s}}\,ds$$

c). Integrate the first term in the integral ABOVE BY PARTS AND show THAT ~~[crossed out]~~ PART OF IT CANCELS THE TERMS PROPORTIONAL TO η'.

d). EVALUATE THE INTEGRATED TERM TO SHOW THAT

$$\nabla^2\psi = -\frac{\rho}{\varepsilon_0}$$

30 POINTS

(2) PLOT $\log(\lambda)$ vs. $\log(\text{ion energy})$ for a heavy ion beam (mass 200) between 10 keV and 1 GeV. (λ is the maximum transportable line charge density assuming negligible emittance. Ion energy $\equiv qV$.) Plot for each of the following focusing devices:

a). Solenoids

b). Electric Quads

c). Magnetic Quads

d). Einzel lenses

Assume $B_{solenoid} = B_{quad} = 2T$

$\Phi_{einzel\ lens} = \Phi_{quad} = \pm 100\ kV$

$r_{beam} \leq 6\ cm$

$L_{einzel} \geq r_{beam}$

Occupancy $\eta = 0.7$

$\frac{r_{beam}}{r_{pipe}} = 0.7$

(Use formulas given in class notes on FINAL PAGE OF JOHN'S 6/17/2008 LECTURE).