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# Intrabeam collisions, gas and electron effects in intense beams

1. Beam/beam coulomb collisions
2. Beam/gas scattering
3. Charge changing processes
4. Gas pressure instability
5. Electron cloud processes
6. Electron-ion instability

Gas and electron effects

-Effects are quite different depending on q, m of species being accelerated

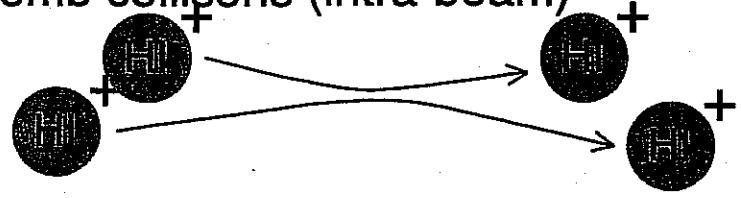
-Circular accelerators vs. Linacs
($t_{residence}$ ~ ms to days vs. 10's of $\mu$s)

-Long pulse vs. short pulse
($t_{pulse}$ ~ 10's of $\mu$s vs. 10's of ns)

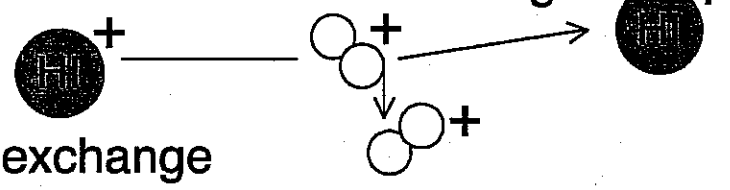
| | | |
|---|---|---|
| + Heavy ion | Residual gas molecule | $e^-$ electron |

## Processes:

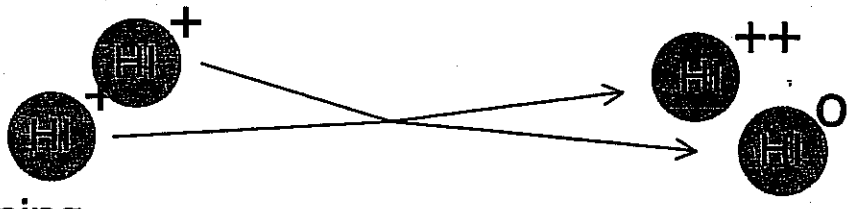
1. Coulomb collisons (intra-beam)



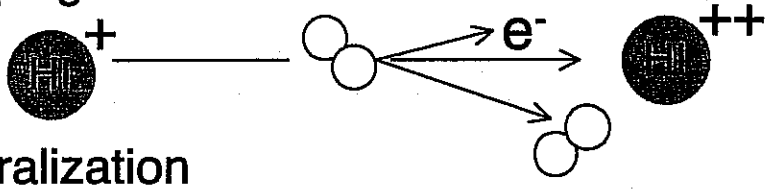
2. Coulomb collisions with residual gas



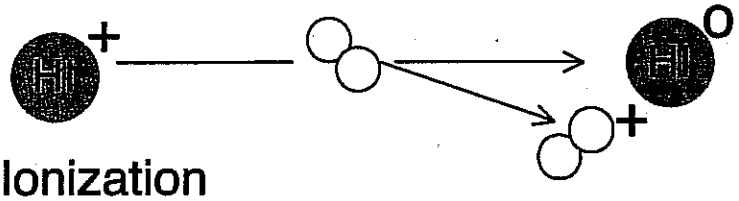
3. Charge exchange



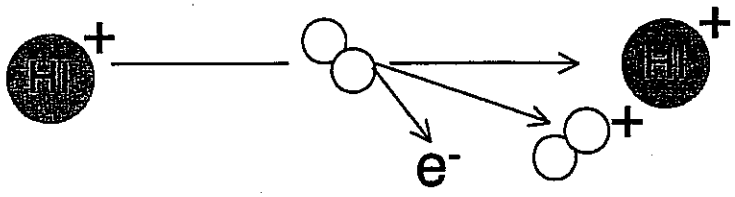
4. Stripping



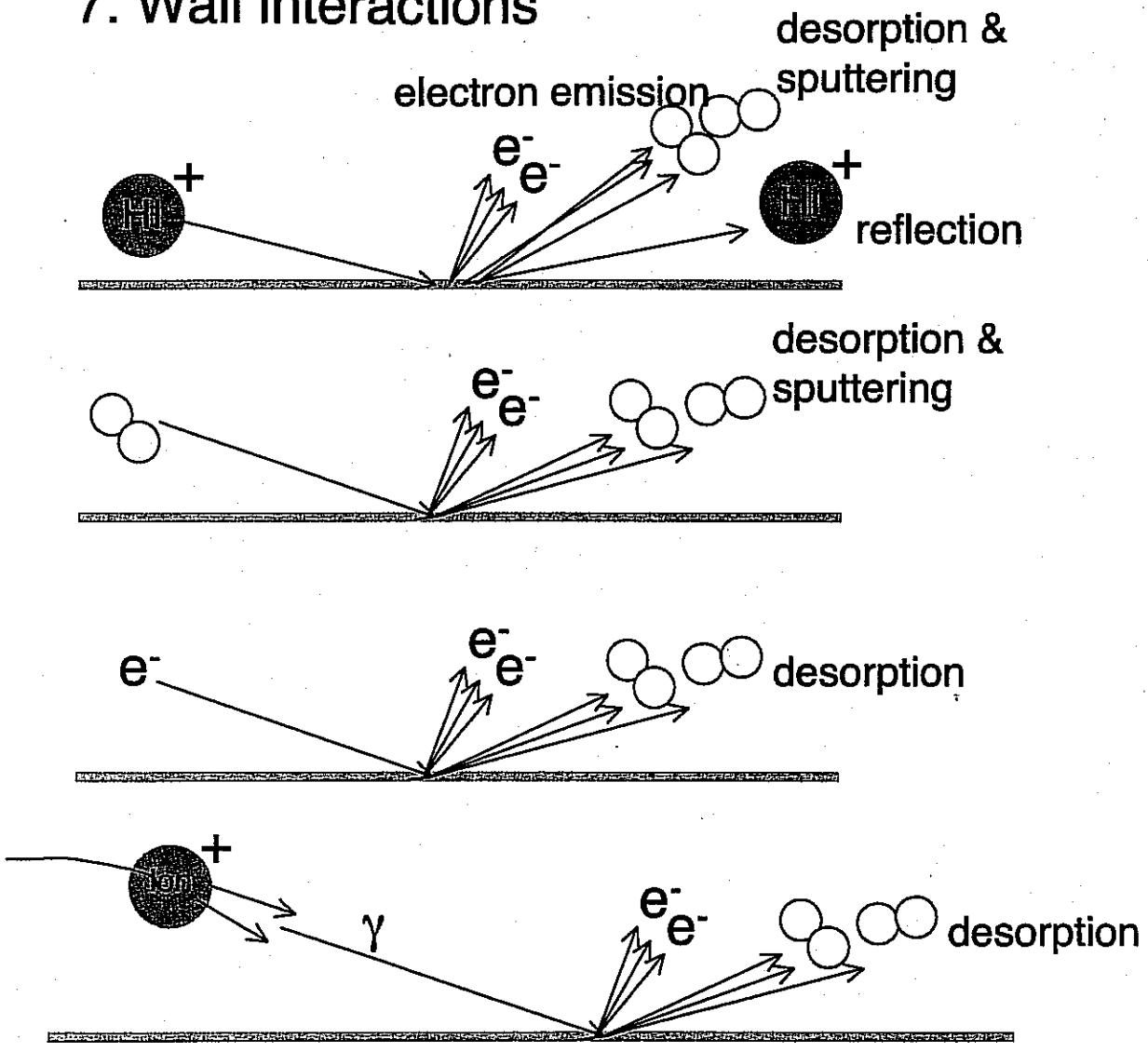
5. Neutralization



6. Gas Ionization



# 7. Wall interactions




| | |
|---|---|
| γ | synchrotron photon |

## 1. COLLISIONS WITHIN BEAM  REISER 6.4

CONSIDER EFFECTS OF COULOMB COLLISIONS IN A CONTINUOUS BEAM PROPAGATING THROUGH A SMOOTH FOCUSING CHANNEL WITH $T_{\perp 0} \neq T_{\parallel 0}$

(IF $T_{\perp 0} = T_{\parallel 0} \Rightarrow$ BEAM ALREADY RELAXED).

FROM ICHIMARU & ROSENBLUTH, PHYS FLUIDS 13, 2778, (1970):

$$\frac{dT_\perp}{dt} = -\frac{1}{2}\frac{dT_\parallel}{dt} = \frac{-(T_\perp - T_\parallel)}{\tau}$$

(SINCE $T_x = T_y = T_\perp$, $T_\parallel$ CHANGES AT TWICE THE RATE OF $T_\perp$)
(SINCE $2k_B T_\perp + k_B T_\parallel = \text{const}$)

$\tau$ = RELAXATION TIME

$$= \frac{15\,(k_B T_{eff}/mc^2)^{3/2}(4\pi\epsilon_0)^2 m^2 c^3}{8\pi^{1/2} q^4 \ln\Lambda\, n} = \left(\frac{15\,\pi^{1/2}}{8\ln\Lambda}\right)\nu_c^{-1}$$

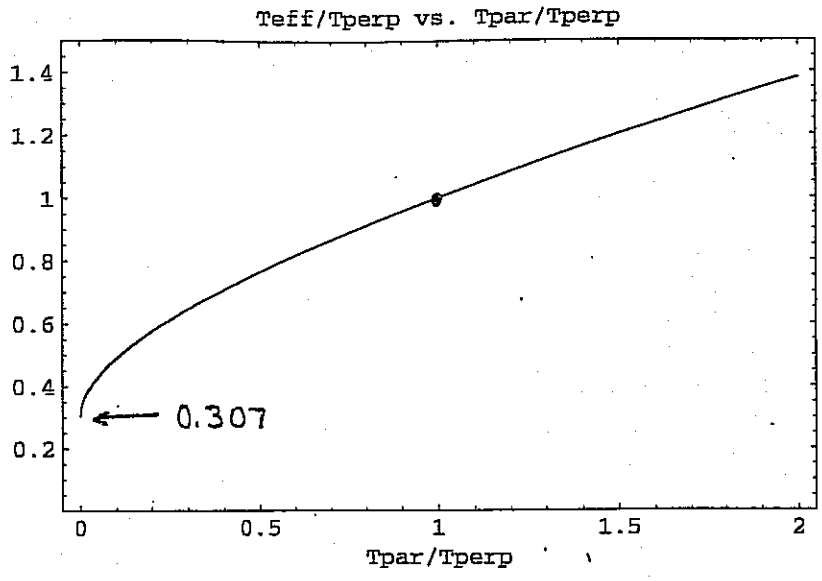
COULOMB COLLISION APPROXIMATE RATE FOR LARGE ANGLES (PAGE 9 OF INTRODUCTION NOTES)

$$\ln\Lambda = \begin{cases} \ln \dfrac{(\epsilon_0 k T_{eff})^{3/2} 12\pi}{q^3 n^{1/2}} & \text{for } \lambda_D < r_b \\[2ex] \ln \dfrac{12\pi\epsilon_0 k T_{eff} r_b}{q^2} & \text{for } \lambda_D > r_b \end{cases}$$

$$T_{eff} = T_\perp \left[\frac{15}{4}\int_{-1}^{1} \frac{\mu^2(1-\mu^2)\,d\mu}{\left[(1-\mu^2) + \mu^2 (T_\parallel/T_\perp)\right]^{3/2}}\right]^{-2/3}$$

$T_{eff}$ is an appropriate average of $T_\perp$ & $T_\parallel$

$\frac{T_{eff}}{T_\perp}$



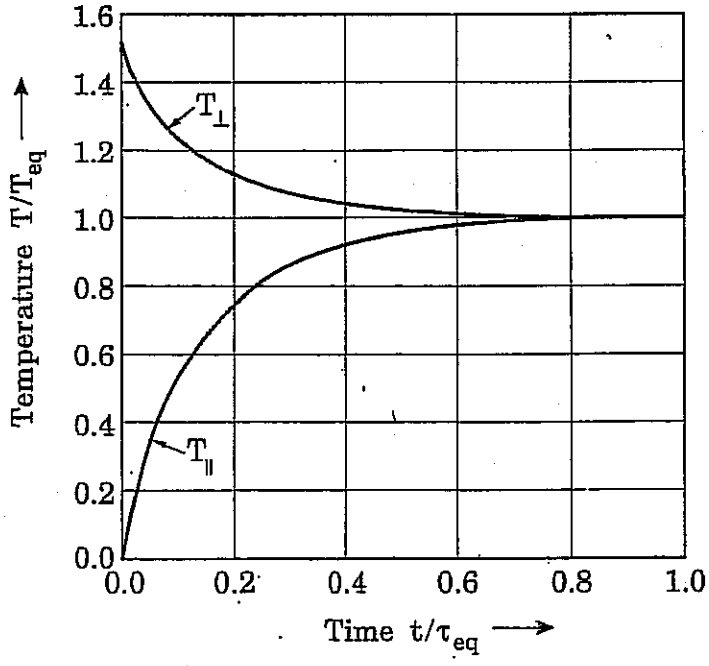

FOR $T_\parallel = 0$

$$T_\perp = \frac{2}{3} T_{\perp 0}\left(1 + \frac{1}{2} e^{-3t/\tau_{\text{eff}}}\right), \tag{6.156a}$$

(APPROXIMATE SOLUTIONS)

$$T_\parallel = \frac{2}{3} T_{\perp 0}\left(1 - e^{-3t/\tau_{\text{eff}}}\right), \tag{6.156b}$$

$\tau_{eff} = 0.42\,\tau_{eq}$




FROM REISER p.527

$\tau_{eq} = \tau(T_{eq})$

## BOERSCH EFFECT

AREN'T COLLISIONS NEGLIGIBLE? (NOT ALWAYS)

PUTTING IN NUMBERS:

FOR IONS:

$$\tau_{eff} = 4.3 \cdot 10^{-4}\,s \; \frac{(A^{1/2})}{Z^4} \left(\frac{kT_{eff}}{1\,eV}\right)^{3/2} \left(\frac{15}{\ln\Lambda}\right)\left(\frac{10^{10}\,cm^{-3}}{n}\right)$$

$$\ln\Lambda = \ln\left[\frac{1.5 \cdot 10^5\,(kT/1\,eV)^{3/2}}{Z^3\,(n/10^{10}\,cm^{-3})}\right]$$

EXAMPLE: 2 MeV INJECTOR

$\tau_{eff} \simeq 8.8 \cdot 10^{-4}\,s$ for $A = 39$, $Z = 1$, $n = 10^{10}\,cm^{-3}$, $kT_{eff} = 0.3\,eV$, $\ln\Lambda = 8.5$

$$t_{transit} \simeq \frac{2d}{V} \simeq \frac{2\,(2m)}{(.01)\,3 \cdot 10^8} = 1.3\,\mu s$$

So $\tau_{eff} >> t_{transit}$ $\Rightarrow$ collisions are rare BUT

$$T_{\substack{accel\\cool}} = \frac{1}{2}\left(\frac{kT_0}{qV}\right) kT_0 = 2.5 \cdot 10^{-9}\,eV \quad \text{for } \begin{matrix} kT_0 = 0.1\,eV \\ qV = 2\,MeV \end{matrix}$$

$$T_{\parallel\,collisions} \simeq \frac{2}{3} T_{\perp 0}\left(1 - \exp\left(-3t/\tau_{eff}\right)\right) \simeq 2T_{\perp 0}\left(\frac{t_{transit}}{\tau_{eff}}\right) = .006\,eV \quad \text{for } T_{\perp 0} = 1\,eV$$

SO $T_H$ FROM "BOERSCH EFFECT"

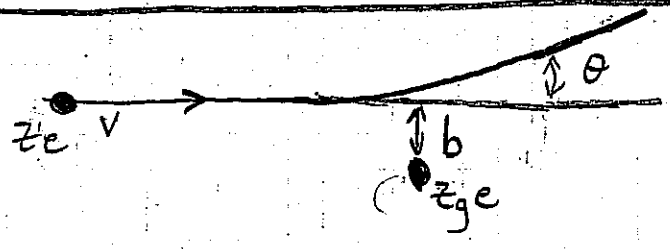
$\gg$ $T_{\parallel}$ FROM LONGITUDINAL COOLING

100 SHEETS
U.S.A.

# COULOMB COLLISIONS IN RESIDUAL GAS (REISER 6.4.3)

JACKSON CHAPTER 13

(RUTHERFORD SCATTERING)




$$\frac{dp_x}{dt} = \frac{z z_g e^2}{4\pi\epsilon_0 r^2}\frac{b}{r} \Longrightarrow \Delta p = \int_{-\infty}^{\infty} \frac{dp_x}{dt}\frac{dt}{dz}\,dz$$

$$= \frac{z z_g e^2 b}{4\pi\epsilon_0 v}\underbrace{\int_{-\infty}^{\infty}\frac{dz}{(z^2+b^2)^{3/2}}}_{2/b^2}$$

$$= \frac{2 z z_g e^2}{4\pi\epsilon_0 v b}$$

$$\Delta p\,\theta \cong \frac{\Delta p}{p} = \frac{2 z z_g e^2}{4\pi\epsilon_0 p v b} \Rightarrow \frac{db}{d\theta} \sim \frac{1}{\theta^2}$$

DIFFERENTIAL CROSS SECTION FOR SCATTERING WITH IMPACT PARAMETER $b$ INTO SOLID ANGLE $d\Omega$ AT ANGLE $\theta$ SATISFIES

$$\underbrace{2\pi b\,db}_{\text{AREA}} = \frac{d\sigma}{d\Omega}\underbrace{2\pi\sin\theta\,d\theta}_{\text{SOLID ANGLE}}$$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \left(\frac{2 z z_g e^2}{4\pi\epsilon_0 p v}\right)^2 \frac{1}{\theta^4}$$

ELECTRON SCREENING PUTS CUTOFF AT SMALL $\theta$ (LARGE $b$) SO BETTER TO USE

$$\frac{d\sigma}{d\Omega} = \left(\frac{2 z z_g e^2}{4\pi\epsilon_0 p v}\right)^2 \frac{1}{(\theta^2+\theta_{min}^2)^2}$$

AVERAGE ANGLE SQUARED FOR A SINGLE SCATTERING IS:

$$\bar{\theta}^2 = \frac{\int \theta^2 \frac{d\sigma}{d\Omega} 2\pi \sin\theta \, d\theta}{\int \frac{d\sigma}{d\Omega} 2\pi \sin\theta \, d\theta} \cong \frac{\int_0^{\theta_{max}} \frac{\theta^3}{(\theta^2+\theta_{min}^2)^2} d\theta}{\int_0^{\theta_{max}} \frac{\theta}{(\theta^2+\theta_{min}^2)^2} d\theta}$$

$$\cong 2\theta_{min}^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right)$$

ASSUMES $\theta_{max}^2 \gg \theta_{min}^2$

& $\ln\left(\frac{\theta_{max}}{\theta_{min}}\right) \gg 1$

## MULTIPLE COLLISIONS

AFTER TRAVERSING DISTANCE $s$, AND UNDERGOING $N_s$ COLLISIONS, THE MEAN SQUARE ANGLE $\overline{\Theta^2}$ $\left[\sigma_s = \pi\left(\frac{2Z_g e^2}{4\pi\epsilon_0 pv}\right)^2 \frac{1}{\theta_{min}^2}\right]$

$$\overline{\Theta^2} = N_s \bar{\theta}^2 = n_g \sigma_s s \bar{\theta}^2$$

$$= 16\pi n_g \left(\frac{Z Z_g e^2}{4\pi\epsilon_0 m c^2 \gamma p^2}\right)^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right) s$$

JACKSON ARGUES $\theta_{max}$ ARISES FROM DISTRIBUTED NATURE OF NUCLEUS (NOT POINT CHARGE) AND $\theta_{min}$ ARISES FROM SCREENING OF ELECTRONS OR UNCERTAINTY PRINCIPLE

$$\ln\frac{\theta_{max}}{\theta_{min}} \simeq \ln\left(204 Z_g^{-1/3}\right)$$

$$\Delta\langle x'^2\rangle = \frac{1}{2}\overline{\Theta^2}$$ [since $\Delta\langle x'^2\rangle + \Delta\langle y'^2\rangle = \Delta\overline{\Theta^2}$]

$$\epsilon = 4\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

FOR A MATCHED BEAM

$$k_p^2\langle x^2\rangle = \langle x'^2\rangle$$ where $k_p$ = depressed betatron wavenumber

$$\Rightarrow \quad \epsilon = \frac{4\langle x'^2\rangle}{k_p}$$

$$\Delta\epsilon = \frac{4\,\Delta\langle x'^2\rangle}{k_p} = \frac{2\,\overline{\Theta^2}}{k_p}$$

$$\Rightarrow \boxed{\frac{d\epsilon}{ds} = \frac{32\pi}{k_p} n_g \left(\frac{Z Z_g e^2}{4\pi\epsilon_0 mc^2\gamma\beta^2}\right)^2 \ln\left(204 Z_g^{-1/3}\right)}$$

IN TERMS OF NORMALIZED EMITTANCE:

$$\boxed{\frac{d\epsilon_N}{ds} = \frac{32\pi}{k_p} n_g \left(\frac{Z Z_g e^2}{4\pi\epsilon_0 mc^2}\right)^2 \frac{1}{\gamma\beta^3} \ln\left(204 Z_g^{-1/3}\right)}$$

<u>Example:</u> $n_g = 10^{-7}$ torr $= 3.5\cdot 10^{9}\ \mathrm{cm}^{-3} = 3.5\cdot 10^{15}\ \mathrm{m}^{-3}$

$k_{p0} = 2.5\ \mathrm{m}^{-1}$ $\quad k_p = 0.25\ \mathrm{m}^{-1}$

$Z_g = 7$, $Z = 19$, $A = 39$, $\beta = 0.01$, $\epsilon_N = 1\cdot 10^{-6}$ m-rad

$\Rightarrow \frac{d\epsilon_N}{ds} = 3.7\cdot 10^{-11}\ \mathrm{m}^{-1}$ $\Rightarrow$ Need 27 km to equal original emittance!

(But more important for rings & low mass!)

# PRESSURE "BUMPS" AND BEAM LOSS FROM CHARGE CHANGING COLLISIONS

REFERENCE: WORKSHOP ON BEAM INDUCED PRESSURE RISE IN RINGS, BNL, Dec. 2003.

$\sigma_s$ = STRIPPING CROSS SECTION

$\sigma_{ce}$ = CHARGE EXCHANGE CROSS SECTION

$\sigma_i$ = IONIZATION CROSS SECTION

$V_{cm}$ = mean ion velocity in ion beam frame

## (1) BEAM LOSS

$$\frac{dn_b}{dt} = -\sigma_s V_i n_b \bar{n} - \sigma_{ce} V_{cm} n_b^2 - \left.\frac{\partial n_b}{\partial t}\right|_{HALO}$$

## (2) GAS EVOLUTION

$\bar{n}$ = average gas density

$$\frac{d}{dt}\bar{n} = \underbrace{\eta_G \sigma_i V_i n_b \bar{n}}_{\text{IONIZATION}} \left(\frac{V_{beam}}{V_{pipe}}\right) + \underbrace{\eta_{HI} \sigma_s V_i n_b \bar{n}}_{\text{STRIPPING}} \left(\frac{V_{beam}}{V_{pipe}}\right) + \underbrace{\eta_{HI} \sigma_{ce} V_{cm} n_b^2}_{\text{CHARGE EXCHANGE}} \left(\frac{V_{beam}}{V_{pipe}}\right) + \underbrace{q}_{\text{OUTGASSING}} - \underbrace{(S/A_p)\bar{n}}_{\text{PUMPING}}$$

VOLUME OF BEAM

VOLUME OF PIPE

$S$ = Effective linear pumping rate $[m^3 s^{-1}/m]$ $\qquad A_p = \pi r_p^2$ = AREA OF PIPE

$q$ = OUTGASSING rate $= \frac{2\pi r_p Q}{\pi r_p^2} = \frac{2Q}{r_p}$; $\quad Q = \frac{\#}{m^2 s}$

$\eta_G$ = GAS MOLECULES DESORBED PER INCIDENT RESIDUAL GAS ION

$\eta_{HI}$ = GAS MOLECULES DESORBED PER INCIDENT IONIZATION STRIKING WALL

$\left( V_{beam}/V_{pipe} \rightarrow \left(\frac{n_b}{n_{pipe}}\right) \nu_{rep} \Delta t_{beam} \text{ for a rep rated linac} \right)$

If we take $n_b \simeq$ constant

then we may express gas evolution equation as:

$$\frac{d\bar{n}}{dt} = \frac{\bar{n}}{\tau} + q_{eff}$$

with solution:

$$\bar{n} = (\bar{n}_0 + \tau q_{eff}) \exp[t/\tau] - \tau q_{eff}$$

HERE $$\tau = \frac{1}{(\eta_g \sigma_i + \eta_{HI} \sigma_s)\left(\frac{V_{beam}}{V_{pipe}}\right) n_b V_i - S/A_p}$$

$$q_{eff} = q + \eta_{HI} \sigma_{CE} V_{cm} n_b^2 \left(\frac{V_{beam}}{V_{pipe}}\right)$$

<u>EQUILIBRIUM</u> REACHED IF $\tau < 0$ (i.e. pumping exceeds desorption).

$$\Rightarrow \bar{n} = -\tau q_{eff} = \frac{q + \eta_{HI} \sigma_{CE} V_{cm} n_b^2 (V_{beam}/V_{pipe})}{S/A_p - (\eta_g \sigma_i + \eta_{HI} \sigma_s)\left(\frac{V_{beam}}{V_{pipe}}\right) n_b V_i}$$

<u>INSTABILITY</u> IF $$n_b V_i \geq \frac{\frac{S}{A_p}\left(\frac{V_{pipe}}{V_{beam}}\right)}{\eta_g \sigma_i + \eta_{HI} \sigma_s}$$

Instability first observed on the ISR proton storage ring, limiting current in mng, in 1970's.

If $l_{beam} = l_{pipe}$

INSTABILITY CRITERION MAY BE WRITTEN

$$I > \frac{ZeS}{\eta_g \sigma_i + \eta_{HI} \sigma_s}$$

<u>EXAMPLE:</u> If $S = 100\ \ell\, s^{-1} m^{-1} = 0.1\ m^3 s^{-1} m^{-1}$

ISR

$\eta_g = 4$

$\sigma_i = 10^{-22}\ m^2 = 10^{-16}\ cm^2$ ; $\sigma_s = 0$

$Z = 1$ (protons)

$\Rightarrow I \leq 40$ Amperes

(PRESSURE RUNAWAYS WERE OBSERVED ON the ISR AT 14-18A, (BENVENUTI et al, IEEE Trans. on Nuc. Sci. NS-24, 1773, 1977)

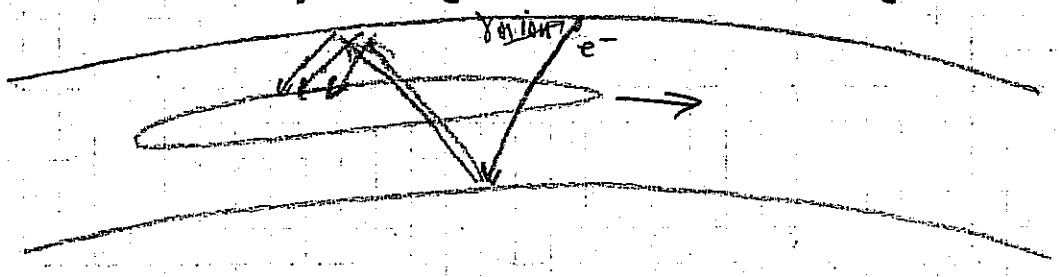
SEE "BEAM INDUCED PRESSURE RISE IN RINGS"
13th ICFA BEAM DYNAMICS MINI WORKSHOP, BNL, Dec. 9-12, 2003.
WEBSITE: http://www.c-ad.bnl.gov/icfa

# "ELECTRON CLOUD EFFECTS"

REFERENCE: CERN e-CLOUD WORKSHOP
http://wwwslap.cern.ch/collective/ecloud02/
→ proceedings.html

## BASIC IDEA

IN ION storage rings or collider rings:




ELECTRONS ARE ATTRACTED TO POSITIVE POTENTIAL OF BEAM & ACCUMULATE

SOME SYMPTOMS:

1. BEAM LOSS & pressure rise
2. HIGH FREQUENCY CENTROID oscillations

SOME ACCELERATORS WHICH SHOW EVIDENCE OF $e^-$ EFFECTS

1. LANL PSR
2. CERN PS & SPS
3. BNL RHIC

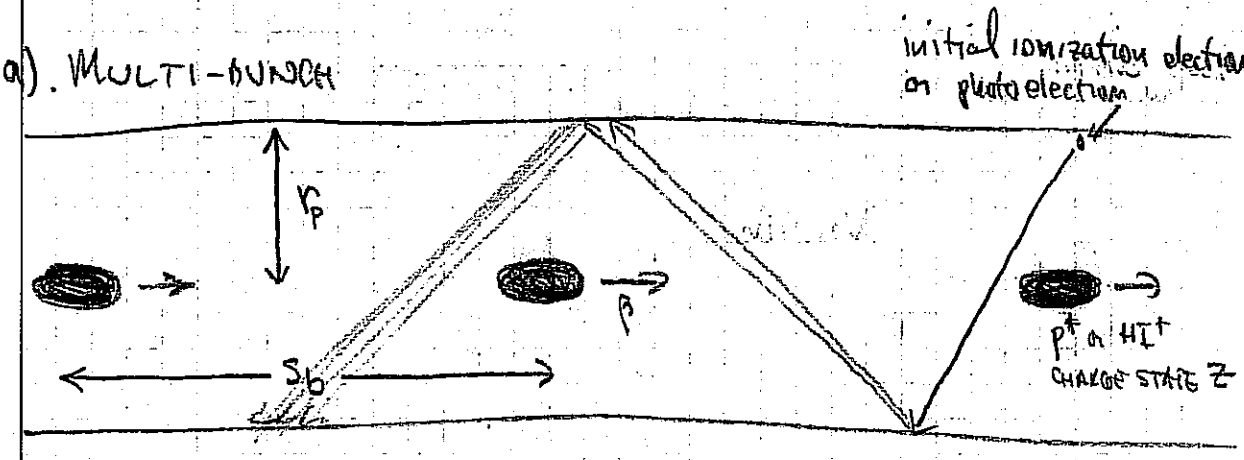
COULD LIMIT PLANNED / UNDER CONSTRUCTION:

1. SNS ACCUMULATOR RING
2. LHC

cf. "Electron-cloud effects in HIGH INTENSITY PROTON ACCELERATORS" J. Wei & R. Macek, CERN

## BEAM INDUCED MULTIPACTING

a). MULTI-BUNCH




USING COULOMB COLLISION FORMULA FROM PAGE 9:

$$\Delta p_x \cong \frac{2 Z N_b e^2}{4\pi\epsilon_0 v r_p}$$

$N_b$ = Number of ions of charge $Z$ in bunch

$$\Delta E_e = m_e c^2\left[\sqrt{\frac{\Delta p_x^2}{m^2c^2}+1} - 1\right] = m_e c^2\left[\sqrt{\left(\frac{2 r_e Z N_b}{\beta r_p}\right)^2 + 1} - 1\right]$$

$$\cong 2 r_e^2 m_e c^2 \frac{Z^2 N_b^2}{\beta^2 r_p^2}$$

for $\Delta E_e \ll m_e c^2$ $\left(\text{or } \frac{2 r_e Z N_b}{\beta r_p} \ll 1\right)$

$$\left(\text{where } r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2} \simeq 2.8 \times 10^{-15}\,\text{m}\right)$$

## DEFINE A MULTIPACTING PARAMETER $\mathcal{J}_m$

$$\mathcal{J}_m = \frac{\text{TIME FOR ELECTRON TO CROSS PIPE}}{\text{TIME BETWEEN BUNCHES}} = \frac{2 r_p}{s_b}\frac{\beta}{\beta_e}$$

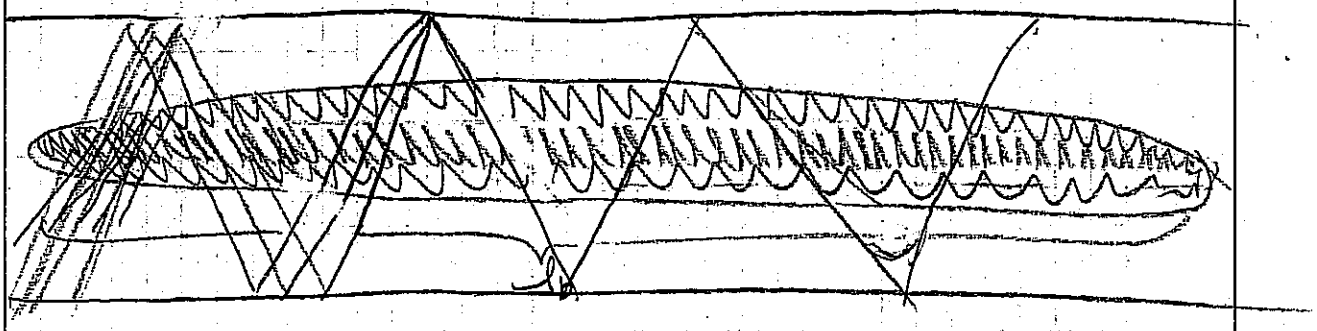
$$\cong \frac{\beta^2 r_p^2}{Z N_b r_e s_b}$$

RESONANCE CONDITION:

$$\mathcal{J}_m = 1$$

$s_b$ = distance between bunches

## b). SINGLE-BUNCH BEAM-INDUCED MULTIPACTING




$$\zeta_s = \frac{r_p\,\beta}{l_b\,\beta_e} = \frac{\text{time for electrons to cross pipe}}{\text{passage time for half of the bunch}}$$

Recall:

$$\varphi = \begin{cases} \frac{\lambda}{2\pi\epsilon_0}\left[\frac{1}{2}\left(1-\frac{r^2}{r_b^2}\right)+\ln\frac{r_p}{r_b}\right] & 0<r<r_b \\ \frac{\lambda}{2\pi\epsilon_0}\left[\ln\frac{r_p}{r}\right] & r_b<r<r_p \end{cases}$$

$$\frac{1}{2}m_e v_e^2 + q\varphi \simeq \text{const} \simeq 0$$

$$\beta_e \sim \frac{1}{2}\sqrt{\frac{2q\varphi}{m_e c^2}} \sim \sqrt{\frac{N_0 Z e^2}{l_b\, 4\pi\epsilon_0 m_e c^2}} \sim \sqrt{\frac{Z r_e N_0}{l_b 4\pi\epsilon_0}}$$

(AVERAGE $e^-$ VELOCITY)

$$\Rightarrow \zeta_s = \frac{\beta r_p}{\sqrt{r_e l_b N_0 Z}}$$

THE ENERGY GAIN OF THE ELECTRON, RELIES ON THE DENSITY CHANGING OVER THE COURSE OF THE BUNCH.

$$\Delta E_e \sim \frac{m_e c^2}{2}\left[\frac{Z r_e N_0(s)}{l_b}\right] - \frac{m_e c^2}{2}\left[\frac{Z r_e N_0(s+\Delta s)}{l_b}\right]$$

$$\sim \frac{m_e c^2}{2}\left(\frac{\partial N_0}{\partial z}\Delta z\right)\left(\frac{Z r_e}{l_b}\right)$$

$$\Delta E_e \sim \frac{m_e c^2}{2}\left(\frac{\partial N_0}{\partial z}\Delta z\right)\left(\frac{Z r_e}{l_b}\right)$$

$$\Delta z = \frac{r_p}{\gamma_e}\beta\epsilon = \beta r_p\sqrt{\frac{l_b}{Z r_e N_0}}\,; \qquad \frac{\partial N_0}{\partial z} \sim \frac{N_0}{l_b}$$

So $\Delta E_e \approx m_e c^2\left(\frac{Z N_0 r_e}{l_b^3}\right)^{1/2}\beta r_p$

$\xi \leq 1 \Rightarrow$ Electron build up possible within bunch

WHAT IS STEADY STATE ELECTRON DENSITY?

Electrons can build up until $E_r$ at pipe $\sim 0$.

$$\Rightarrow \lambda_e = \lambda_J$$

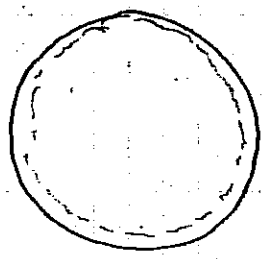
$$\pi r_p^2 n_e = \pi r_b^2 Z n_i$$

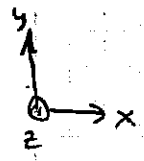
$$n_e = \left(\frac{r_b}{r_p}\right)^2 Z n_i$$

# ELECTRON - ION INSTABILITY

(SEE ALSO R.C. DAVIDSON & H. QIN, Physics of Intense Charged Particle Beams in High Energy Accelerators, p 503 FOR KINETIC TREATMENT).

CONSIDER A UNIFORM DISTRIBUTION OF ELECTRONS (AT REST) WHICH HAS THE SAME RADIUS (OR SLIGHTLY SMALLER RADIUS) AS A UNIFORM DENSITY ION BEAM, THAT IS MOVING AT VELOCITY $V_z$ (OUT OF THE PLANE OF THE PAPER).






$$E_x = \frac{\lambda_i(r)(x-x_i)}{2\pi\epsilon_0 r_0 r} = \frac{\rho_i}{2\epsilon_0}(x-x_i)$$

THE EQUATION OF MOTION FOR THE CENTROID OF THE ELECTRONS IS OBTAINED FROM

$$m_e\ddot{x} = \frac{-e\rho_i}{2\epsilon_0}(x-x_i) + \frac{e\rho_e}{2\epsilon_0}(x-x_e)$$

or

$$\frac{d^2x_e}{dt^2} = -\frac{\omega_{pi}^2}{2}\left(\frac{m_i}{q}\frac{e}{m_e}\right)(x_e - x_i)$$

here $\omega_{pi}^2 = \dfrac{q^2 n_i}{\epsilon_0 m_i} = \dfrac{q\rho_i}{\epsilon_0 m_i}$

(THE CENTER OF oscillation for the electrons is the center of the ion beam).

$x_e$ = centroid of electron beam

$x_i$ = centroid of ion beam

THE EQUATION OF MOTION FOR THE CENTROID OF THE IONS IS GIVEN BY

$$\frac{d^2 x_i}{dt^2} = -\omega_{po}^2 x_i - \left[\frac{m_e N_e}{m_i N_i}\right]\left(\frac{\omega_{pi}^2}{2}\frac{m_i}{q}\frac{e}{m_e}\right)(x_i - x_e)$$

THE TOTAL MOMENTUM KICK TO EACH SPECIES MUST BE EQUAL & OPPOSITE

$$\Rightarrow \frac{d^2 x_i}{dt^2} = -\omega_{po}^2 x_i - f\,\frac{\omega_{pi}^2}{2}(x_i - x_e)$$

HERE $f \equiv \dfrac{e N_e}{q N_i}$ = fractional neutralization

Now $\dfrac{d}{dt}$ = total derivative $= \dfrac{\partial}{\partial t} + v_z \dfrac{\partial}{\partial z}$

$\Rightarrow$ THE ION & ELECTRON EQUATIONS MAY BE WRITTEN

$$\left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z}\right)^2 x_i = -\omega_{po}^2 x_i - f\,\frac{\omega_{pi}^2}{2}(x_i - x_e)$$

$$\frac{\partial^2}{\partial t^2} x_e = -\frac{\omega_{pi}^2}{2}\left(\frac{m_i}{q}\frac{e}{m_e}\right)(x_e - x_i)$$

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Now let $x_e = x_e \exp[i(\omega t - kz)]$; $\quad x_i = x_i \exp[i(\omega t - kz)]$

$$\Rightarrow (-\omega^2 + 2\omega k v_z - k^2 v_z^2)\, x_i = -\omega_{p0}^2 x_i - f\frac{\omega_{pi}^2}{2}(x_i - x_e)$$

$$-\omega^2 x_e = -\frac{\omega_{pi}^2}{2}\left(\frac{m_i}{m_e}\frac{e}{q}\right)(x_e - x_i)$$

$$\Rightarrow \left[(\omega - kv_z)^2 - \omega_{p0}^2 - f\frac{\omega_{pi}^2}{2}\right] x_i = -\frac{f\omega_{pi}^2}{2} x_e$$

$$\left[\omega^2 - \frac{\omega_{pi}^2}{2}\left(\frac{m_i}{m_e}\frac{e}{q}\right)\right] x_e = -\frac{\omega_{pi}^2}{2}\left(\frac{m_i}{m_e}\frac{e}{q}\right) x_i$$

Multiplying the above equations and dividing by $x_e x_i$, yields the dispersion relation:

$$\underbrace{\left[(\omega - kv_z)^2 - \omega_{p0}^2 - f\frac{\omega_{pi}^2}{2}\right]}_{\text{ION BETATRON FREQUENCY (INCREASED BY SPACE CHARGE OF ELECTRONS)}}\underbrace{\left[\omega^2 - \frac{\omega_{pi}^2}{2}\left(\frac{m_i}{m_e}\frac{e}{q}\right)\right]}_{\text{ELECTRON OSCILLATING IN POTENTIAL WELL OF ION}} = \underbrace{+\frac{f\omega_{pi}^4}{4}\left(\frac{m_i}{m_e}\frac{e}{q}\right)}_{\text{COUPLING}}$$

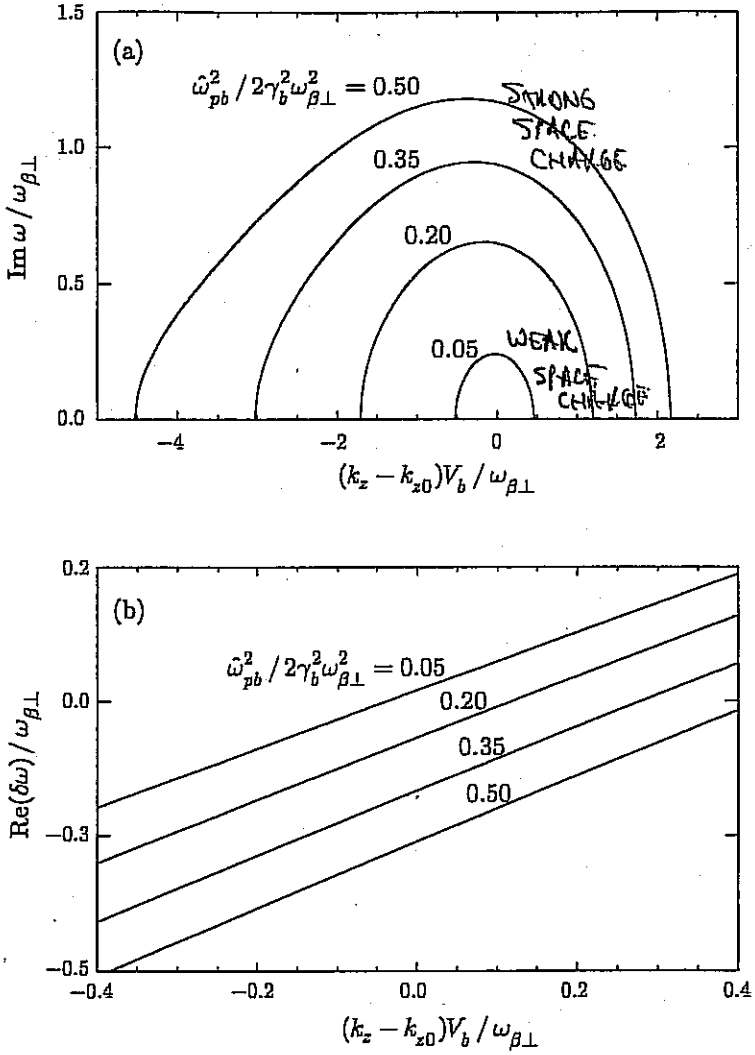
So a beam with high spatial frequency undergoing betatron oscillations in the comoving frame, $kv_z - \omega \cong \sqrt{\omega_{p0}^2 + \frac{f\omega_{pi}^2}{2}}$ will resonate with electrons oscillating in the ion well if

$$\omega \cong \frac{\omega_{pi}}{\sqrt{2}}\sqrt{\frac{m_i e}{m_e q}}$$

Giving rise to instability!

FROM DAVIDSON & QIN (22)
PHYSICS OF INTENSE CHARGED PARTICLE BEAMS, 2001.






$\omega_{\beta\perp}^2 \equiv \omega_{\beta 0}^2$

$\frac{\hat{\omega}_{pb}^2}{2V_p^2\omega_{\beta\perp}^2} = \left(1 - \frac{\sigma^2}{\sigma_0^2}\right)$

**Figure 10.11.** Plots of (a) normalized growth rate ($Im\omega/\omega_{\beta\perp}$), and (b) normalized real frequency $(Re\omega - \omega_e)/\omega_{\beta b}$ versus shifted axial wavenumber $(k_z - k_{z0})V_b/\omega_{\beta\perp}$ obtained from the dispersion relation (10.103) for the unstable branch with positive real frequency. System parameters correspond to $v_{T\|b} = 0 = v_{T\|e}$, $m_b/m_e = 1836$ (protons), $(\gamma_b - 1)m_bc^2 = 800$ MeV, $r_b/r_w = 0.5$, and $f = 0.1$. Curves are shown for several values of normalized beam intensity $\hat{\omega}_{pb}^2/2\gamma_b^2\omega_{\beta\perp}^2$ ranging from 0.05 to 0.5.

$k_{z0} V_z = \omega \mp \sqrt{\omega_{\beta 0}^2 + f\omega_p^2/2}$ ; $\omega = \frac{\omega_{pi}}{2}\sqrt{\frac{m_i e}{m_e q}}$

FROM DAVIDSON & QIN, 2001 (23)



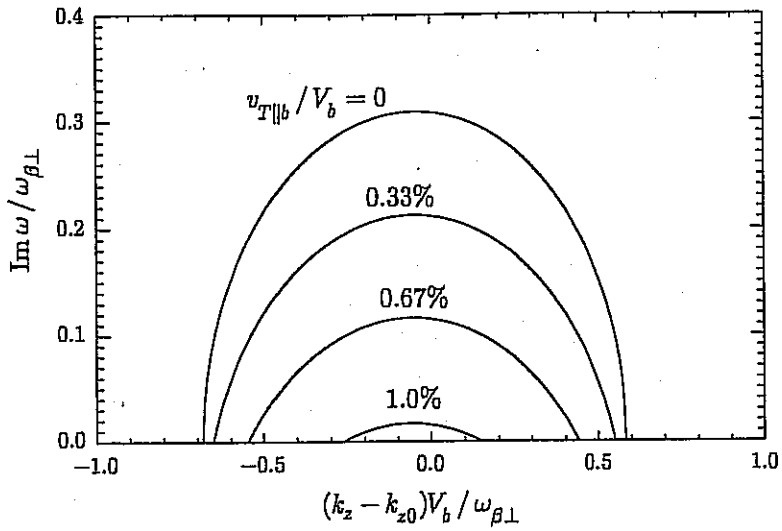



**Figure 10.12.** Plot of normalized growth rate ($Im\omega/\omega_{\beta\perp}$), and normalized real frequency $(Re\omega - \omega_e)/\omega_{\beta\perp}$ versus positive real frequency. System parameters correspond to $\hat{\omega}_{pb}^2/2\gamma_b^2\omega_{\beta\perp}^2 = 0.07$, $v_{T\|e} = v_{T\|b}$, $m_b/m_e = 1836$ (protons), $(\gamma_b - 1)m_bc^2 = 800$ MeV, $r_b/r_w = 0.5$, and $f = 0.1$. Curves are shown for several values of normalized ion thermal spread $v_{T\|b}/V_b$ ranging from 0 to 0.01.

velocity $V_b$ [Eq. (10.105)], it is expected that Landau damping by parallel ion kinetic effects can have a strong stabilizing influence at modest values of $v_{T\|b}/V_b$. That this is indeed the case is evident from Fig. 10.12, which shows a substantial reduction in maximum growth rate and eradication of the instability over the instability bandwidth as $v_{T\|b}/V_b$ is increased from 0 to 0.01.

We now make use of the nonlinear particle perturbative simulation method [60, 61] described in Sec. 8.5 to illustrate several important properties of the two-stream instability in intense beam systems (Sec. 10.4.2).

PREVENTIVE MEASURES (from J. Weid & L. Maccek, CERN electron cloud workshop 2003).

- SUPPRESS ELECTRON GENERATION
  - SURFACE TREATMENT OF THE VACUUM PIPE
  - KICKER MAGNETS IN GAPS
  - VACUUM PORTS SCREENED TO REDUCE E-field
  - CLEARING ELECTRODES
  - HIGH VACUUM
  - SOLENOIDS - TO REDUCE MULTIPACTING

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## SUMMARY OF ELECTRON, GAS, PRESSURE, & SCATTERING EFFECTS

1. COULOMB COLLISIONS WITHIN BEAM CAN TRANSFER ENERGY FROM ⊥ TO ∥ AND PROVIDE LOWER LIMIT ON $T_{\parallel}$, HIGHER THAN FROM ACCELERATIVE COOLING.

2. COULOMB INTERACTIONS WITH RESIDUAL GAS NUCLEI PROVIDE A SOURCE OF EMITTANCE GROWTH (BUT NOT IMPORTANT FOR HIGHER MASS AND LINAC RESIDENCE TIMES).

3. PRESSURE INSTABILITY FROM DESORPTION OF RESIDUAL GAS BY STRIKED BEAM IONS HITTING WALL OR BEAM-IONIZED RESIDUAL GAS ATOMS, FORCED TO WALL BY E-field of BEAM. LIMITS CURRENT IN RINGS OR HIGH REP RATE LINAC.

4. ELECTRONS CAN CASCADE AND REACH A "QUASI" EQUILIBRIUM POPULATION OF SIMILAR LINE CHARGE TO THE ION BEAM. ELECTRON-ION TWO STREAM INSTABILITY IS UNSTABLE, AND CAN LEAD TO TRANSVERSE INSTABILITY, SIMILAR TO WHAT IS OBSERVED IN SOME PROTON RINGS.

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