Tutorial on Induction Accelerators with Application to the IRE

Tutorial Seminar Series on Fusion Energy

Lawrence Livermore National Laboratory

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Outline

I. Accelerator architecture

II. Components of an Induction Accelerator

III. Transverse and longitudinal beam physics

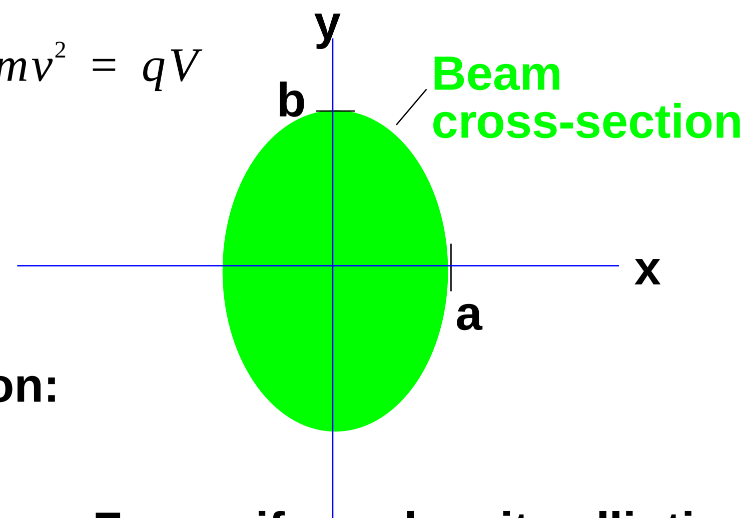
IV. Limits

V. Drift compression and final focus limits in the IRE

VI. Beam Focusability

Transverse physics: The envelope equation

$$m\frac{d^2x}{dt^2} = -(mv^2)kx + qE_x \qquad \text{here } \frac{1}{2}mv^2 = qV$$



1. Change independent variable:

$$dz = v_z\, dt$$

2. Average over distribution function:

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3. Define: $a=2(\langle x^2\rangle)^{1/2}$, $b=2(\langle y^2\rangle)^{1/2}$

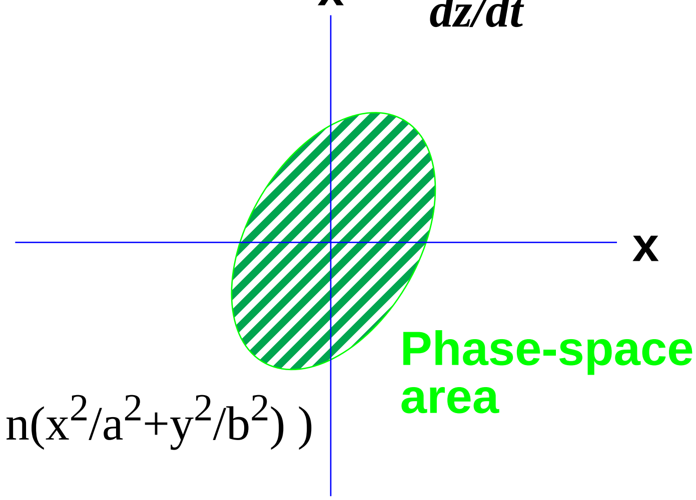
For uniform density elliptical beam:

$$E_x = \frac{\lambda}{\pi\varepsilon_0}\frac{x}{a(a+b)} \qquad E_y = \frac{\lambda}{\pi\varepsilon_0}\frac{y}{b(a+b)}$$

$$\frac{d^2a}{dz^2} = -ka + \frac{2K}{a+b} + \frac{\varepsilon_x^2}{a^3}$$

$$x' = \frac{dx/dt}{dz/dt}$$

where $K \equiv \dfrac{\lambda}{4\pi\varepsilon_0 V}$ **= Perveance**

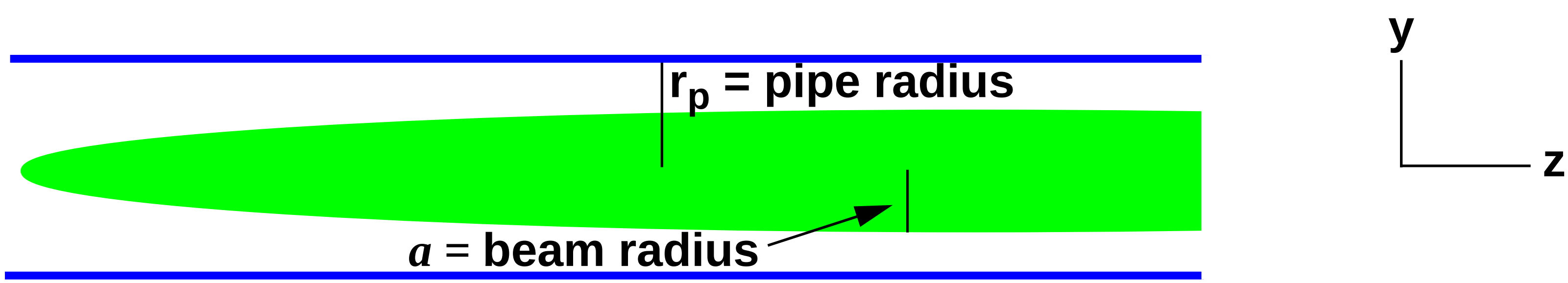


$\varepsilon_x \equiv 4\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$ **= (x-x' phase space area)/π**

= rms emittance

(Envelope equation is valid generally if $n(x,y) = n(x^2/a^2 + y^2/b^2)$ **)**

Longitudinal physics: $E_z = -g\frac{\partial\lambda}{\partial z}$



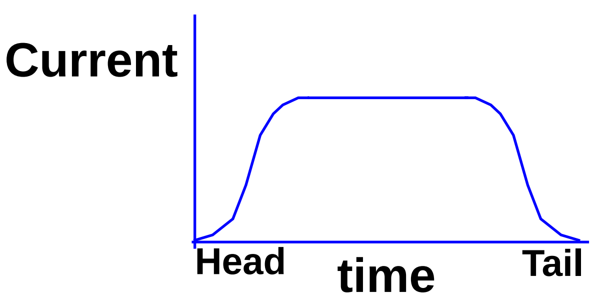
$$E_r = \frac{\lambda(r)}{2\pi\varepsilon_0 r}$$ **(from Gauss law)**

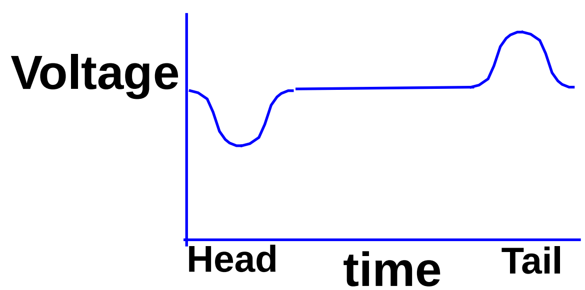
for $\frac{\partial}{\partial r} >> \frac{\partial}{\partial z}$ $\quad \phi = \int E_r dr = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{2} + \ln\frac{r_p}{a}\right)$

=> $$E_z = -\frac{\partial\phi}{\partial z} \cong -\frac{1}{2\pi\varepsilon_0}\left[\ln\frac{r_p}{a}\right]\frac{\partial\lambda}{\partial z} \equiv -g\frac{\partial\lambda}{\partial z}$$

($\frac{\lambda}{a^2}$ assumed constant)

=> To confine beam longitudinally, “ears” are applied to beam at each gap.

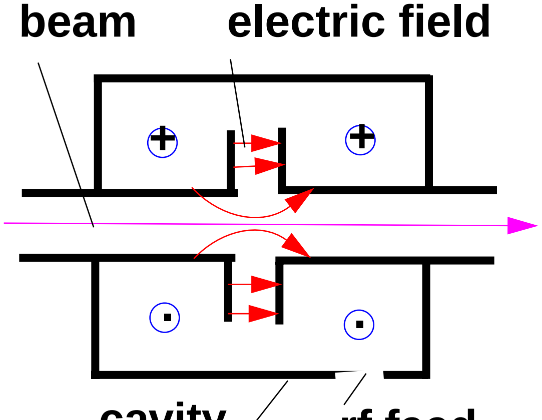


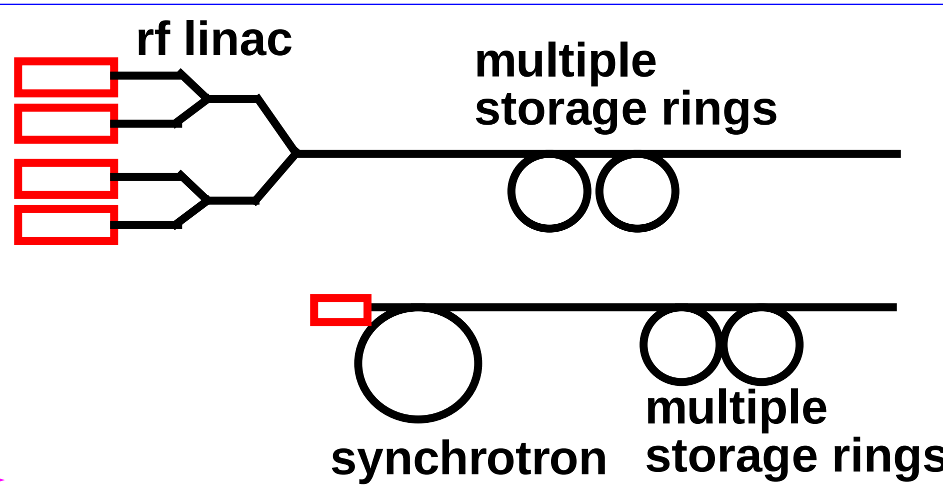


There are two principle methods of acceleration

1. r.f. acceleration

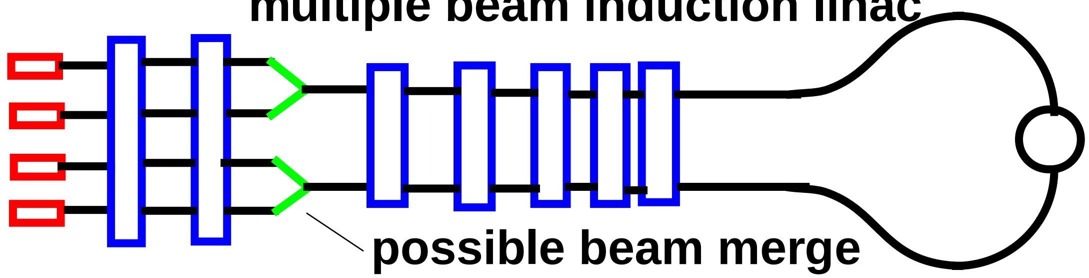
(Approach in Europe and Japan)



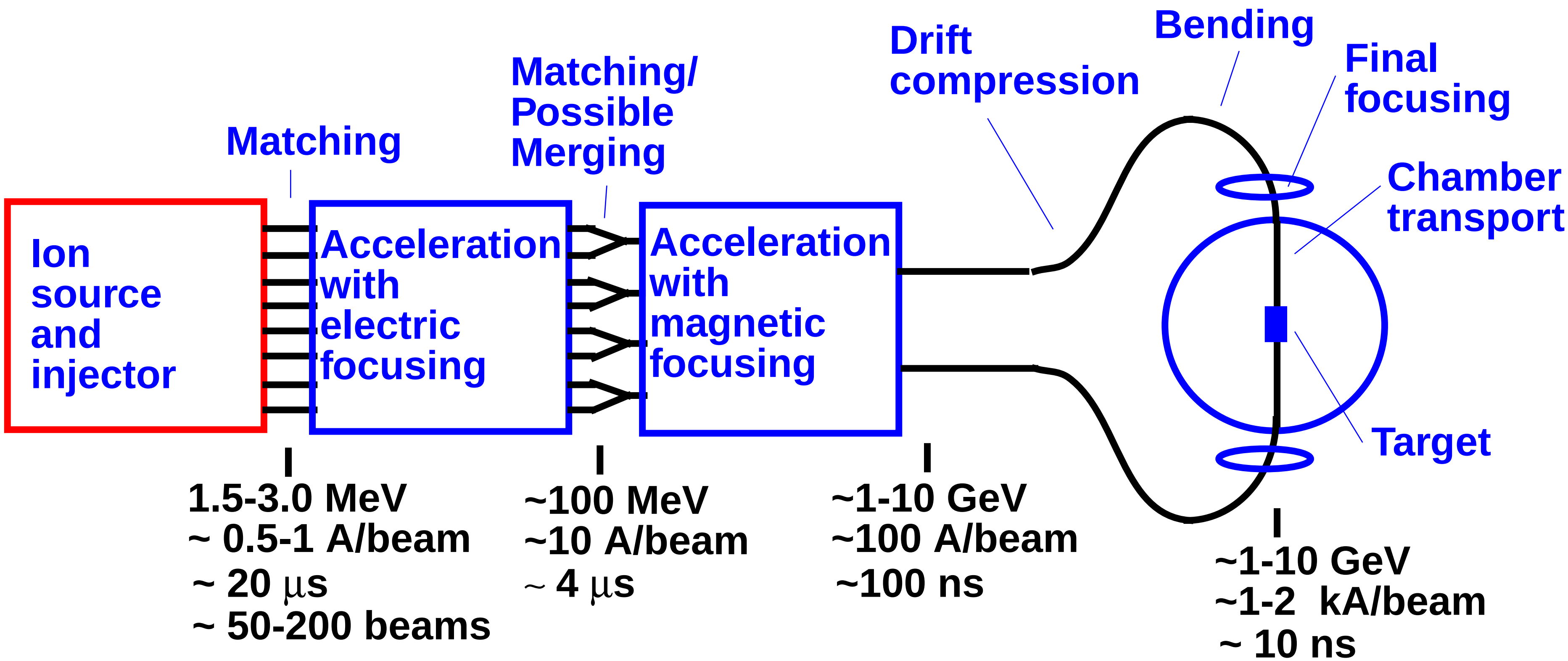


2. Induction acceleration

(U.S. approach)



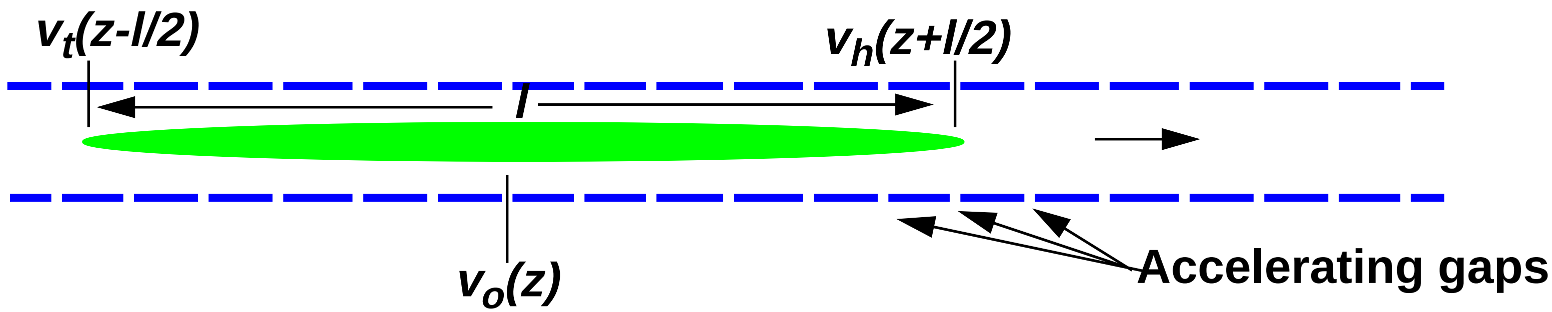
Induction acceleration for HIF consists of several subsystems and a variety of beam manipulations



For "example IRE:"

1.6 MeV	9.4 MeV	200 MeV	200 MeV
0.8 A/beam	1.9 A/beam	15.3 A/beam	1.0 - 0.26 kA/beam
6.7 µs	2.8 µs	335 ns	5-20 ns
32 beams			

A velocity tilt is required to compress beam (or even keep beam together while accelerating)



Also, $qV = \frac{1}{2}mv^2$ **=>** $\frac{1}{V}\frac{dV}{dz} = \frac{2}{v_o}\frac{dv_o}{dz}$

$$\frac{dl}{dt} = \frac{dl}{dV}\frac{dV}{dz}v_0 = v_h\left(z+\frac{l}{2}\right) - v_t\left(z-\frac{l}{2}\right)$$

Velocity tilt $\equiv \frac{\Delta v}{v_o} \equiv -\frac{v_h(z)-v_t(z)}{v_o} = \frac{dv_o}{dz}\frac{l}{v_o} - \frac{v_h\left(z+\frac{l}{2}\right) - v_t\left(z-\frac{l}{2}\right)}{v_o}$

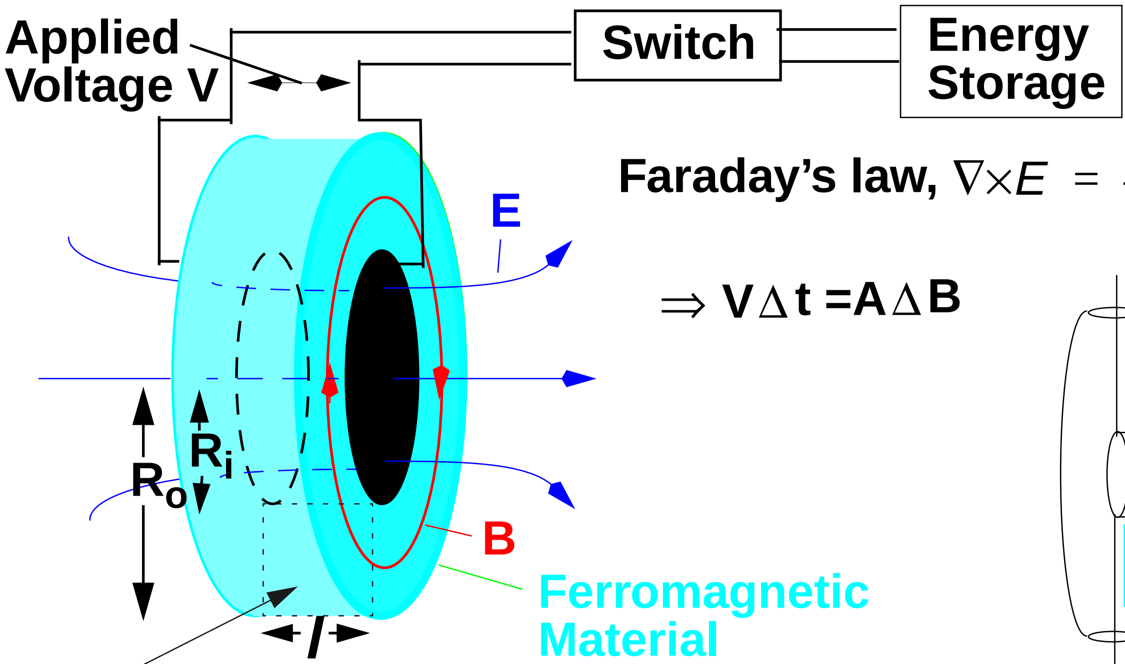
$$= \frac{l}{V}\frac{dV}{dz}\left(\frac{1}{2} - \frac{V}{l}\frac{dl}{dV}\right)$$

Even if dl/dV=0, dV/dz >0 requires tilt >0.

But,

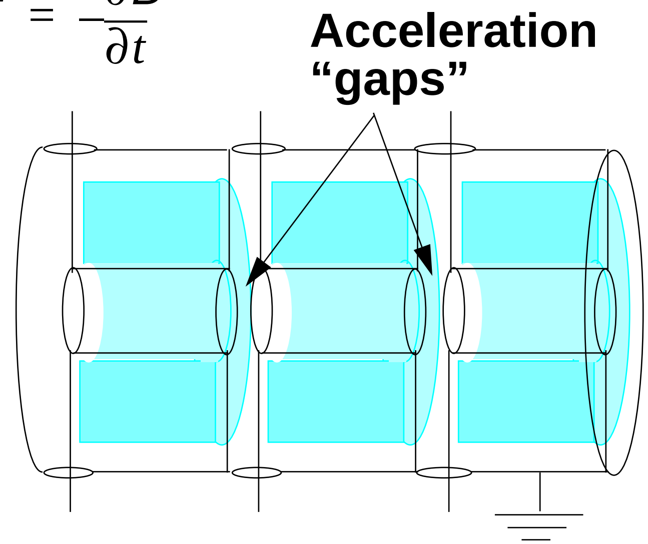
$\frac{\Delta v}{v_o}$ **< ~0.3 in electrostatic section so head σ_0 < 85^o and tail remains within pipe (σ_0 ~ 1/v^2).**

Induction acceleration: Volt-second limits



Faraday's law, $\nabla \times E = -\dfrac{\partial B}{\partial t}$

$\Rightarrow$ **$V \Delta t = A \Delta B$**



Cross-sectional area A
$A = (R_o - R_i)\ l$

Volt-seconds per m: $(dV/dz)\ \Delta t = (R_o - R_i)\ \ \Delta B\ \ f_{radial}\ \ f_{longit.}$

$\sim$1 m $\sim$2.5T $\sim$0.8 $\sim$0.8

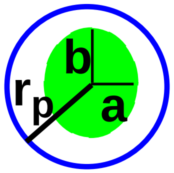
$$\boxed{(dV/dz)\ \Delta t <\sim 1.6\ \text{V-s/m}}$$

Envelope instabilities set upper limit on "single particle" phase advance σ_0

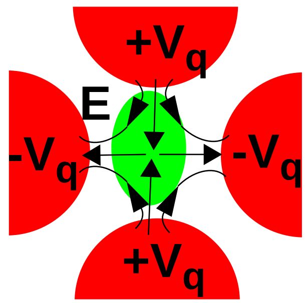
Experimental data (Tiefenback, 1986) from LBL's Single Beam Transport Experiment (SBTE)

$$\boxed{\sigma_0 < 85^o}$$

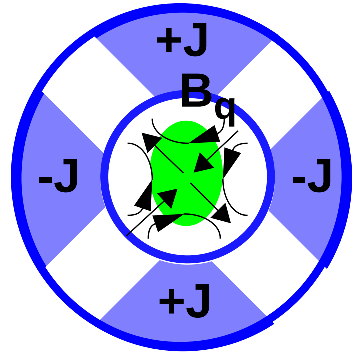
Focusing systems scale differently w/ ion energy qV, charge/mass ratio q/m, and lattice period 2L



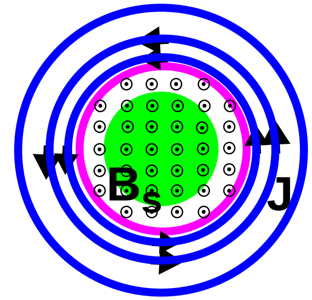
Electric quads



Magnetic quads



Solenoids



Envelope equation:

$$a'' = \frac{\varepsilon^2}{a^3} + \boxed{\frac{2\,K}{a+b} \pm \frac{V_q}{V}\,\frac{a}{r_p^2}} \quad \boxed{\frac{2\,K}{a+b} \pm \left(\frac{qB_q^2}{2mV}\right)^{1/2} \frac{a}{r_p}} \quad \boxed{\frac{K}{a} + \frac{\omega^2 a}{v_z^2} - \frac{\omega\omega_c a}{v_z^2}}$$

smooth limit, averaging over L

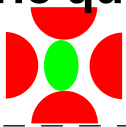
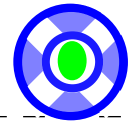
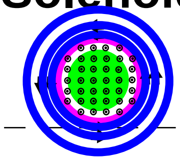
setting $\omega=\omega_c/2$

$$a'' = \frac{\varepsilon^2}{a^3} + \frac{K}{a} - k^2\,a$$

$$k^2 = \left(\frac{\sigma_0}{2L}\right)^2 \cong \boxed{\frac{1}{4r_p^2}\left(\frac{\eta L}{r_p}\right)^2 \frac{V_q^2}{V^2}} \quad \boxed{\frac{1}{8}\left(\frac{\eta L}{r_p}\right)^2 \left(\frac{qB_q^2}{mV}\right)} \quad \boxed{\frac{\eta}{8}\left(\frac{qB_s^2}{mV}\right)}$$

(where K= perveance=$\lambda/4\pi\varepsilon_0 V$, ω=rotation freq.,ω_c = cyclotron freq., σ_0 = undepressed phase advance., ε = emittance, and $mv_z^2/2$ =qV)

Scaling of line charge density λ_b with ion energy qV, charge/mass ratio q/m, and lattice period 2L

Electric quads	Magnetic quads	Solenoids
		

$$k^2 = \left(\frac{\sigma_0}{2L}\right)^2 \cong \quad \frac{1}{4r_p^2}\left(\frac{\eta L}{r_p}\right)^2 \frac{V_q^2}{V^2} \qquad \frac{1}{8}\left(\frac{\eta L}{r_p}\right)^2 \left(\frac{qB_q^2}{mV}\right) \qquad \frac{\eta}{8}\left(\frac{qB_s^2}{mV}\right)$$

Envelope/lattice instability limit: $\sigma_0 \lesssim \pi/2$

Maximum line charge density per beam λ_b (found from K/a = k^2 a)

$$0.9\ \frac{\mu C}{m}\left(\frac{V_q}{80\ kV}\right)\left(\frac{\sigma_0}{1.4}\right)\left(\frac{a/r_p}{0.7}\right)^2\left(\frac{\eta}{0.7}\right)$$

$$\lambda_b = \quad 1\ \frac{\mu C}{m}\left(\frac{\eta}{0.7}\right)\left(\frac{\sigma_0}{1.4}\right)\left(\frac{a/r_p}{0.7}\right)^2\left(\frac{B_q}{2T}\right)\left(\frac{q/m}{1/200}\right)^{1/2}\left(\frac{V}{2MeV}\right)^{1/2}\left(\frac{r_p}{6cm}\right)$$

$$.03\ \frac{\mu C}{m}\left(\frac{\eta}{0.7}\right)\left(\frac{a/r_p}{0.7}\right)^2\left(\frac{B_s}{2T}\right)^2\left(\frac{q/m}{1/200}\right)\left(\frac{r_p}{6cm}\right)^2$$

for (magnetic quads):

$$I_b = \lambda_b v = 1.4\ A\left(\frac{\eta}{0.7}\right)\left(\frac{\sigma_0}{1.4}\right)\left(\frac{a/r_p}{0.7}\right)^2\left(\frac{B_q}{2T}\right)\left(\frac{q/m}{1/200}\right)\left(\frac{V}{2MeV}\right)\left(\frac{r_p}{6cm}\right)$$

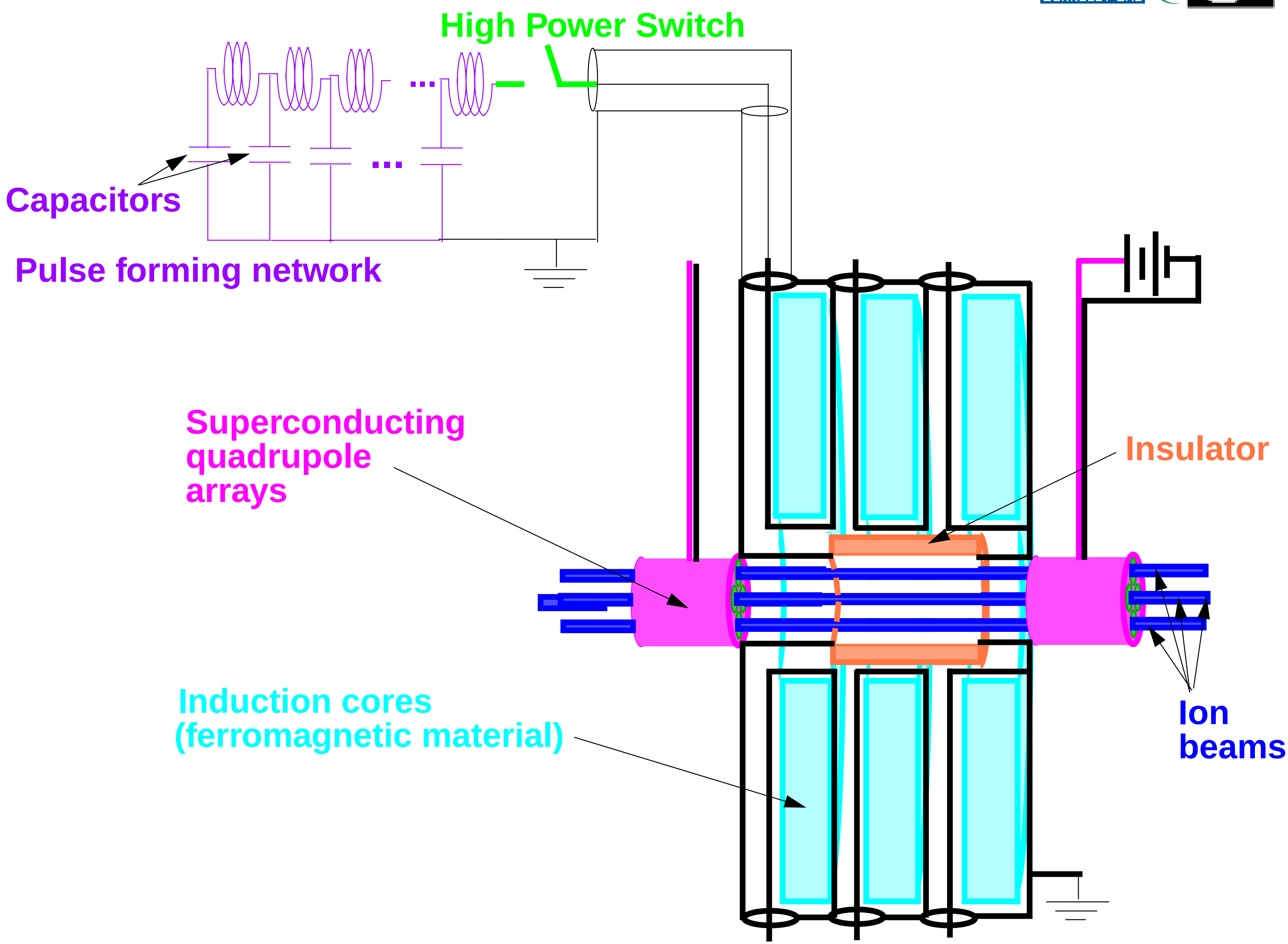
Conclusions

1. Space charge and phase space considerations dictate much of the architecture of the accelerator.

2. Focusability is the key scientific issue and is amenable to simulations and experiment.

3. The example IRE would serve to test the limits encountered in both architecture and focusability.

Major components of an induction linac



In an induction linac, certain limits constrain design

Phase advance per lattice period $\sigma_0 < \sim 85^o$ (to avoid envelope/lattice instabilities)

Space charge is limited by external focusing $K < (\sigma_0 a/2L)^2$ where K is the perveance (proportional to line charge density over beam Voltage), a is the average beam radius and L is the half-lattice period.

Velocity tilt $\Delta v/v <\sim$ 0.3 for electrostatic quads (larger for magnetic quads) to avoid mismatches at head and tail of beam, and to ensure tail radius within pipe and head σ_0 within limit)

Volt-seconds per meter $(dV/ds)\ l/v_0 <\sim$ 1.5-2.0 V-s/m (for "reasonable" core sizes)

Voltage gradient $dV/ds <\sim$ 1-2 MV/m (to avoid breakdown in gaps)

In “analytic” design, accelerator has three sections

Three sections:

1. Electrostatic quads; constant bunch length (load and fire)

2. Magnetic quads with bunch compression

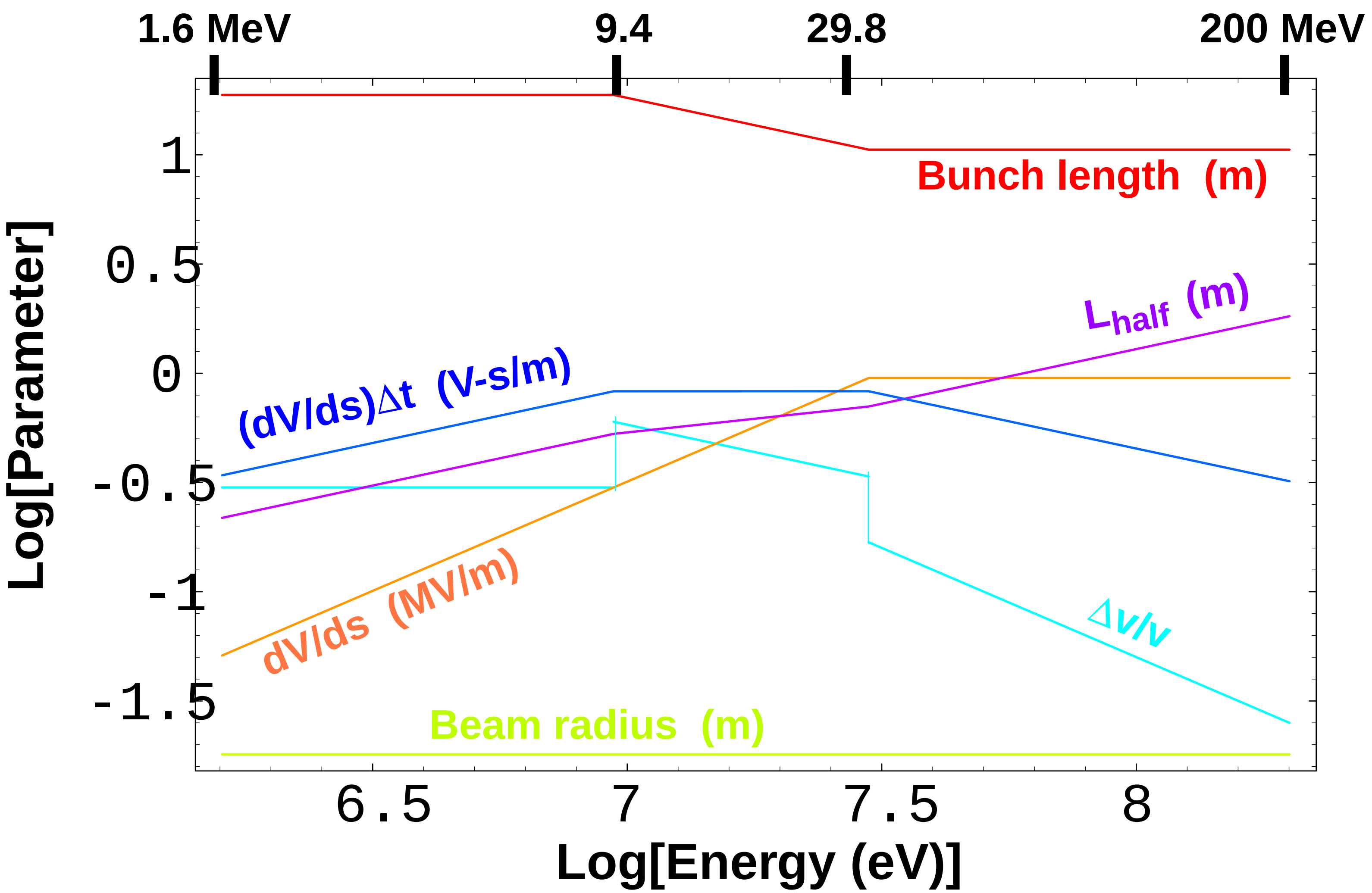
3. Magnetic quads; const. bunch length and maximum acceleration gradient

1. Constant bunch length => $dl/dV=0$, maximum velocity tilt ($\Delta v/v=0.3$) => $dV/ds \sim V$; Maximum space charge => $L \sim V^{1/2}$; Constant σ_0 and constant E' => $\eta \sim$ const.

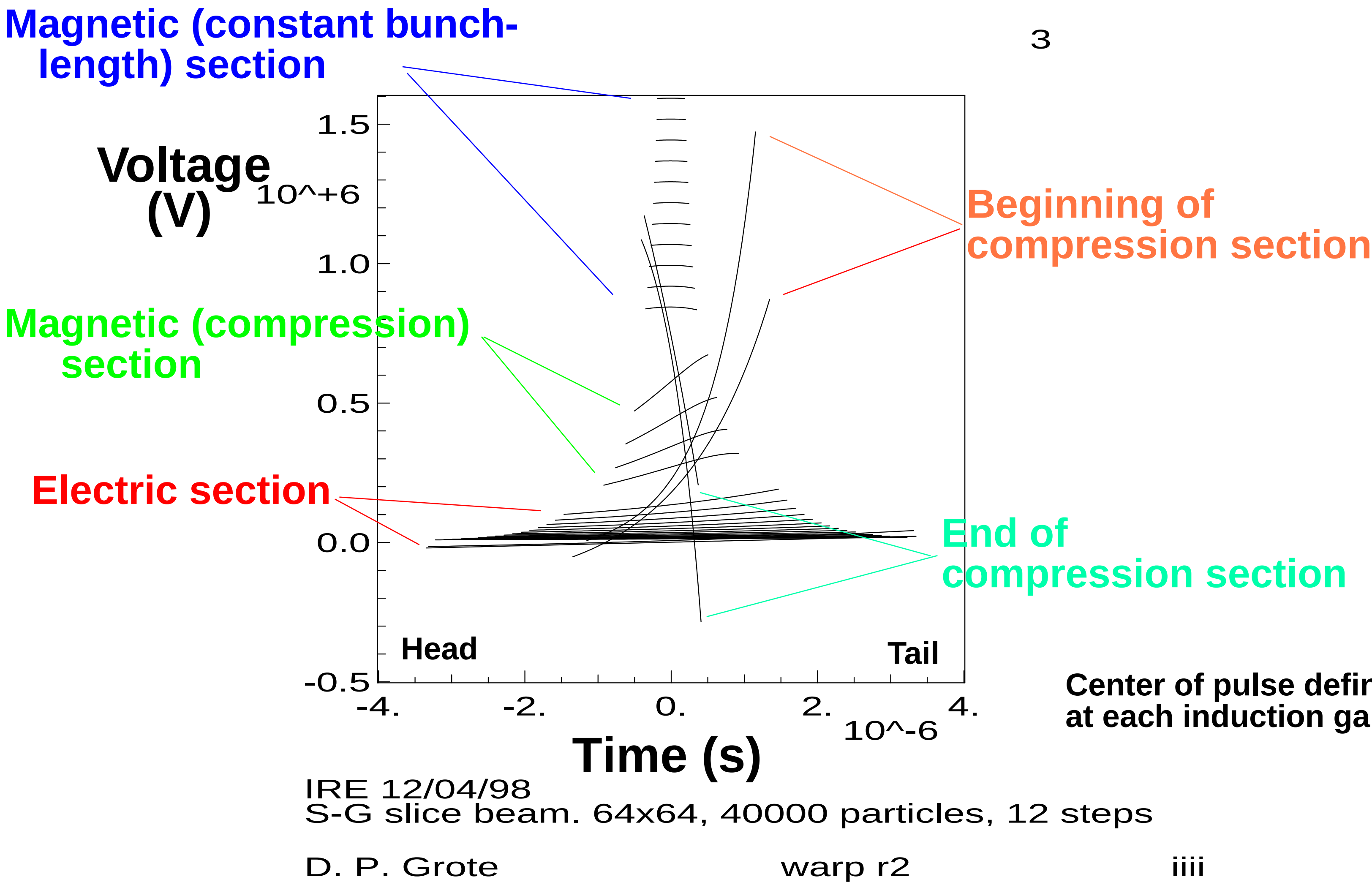
2. Assume velocity tilt such that acceleration and compression give equal contributions => $l \sim V^{-1/2}$; constant volt-seconds per meter => $dV/ds \sim V$; Maximum space charge => $L \sim V^{1/4}$; Constant σ_0 at maximum B' => $\eta \sim$ const.

3. When maximum gradient reached, freeze at max => $dV/ds \sim$ const.; Constant bunch length => $L \sim V^{1/2}$; constant magnet length => $\eta \sim V^{-1/2}$; constant σ_0 => $B' \sim 1/(1-2\eta/3)^{1/2}$; velocity tilt $\Delta v/v \sim 1/V$

Variation with ion energy of some key parameters through the example IRE



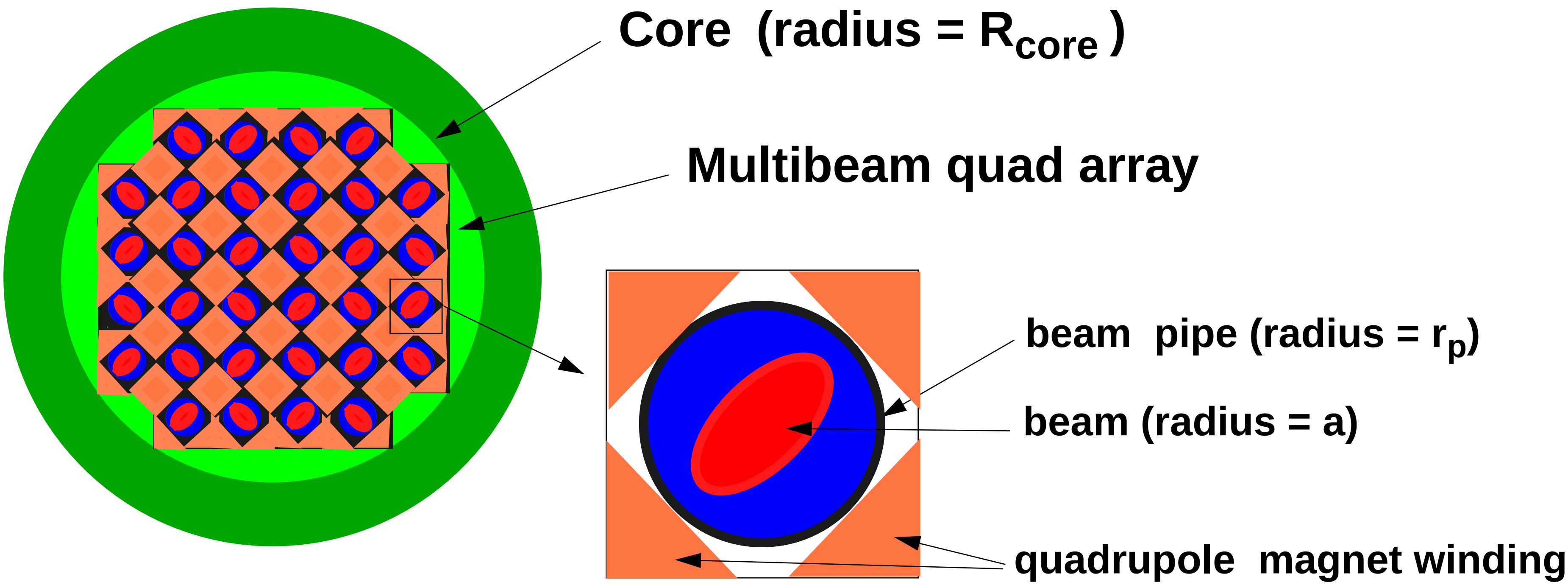
Waveform generation algorithms using CIRCE/ WARP are being developed for 3D simulations



Center of pulse defines t=0 at each induction gap

In addition ear electric fields must be generated to confine the beam longitudinally.

An array of small beamlets increases the total beam current through the core



Current per beam = $I_b \sim a^2 B \beta^2 / r_p$

$r_p \sim a$ (until misalignments require minimum size--better: $r_p = c_1 a + c_2$)

so $I_b \sim a$; N_b = number of beams in array $\sim R^2_{core} / a^2$

Total current through core = $I_{tot} = N_b I_b \sim R^2_{core} / a$ (until misalignments dominate scaling)

Compression, velocity tilt, drift length, and target parameters for 3 IRE final pulse durations

$$\frac{dv}{dt} = -g\left(\frac{q}{m}\right)\frac{\partial\lambda}{\partial z}$$

$$\lambda = \lambda_{max}\left(1 - \frac{4\Delta z^2}{l_b^2}\right)$$

$$g \cong 1.3$$

At end of accelerator: $\lambda_{max} = 0.66 \times 10^{-6}$ **C/m**

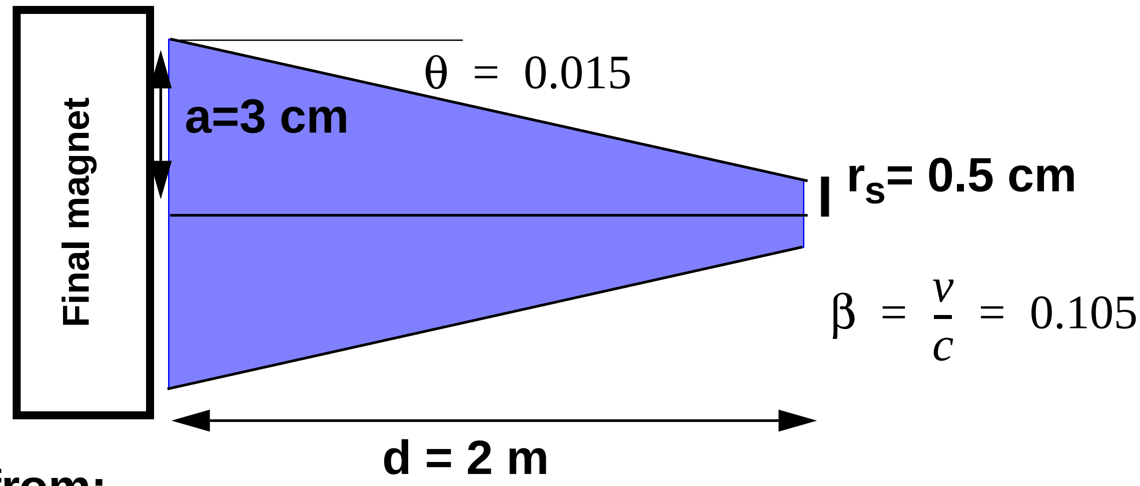
$K_a = 3 \times 10^{-5}$

$l_a = 10.5$ **m**

At target: $r_s = 5$ **mm**

Pulse duration τ_t	Compres-sion ratio C	Velocity tilt $\Delta v/v$	Drift length d (m)	Energy flux F (W/cm^2)	Temp. $k_B T$ (eV)
	l_a/l_t	$[8K_a\, g\, (C-1)]^{1/2}$	$\frac{l_a(1-1/C)}{\Delta v/v}$	$\frac{E}{\pi r_s^2 \tau_t}$	$k_B(F/\sigma)^{1/4}$
5 ns	67	0.145	71.7	7.6×10^{12}	93
10 ns	33.5	0.101	101	3.8×10^{12}	78
20 ns	16.8	0.071	140	1.9×10^{12}	66

Final focus beam quality and neutralization requirement



Spot size from:

Emittance

$\varepsilon_N = 5$ **mm-mrad** $\qquad \delta r_\varepsilon \cong \frac{\varepsilon_n}{\beta\theta} = 3.2$ **mm** $\qquad$ **(**$\varepsilon_N = 1$ **mm-mrad at injector)**

Chromaticity

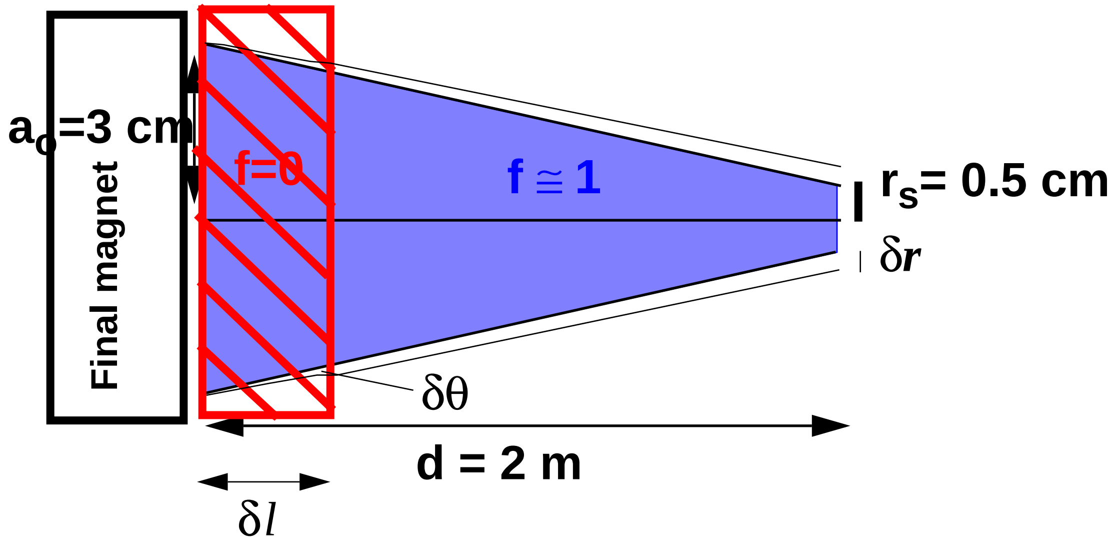
$\frac{\delta p}{p} = 0.01 \qquad \delta r_{\frac{\delta p}{p}} \cong 6 d\theta \frac{\delta p}{p} = 1.8$ **mm**

From perveance, what neutralization is required?

$$\frac{d^2 a}{dz^2} = (1-f) K_a \frac{C}{a} \quad \textbf{=>} \quad 1-f = \frac{\theta^2}{2K_a C \ln\left(\frac{a}{r_s}\right)} \qquad K_a = 3\times 10^{-5}$$

$$(1-f) = \begin{bmatrix} 0.02 \\ 0.04 \\ 0.08 \end{bmatrix} \textbf{ for } C = \begin{bmatrix} 67 \\ 33.5 \\ 16.8 \end{bmatrix};\ \left(\begin{bmatrix} 98 \\ 96 \\ 92 \end{bmatrix} \text{\% neutralization} \right)$$

Effect on spot size of uncertainty in neutralization point



$$\frac{d^2 a_o}{dz^2} = (1-f)K_a\frac{C}{a_o} + \frac{\varepsilon^2}{a_o^3} \quad \Rightarrow \quad \frac{d}{dz}\delta a_o' \cong fK_a\frac{C}{a_o}$$

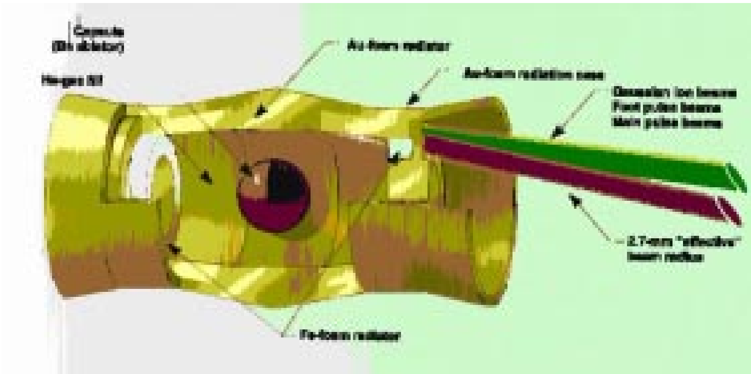
$$\delta\theta = \delta a' = fK_a C\frac{\delta l}{a_o} \quad \Rightarrow \quad \delta r \cong fK_a C\frac{\delta l}{a_o}d$$

If $\delta l \cong a_o$ **=>** $\delta\theta \cong fK_a C$

$$\delta r = dfK_a C = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix} \textbf{ mm} \quad \textbf{for} \quad C = \begin{bmatrix} 67 \\ 33.5 \\ 16.8 \end{bmatrix}$$

Focusability at the target is key scientific issue

Conditions of beam at target are set by hohlraum and implosion physics



Energy in pulse: ~ 3 to 6 MJ

Duration of main pulse: ~ 8 to 10 ns

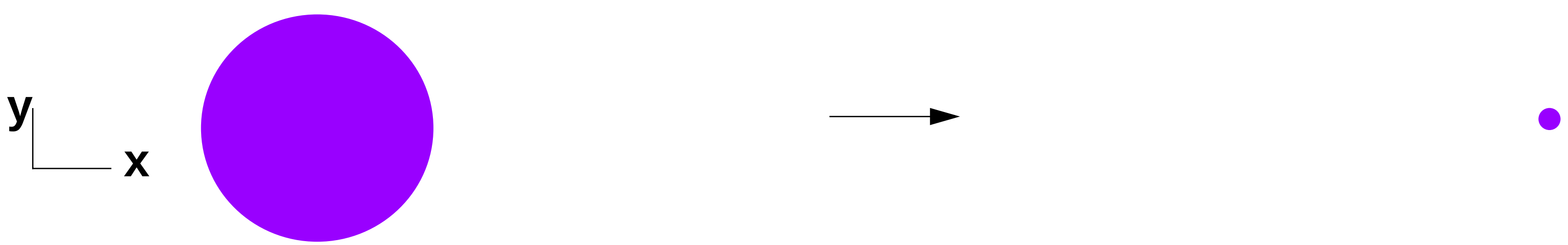
Duration of foot pulse: ~ 30 ns

Spot radius: ~ 1.5 to 3 mm

Transverse and longitudinal compression are required to meet target specifications.

Length of beam just outside of injector ~25-50 m

At target ~ 0.5 - 1 m



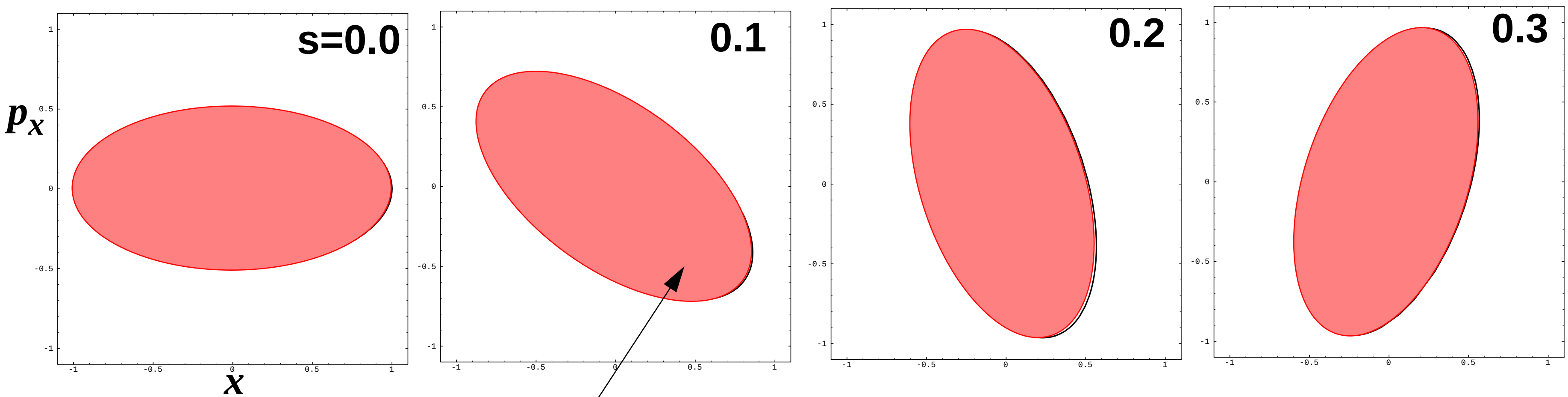
Radius of beam at source ~ 1-3 cm

At target ~ 1.5-3 mm

Compression factors of 10 to 50 in both longitudinal and transverse directions are required.

Emittance constant for linear force profile & matched beams

Linear force profile ($x'' = - k^2 x$) => Phase space area preserved, ellipse stays elliptical.



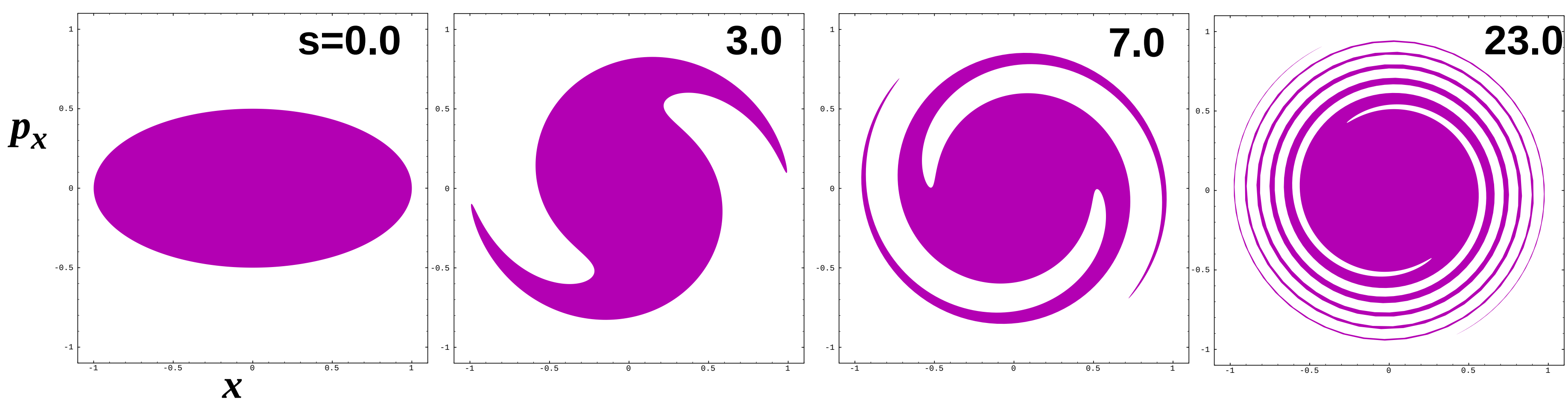
Emittance = phase space area

Emittance constant if forces linear

Here, width of beam is oscillating or “mismatched.”

Non-linear forces (e.g. $x'' = - k^2 x + \varepsilon x^3$) $\Rightarrow$ position-dependent frequency

$\Rightarrow$ phase mixing, increasing effective area $\Rightarrow$ Emittance increases if forces non-linear



Sources of non-linearity and mismatch are well defined

Sources of non-linearities

- External focusing magnets
- Space-charge
- Multiple-beam effects

Sources of mismatch

- Accelerator imperfections
  - Quad strength and placement errors
  - Acceleration waveform errors
  - Bend strength errors
- Velocity tilt

Simulations give reliable and definitive tolerances on each source

Beam loss also constrains design

Avoidance of accelerator activation and heat load on superconducting magnets, limits the amount of beam loss in the accelerator

Mechanisms for beam loss:

- Halo production
- Charge-changing collisions with residual gas
- Charge-changing collisions with other beam ions

Several potential instabilities have been investigated in HIF drivers

Temperature anisotropy instability

After acceleration $T_{||} << T_{\perp}$, internal beam modes are unstable; saturation occurs when $T_{||} \sim T_{\perp}/3$

Longitudinal resistive instability

Module impedance interacts with beam, amplifying space-charge waves that are backward propagating in beam frame

Beam break-up (BBU) instability

High frequency waves in induction module cavities interact transversely with beam

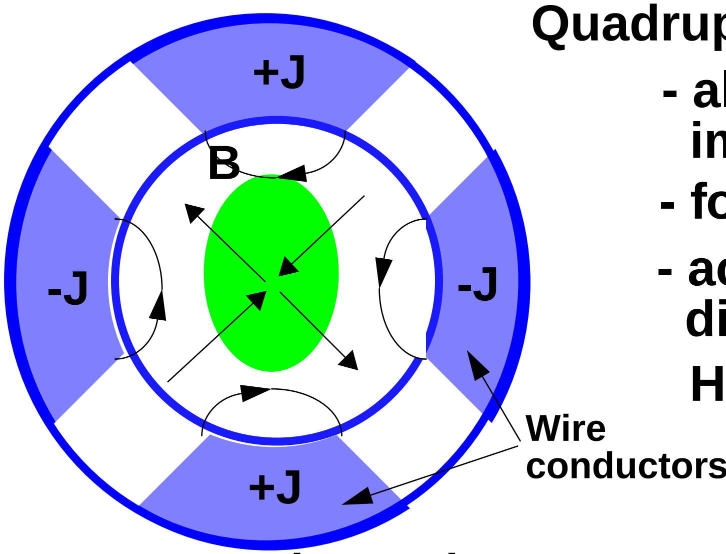
Beam-plasma instability

Beam interacts with residual gas in target chamber

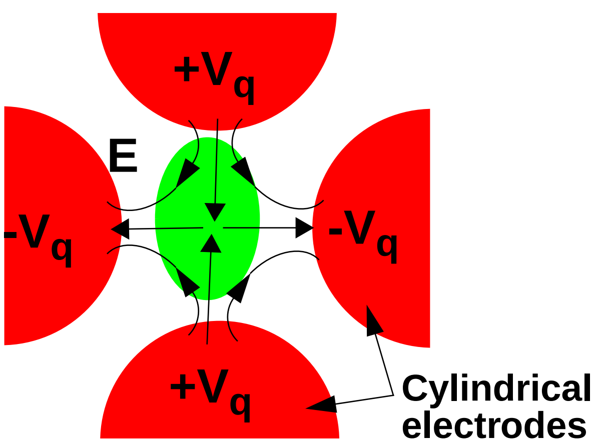
All of these instabilities have known analytic linear growth rates, which constrain the accelerator design (to ensure minimal growth or benign saturation).

Heavy ion accelerators use alternating gradient quadrupoles to focus (confine) the beams (non-neutral plasmas)

Space-charge forces and thermal forces act to expand beam



Magnetic quad

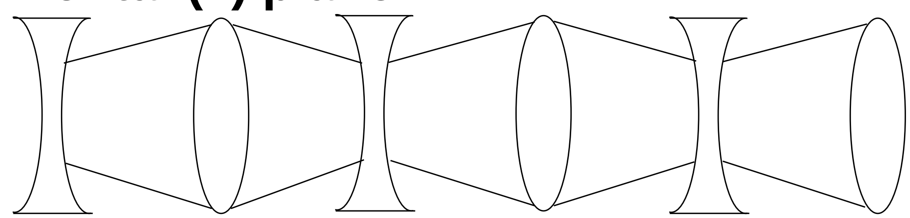


Electric quad

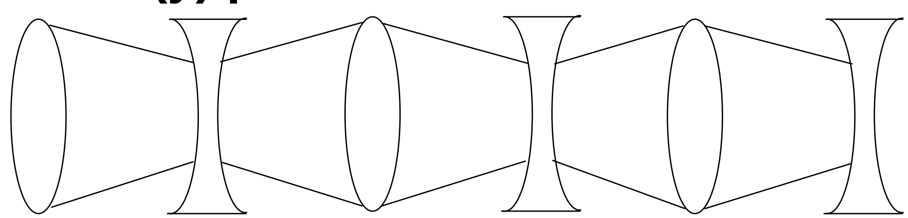
Quadrupoles (magnetic or electric):

- alternately provide inward then outward impulse
- focus in one plane and defocus in other
- act as linear lenses. (Force proportional to distance from axis).

Horizontal (x) plane:

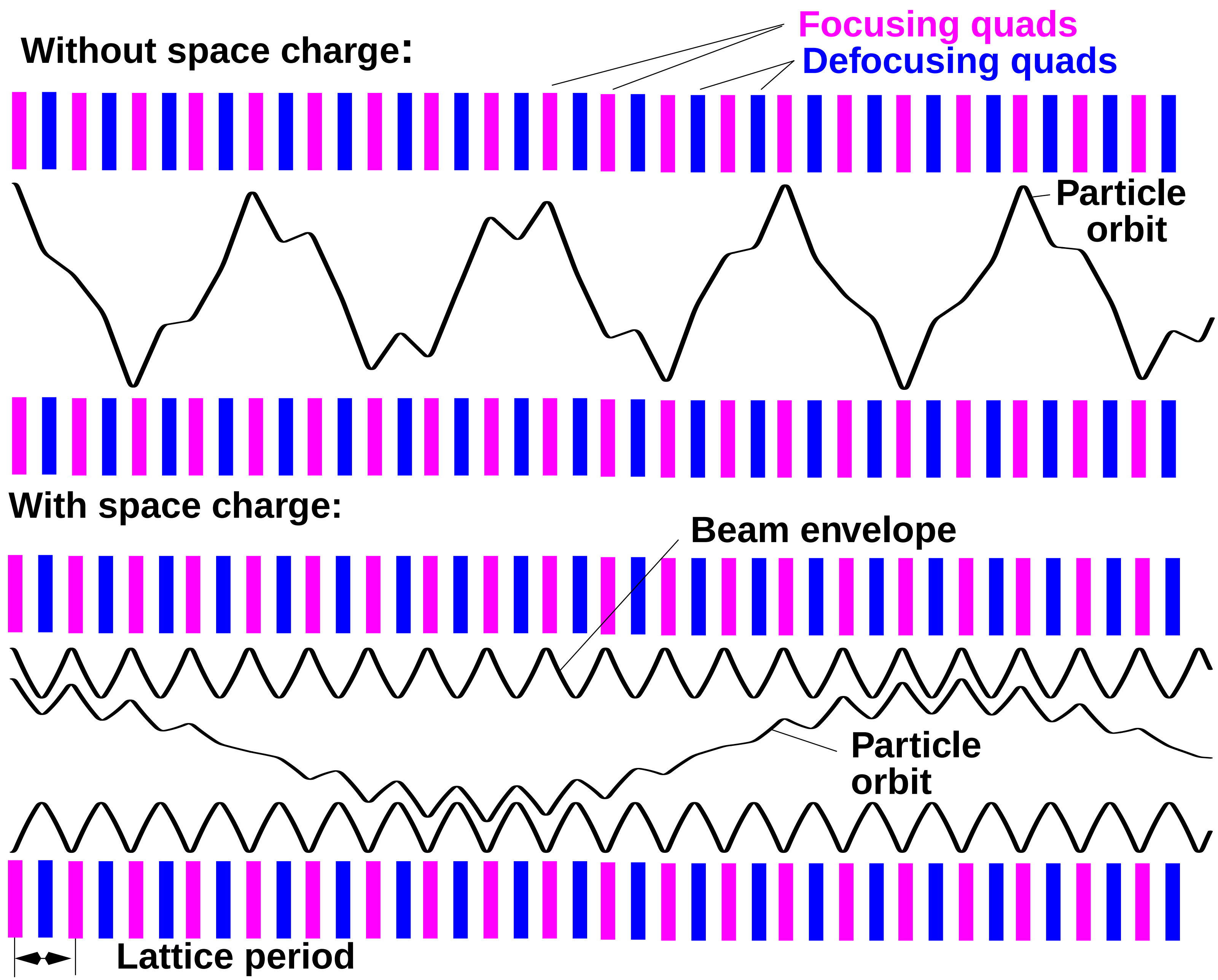


Vertical (y) plane:

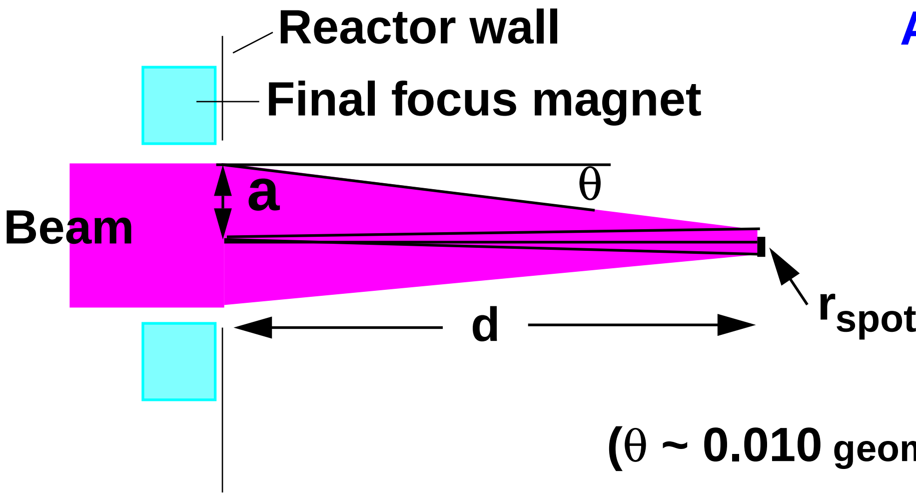


Average displacement is larger in focusing lenses so the net effect is focusing.

Space charge reduces betatron phase advance



Final focus requires high beam quality



At final focus:

r_{spot} <~ 2.7 mm, Δz <~ 0.5 m } beam intensity requirement

$\Delta v_x/v_z$, $\Delta v_y/v_z$ <~ $\theta/2$ <~ 0.005 emittance requirement

$\Delta v_z/v_z$ <~ $r_{spot}/12\theta d$ <~ 0.005 chromatic aberration limit

(θ ~ 0.010 geom. aberration limits; d >~ 5 m reactor constraints)

Back at the injector (typical example):

a ~ 3 cm, Δz ~ 42 m } constrained by transportable space charge limits

$\Delta v_x/c$, $\Delta v_y/c$ ~ 2×10^{-6} source temperature ~ 1.0 eV after injection

$\Delta v_z/v_z$ ~ 0.0005 voltage injection errors; $N_{\text{beam initial}}/N_{\text{beam final}} = 4$

Ratio of final required 6D phase space volume to initial volume:

$$\frac{(r_{spot}^2\ \Delta z\ \Delta p_x\ \Delta p_y\ \Delta p_z N_{beam})_{final}}{(a^2\ \ \Delta z\ \Delta p_x\ \Delta p_y\ \Delta p_z N_{beam})_{initial}} \sim 2000$$

Emittance leeway in each direction

$\varepsilon_{x\ final}/\varepsilon_{x\ initial}(N_{bf}/N_{bi})^{1/2} \sim r_{spot}\ \Delta v_{xfinal}/\ a\ \Delta v_{xinitial}(N_{bf}/N_{bi})^{1/2} \sim 20$

$\varepsilon_{z\ final}/\varepsilon_{z\ initial} \sim \Delta z_{final}\ \Delta v_{zfinal}/\ \Delta z_{initial}\ \Delta v_{zinitial} \sim 5$

Radial electric field and beam potential

Radial electric field at pipe radius ($r_p = 5$ cm)

$$E = \frac{2\lambda}{4\pi\varepsilon_o r_p} = \begin{bmatrix} 16 \\ 8 \\ 4 \end{bmatrix} \textbf{ MV/m for } C = \begin{bmatrix} 67 \\ 33.5 \\ 16.7 \end{bmatrix}$$

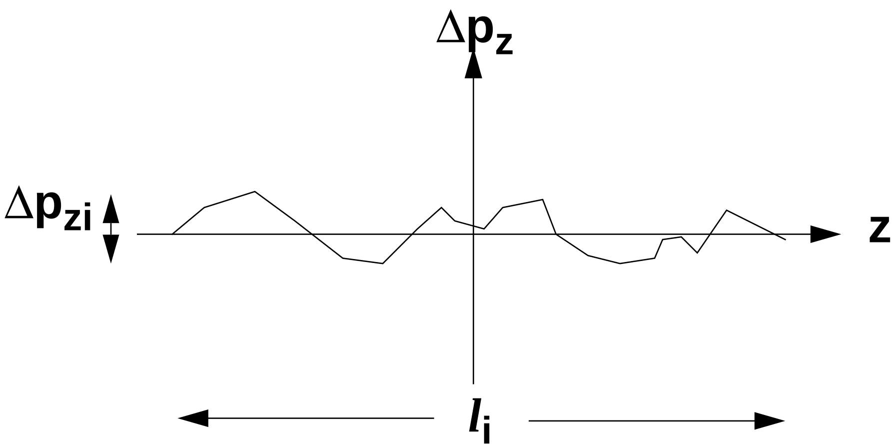
Potential difference, center to beam edge:

$$\Delta\phi = \frac{\lambda}{4\pi\varepsilon_o} = \begin{bmatrix} 400 \\ 200 \\ 100 \end{bmatrix} \textbf{ kV for } C = \begin{bmatrix} 67 \\ 33.5 \\ 16.7 \end{bmatrix}$$

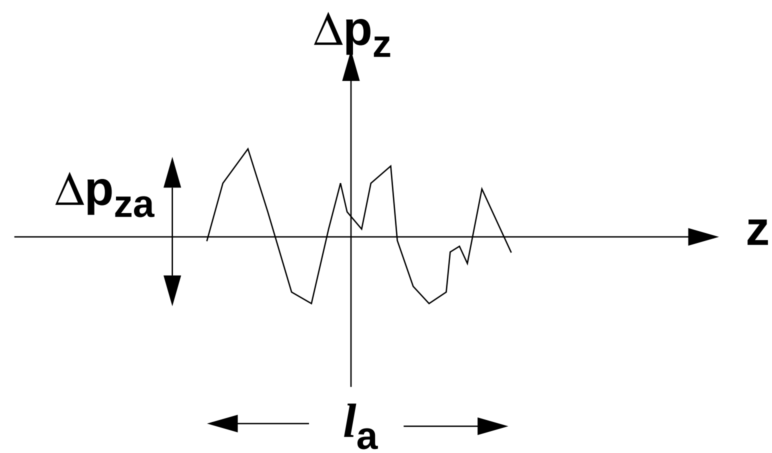
Potential difference, center to pipe:

$$\Delta\phi = \frac{\lambda}{4\pi\varepsilon_o}\left(1 + 2\ln\left(\frac{r_p}{a}\right)\right) = \begin{bmatrix} 800 \\ 400 \\ 200 \end{bmatrix} \textbf{ kV for } C = \begin{bmatrix} 67 \\ 33.5 \\ 16.7 \end{bmatrix}$$

Compression factor sets limits on voltage errors at injector and gaps



Bunch length at ith gap



Bunch length at end of accelerator

Phase space conservation: $\delta p_i l_i = \delta p_a l_a$

Energy/momentum: $\dfrac{p^2}{2m} = qV$ **=>** $\delta p_i = \dfrac{1}{2}\dfrac{p_i}{V_i}\delta V_i$

Also, at each gap

$$\delta V_i^2 = \frac{\Delta V_{gap}}{V_{pulser}} \delta V^2_{pulseri}$$

where $V_{pulser} \cong 10$ **kV**

At end of accelerator:

$$\Rightarrow \quad \frac{\delta p^2_a}{p^2_a} = \sum_{i=1}^{N} \frac{1}{4}\left(\frac{\Delta t_i}{\Delta t_a}\right)^2 \left(\frac{\Delta V_{gap}}{\Delta V_{pulser}}\right)\left(\frac{\delta V_{pulseri}}{V_a}\right)^2 + \frac{1}{4}\left(\frac{\Delta t_o}{\Delta t_a}\right)^2 \left(\frac{\delta V_{injector}}{V_{injector}}\right)^2$$

For equal contributions

$$\frac{\delta p_a}{p_a} = \left[\left(0.011\frac{\delta V_{pulser}}{V_{pulser}}\right)^2 + \left(0.087\frac{\delta V_{injector}}{V_{injector}}\right)^2\right]^{\frac{1}{2}} = \sqrt{2}(0.011)\frac{\delta V_{pulser}}{V_{pulser}}$$

At target:

$$\frac{\delta p_t}{p_t} = C\frac{\delta p_a}{p_a} \quad \Rightarrow \quad \frac{\delta V_{pulser}}{V_{pulser}} = \begin{bmatrix} 9.6\times 10^{-3} \\ 1.9\times 10^{-2} \\ 3.8\times 10^{-2} \end{bmatrix} \text{ and } \frac{\delta V_{injector}}{V_{injector}} = \begin{bmatrix} 1.2\times 10^{-3} \\ 2.4\times 10^{-3} \\ 4.9\times 10^{-3} \end{bmatrix} \text{ for C=} \begin{bmatrix} 67 \\ 33.5 \\ 16.8 \end{bmatrix}$$