Update on Requirements for Target Experiments on NDCX II

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Building 47 Conference Room, LBNL
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Outline

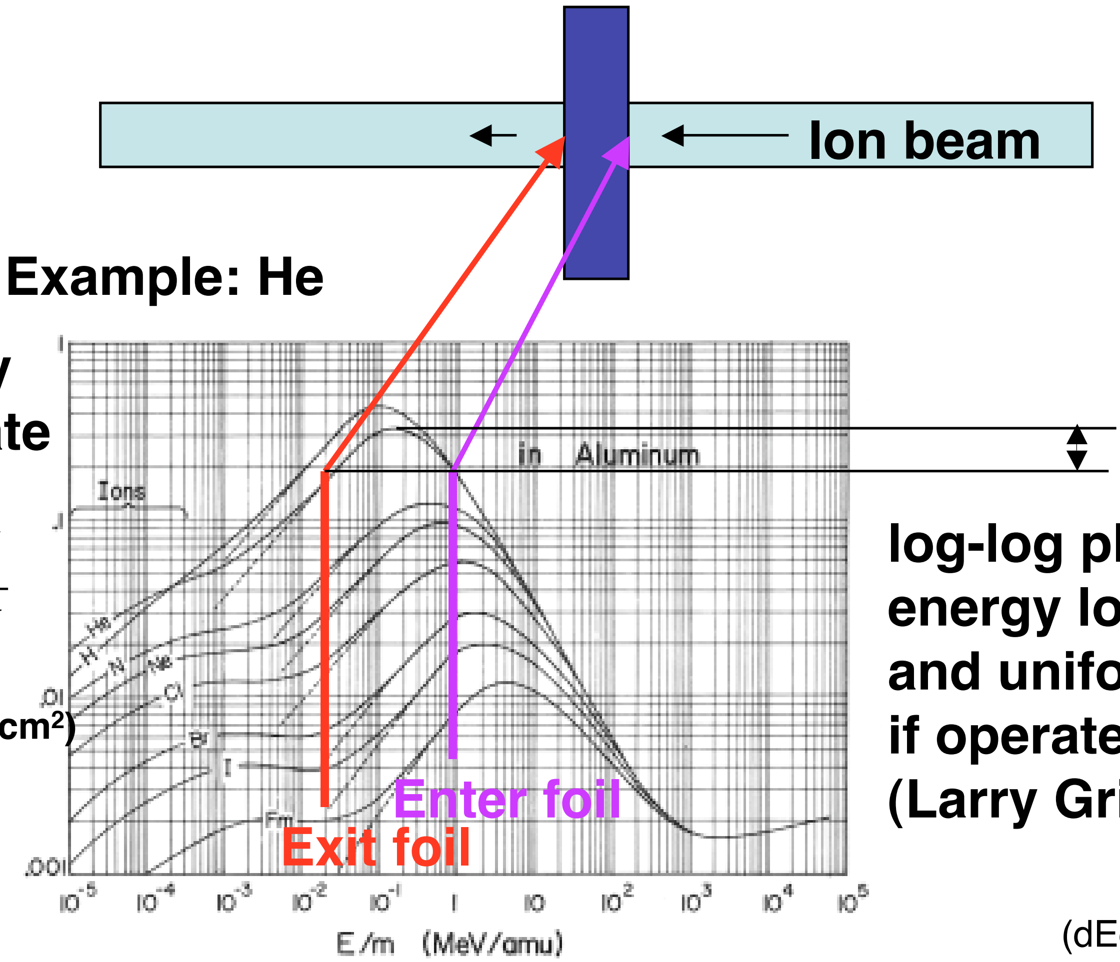
1. **Review of requirements (i.e. emittance, velocity spread $\delta p/p_{rms}$, and energy fluence (J/cm^2)) for Li based NDCX II for Warm Dense Matter application**

   (Update: f_{peak} for parabolic pulse and consideration of a K based NDCX II).

2. **Discussion of requirements based on hydrodynamic experiments**

Strategy: maximize uniformity and the efficient use of beam energy by placing center of foil at Bragg peak

In simplest example, target is a foil of solid or “foam” metal



Energy loss rate

$$-\frac{1}{Z^2}\frac{dE}{dX}$$

(MeV/mg cm^2)

$\Delta dE/dX \propto \Delta T$

log-log plot => fractional energy loss can be high and uniformity also high if operate at Bragg peak (Larry Grisham, PPPL)

Energy/Ion mass **(MeV/amu)**

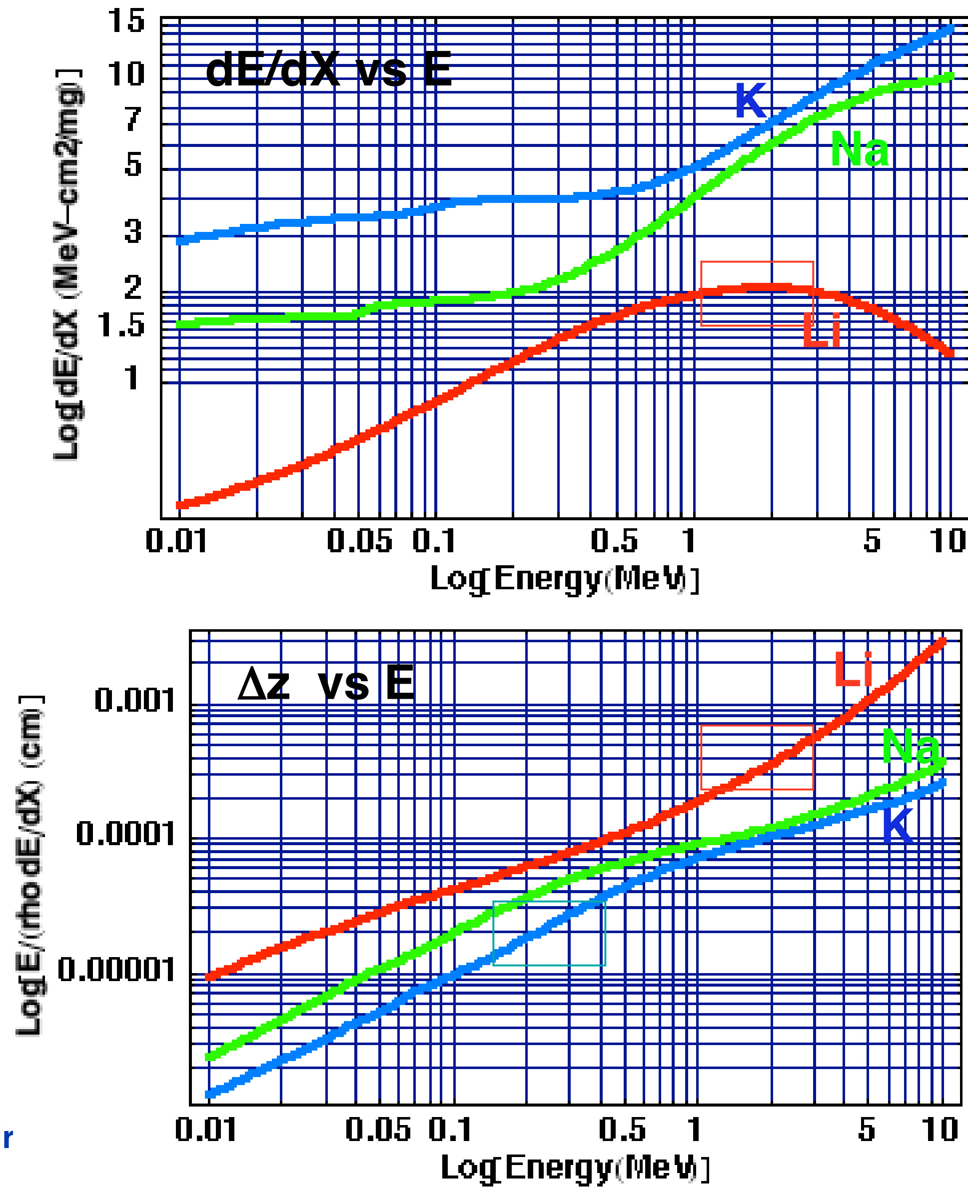
(dEdX figure from L.C Northcliffe and R.F.Schilling, Nuclear Data Tables, **A7**, 233 (1970))

Current NDCX II design is based on Lithium ion

SRIM code results (provided by Igor Kaganovich) gives dE/dX for three ions of interest (K, Na, and Li).

Li ions at ~ 1.8 MeV are at Bragg peak (although K ions at 200 - 400 keV are near inflection point)

Also range of Li ions at ~ 1.8 MeV is ~ 3 μ (a factor of 10 times longer than 400 keV K ions) so hydro time is factor of 10 longer



Target temperature (assuming no hydro motion)

$\mathbf{n_{atom}\, c_v T = \Delta E_{ion}\, N_{ions}/(\pi\, r_{spot}^2\, \Delta z)}$ (for uniform distribution on spot)

For solids at intermediate temps, $c_v = 3k_b$

$\mathbf{\Delta z = \Delta E_{ion}/(\rho\, dE/dX)}$ ΔE_{ion}=change in ion energy between entrance to and exit from foil

So $kT=9.6$ eV $(N_{ions}/10^{13})$ $(1\ \mathrm{mm}/r_{spot})^2$ $(dE/dX/2\ \mathrm{MeV\ cm^2/mg})$ $(A_{targ}/27)$

In terms of micro-coulombs:

$kT=9.6$ eV $(N_{ions}/10^{13})$ $(1\ \mathrm{mm}/r_{spot})^2$ $(dE/dX/2\ \mathrm{MeV\ cm^2/mg})$ $(A_{targ}/27)$

$=6.0$ eV $(Q/1\ \mu C)$ $(1\ \mathrm{mm}/r_{spot})^2$ $(dE/dX/2\ \mathrm{MeV\ cm^2/mg})$ $(A_{targ}/27)$

Expressed in terms of the fluence per unit area, F

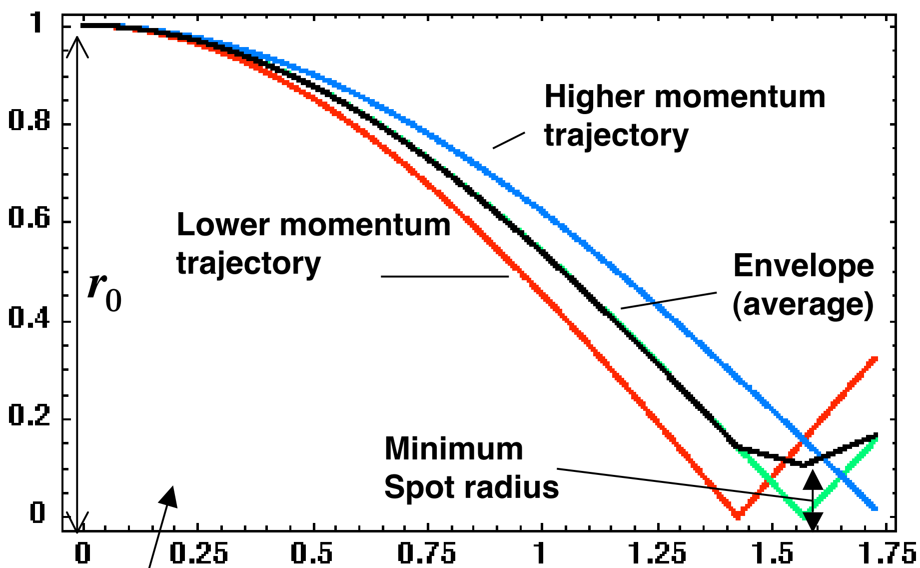
$\mathbf{kT= 0.19}$ eV $\mathbf{(F/\ 1\ J/cm^2)(1\ MeV/E_{entrance})(dE/dX/2\ MeV\ cm^2/mg)\ (A_{targ}/27)}$

This formula is for the energy at foil entrance and dE/dX at foil center. For Lithium at the Bragg Peak, $E_{ioncenter}$=1.88 MeV, dE/dX=2.052 MeV cm^2/mg. The entrance to the foil is approximately 50% higher energy = 2.82 MeV.

Thus to reach 2 eV we need F=29.1 J/cm^2 of Li at $E_{entrance}$ = 2.82 MeV.

The short pulse time and small spot radius place tight constraints on longitudinal and transverse emittance

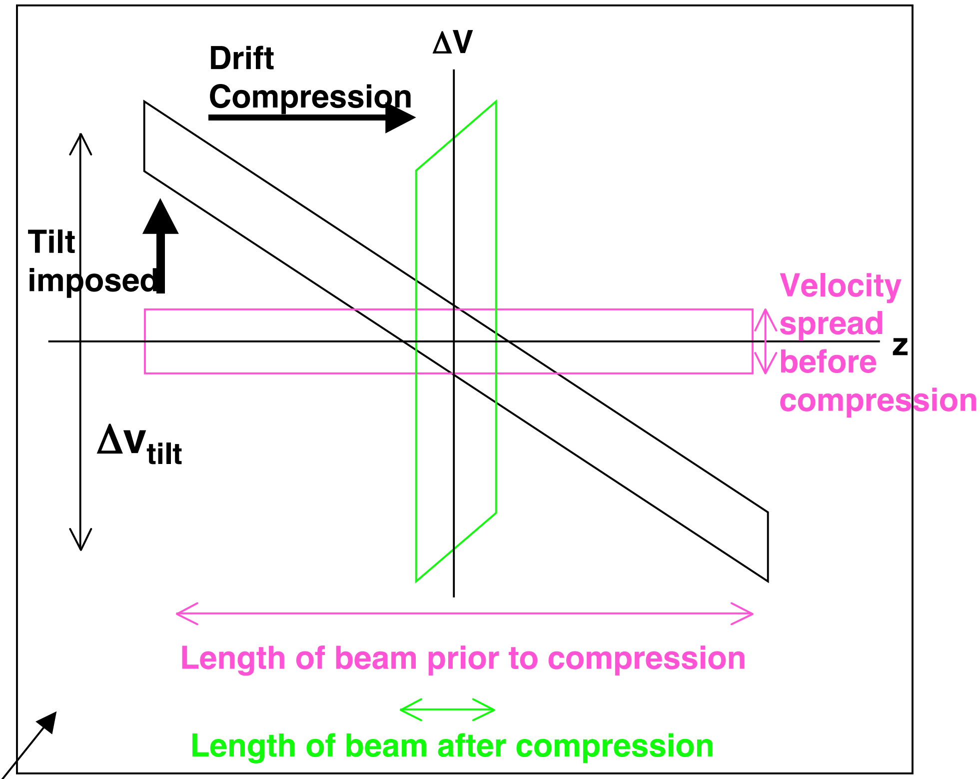
Transversely, spot radius determined by emittance + chromatic aberrations



$$r_{spot}^2 = \frac{4\varepsilon^2 f^2}{\pi^2 r_0^2} + \frac{\pi^2 r_0^2}{4}\left\langle \frac{\delta p^2}{p^2} \right\rangle_{\substack{after\\compress}}$$

$$\left\langle \frac{\delta p^2}{p^2} \right\rangle^{1/2}_{\substack{after\\compress}} = C\left\langle \frac{\delta p^2}{p^2} \right\rangle^{1/2}_{\substack{before\\compress}} = \eta\left(\frac{\Delta v}{v}\right)_{tilt}$$

Longitudinally, phase space undergoes rotation during drift compression; $<(\delta v/v)^2>^{1/2}$ limits final bunch length



C = ratio of initial to final bunch length;
η = conversion factor from tilt to rms
= 0.22 (parabolic) - 0.29 (flattop)

Increasing velocity tilt decreases pulse duration but increases spot radius

If $r_{spot}^2 = \dfrac{4\varepsilon^2 f^2}{\pi^2 r_0^2} + \dfrac{\pi^2 r_0^2}{4}\left\langle \left(\dfrac{\delta p}{p}\right)^2 \right\rangle_{\substack{after\\compress}}$ **then optimum initial beam radius** r_{0_opt} **which minimizes** r_{spot}:

$$r_{0\_opt}^2 = \frac{4\varepsilon f}{\pi^2 \left\langle (\delta p/p)^2 \right\rangle^{1/2}_{\substack{after\\compress}}}$$

Minimum spot radius at r_{0_opt} **is then:**

$$r_{spot\,\min}^2 = 2\varepsilon f \left\langle \left(\frac{\delta p}{p}\right)^2 \right\rangle^{1/2}_{\substack{after\\compress}}$$

$$\left\langle \frac{\delta p^2}{p^2} \right\rangle^{1/2}_{\substack{after\\compress}} = C\left\langle \frac{\delta p^2}{p^2} \right\rangle^{1/2}_{\substack{before\\compress}} = \eta\left(\frac{\Delta v}{v}\right)_{tilt}$$

At maximum compression

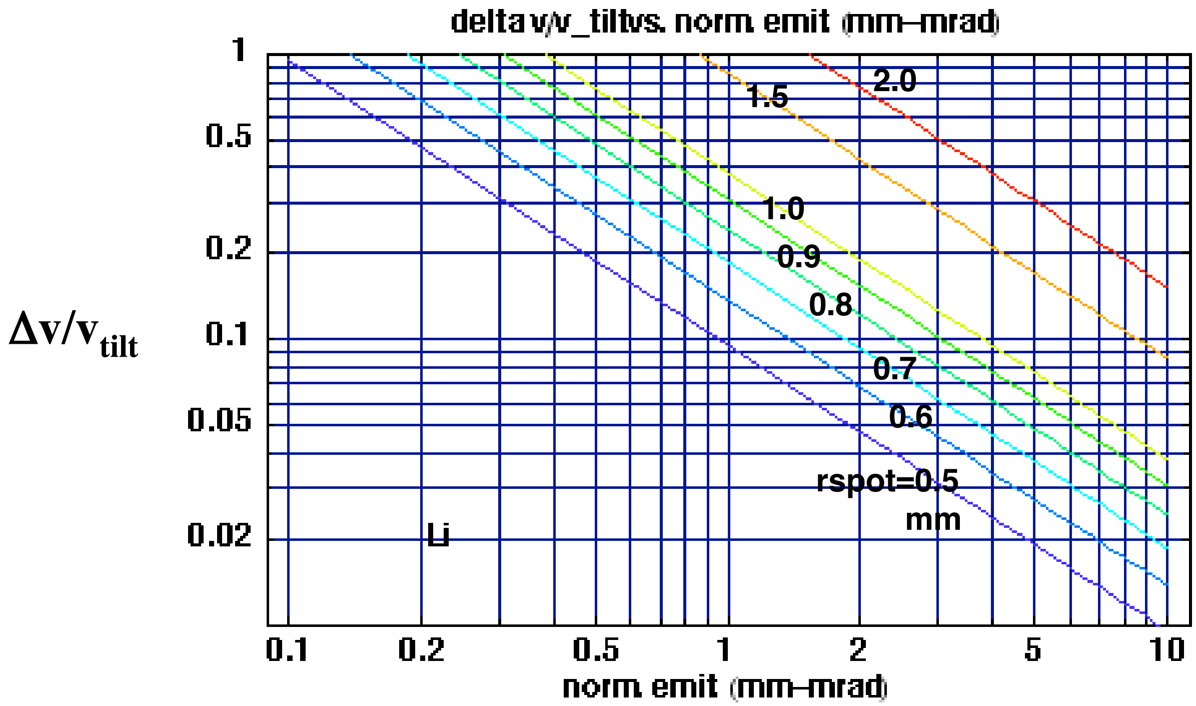
$$r_{spot\,\min}^2 = 2\eta\varepsilon f\left(\frac{\Delta v}{v}\right)_{tilt}$$

$$\Delta t_{\substack{after\\compress}} = \frac{\Delta t_{\substack{before\\compress}} \left\langle \frac{\delta p^2}{p^2} \right\rangle^{1/2}_{\substack{before\\compress}}}{\eta\left(\frac{\Delta v}{v}\right)_{tilt}}$$

Example: for $\Delta v/v_{tilt} = .1$, $\varepsilon_N = 2$ mm-mrad, $\beta=0.047$
$f=0.4$ m, $\eta=0.29$ ==> $r_{spotmin} = 1.0$ mm
For $\Delta t=20\ ns$ **and** $\delta p/p_{rms}=0.1\%$ **(both before compression) yields** $\Delta t = 0.7\ ns$ **(after compression).**

Li spot radius as functions of normalized emittance and velocity tilt

$$r^2_{spot} = 2\eta\varepsilon f \frac{\Delta v_{tilt}}{v}$$



B_solenoid = 15 T is assumed in the above calculations.

Focal length $f = \pi/2k_{\beta 0} = \pi\, mv/qeB = 0.133$ *m (15 T/B) (β/.0295)(A/6.94)(1/q)* *(Li)*

Simulations by Welch (and others subsequently) showed substantial peaking relative to a gaussian with $r_{spot}=r_{spot\_opt}$

Let $n(r)$ **= number of ions per unit area at focal spot, integrated over pulse**
N_0 **= total ions in pulse;** $\Delta = \Delta v/v_{tilt}$

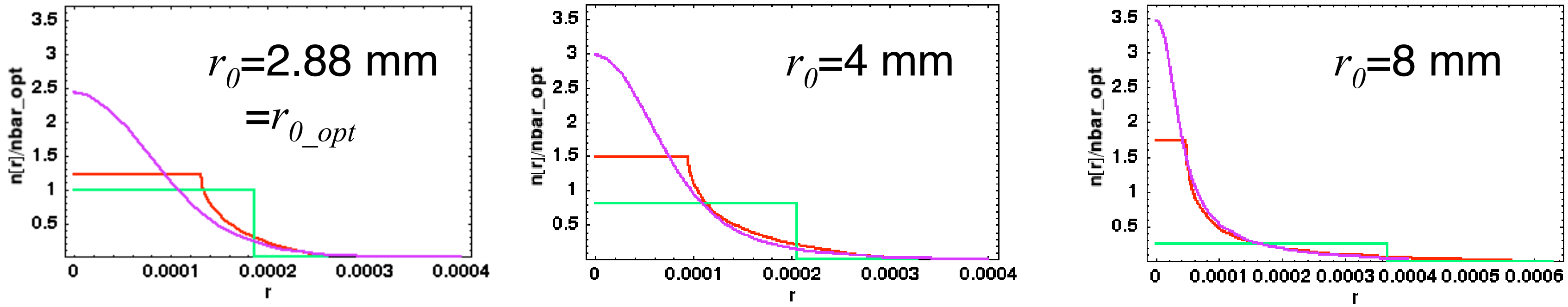
$$n(r) = \int_{-l_b/2}^{l_b/2} \frac{dn(r)}{ds} ds = \int_{-\Delta/2}^{\Delta/2} \frac{dn(r)}{ds}\frac{ds}{d\delta} d\delta = \frac{l_b}{\Delta}\int_{-\Delta/2}^{\Delta/2} \frac{dn(r)}{ds} d\delta$$

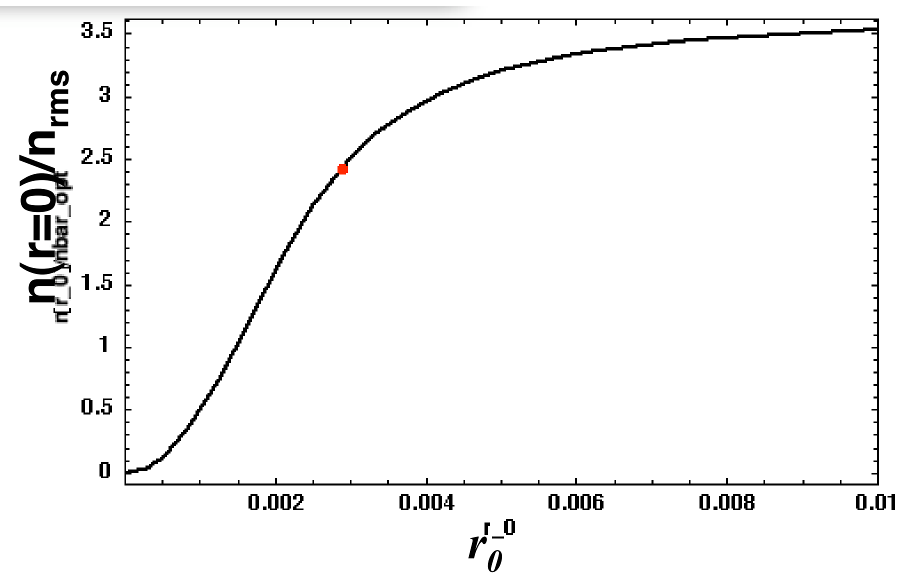
By adding contributions from each velocity class we may calculate the intensity vs. r, by assuming overlapping Gaussians at target

$$\frac{dn(r)}{ds} = \frac{N_0/l_b}{\pi\sigma^2}\exp[-r^2/\sigma^2]$$

Here: $\langle r^2 \rangle_{\substack{each\\slice}} = \sigma^2 = \frac{1}{2}\left(\frac{4f^2\varepsilon^2}{\pi^2 r_0^2} + \frac{\pi^2 r_0^2 \delta^2}{4}\right)$ **and** $2\pi\int_0^\infty n(r) r dr = N_0$

The central intensity [$n(r=0)$] increases as r_0 increases





$$n(r=0) = \frac{4N_0}{\pi f \varepsilon \Delta} \tan^{-1}\left(\frac{\pi^2 \Delta r_0^2}{8 f \varepsilon}\right)$$

$$r_{0\_opt}^2 = \frac{4 f \varepsilon}{\pi^2 \eta \Delta} \qquad r_{spot\_opt}^2 = 2 f \varepsilon \eta \Delta$$

$$n(r=0, r_0 = r_{0\_opt}) = \frac{4N_0}{\pi f \varepsilon \Delta} \tan^{-1}\left(\frac{1}{2\eta}\right) = 2.42 n_{rms}$$

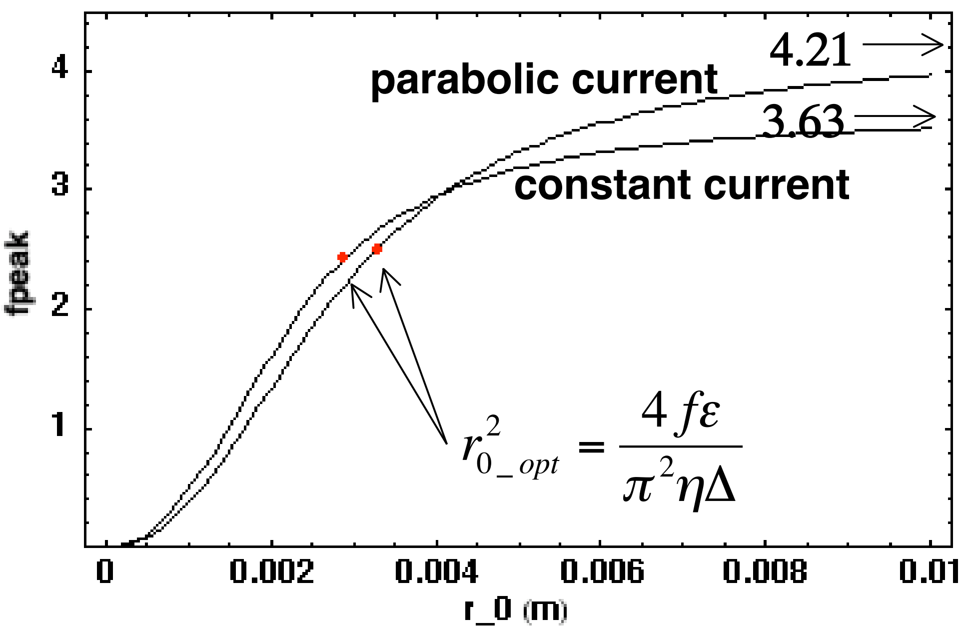
$$n(r=0, r_0 = \infty) = \frac{4N_0}{\pi f \varepsilon \Delta}\left(\frac{\pi}{2}\right) = 3.63 n_{rms}$$

$$\left(\quad n(r=0, rms, opt) = \frac{N_0}{2\pi \eta f \varepsilon \Delta} \equiv n_{rms} \quad \right)$$

values shown are for η=0.29, an initial constant current distribution

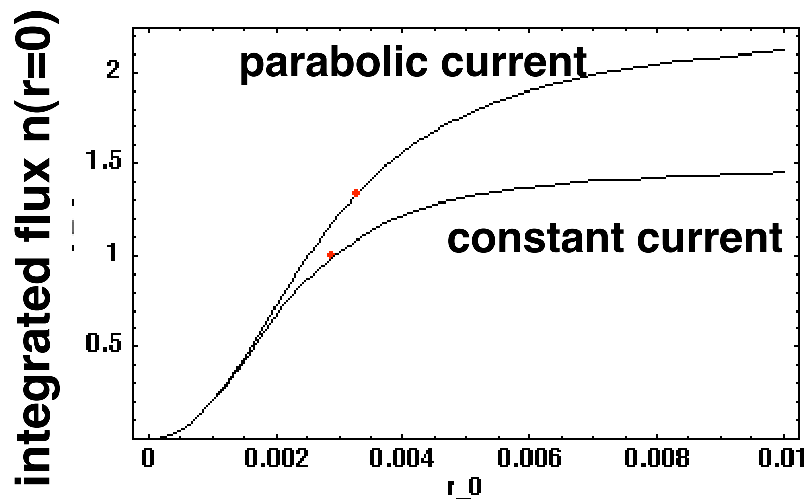
If current is parabolic at time when velocity tilt is applied, "peaking" factor is larger than constant current case

$$r_{spot\_opt}^2 = 2 f \varepsilon \eta \Delta \qquad \eta \equiv \frac{1}{\Delta}\left(\frac{\int_{-l_b/2}^{l_b/2} I(z)(\delta v(z)/v)^2\, dz}{\int_{-l_b/2}^{l_b/2} I(z)\, dz} \right)^{1/2} = \begin{cases} 1/\sqrt{12} \approx 0.29 & \text{constant current} \\ 1/\sqrt{20} \approx 0.22 & \text{parabolic current} \end{cases}$$



$$f_{peak} \equiv \frac{n(r=0)}{N_0 / \pi r_{spot\_opt}^2}$$

$$n(r=0) = \left(\frac{12 N_0}{f \pi^3 \varepsilon \Delta^3}\right)\left(\left(\frac{32 f^2 \varepsilon^2}{\pi^2 r_0^4} + \frac{\pi^2 \Delta^2}{2}\right) \tan^{-1}\left(\frac{\pi^2 r_0^2 \Delta}{8 f \varepsilon}\right) - \frac{4 f \varepsilon \Delta}{r_0^2}\right)$$



(Examples are for ε=5 mm-mrad, *f*=11.0 cm, Δ=0.1)

Requirement on Lithium beam is a requirement on charge in pulse, emittance, and velocity spread

Combining Temperature requirement with relation for spot radius yields:
Temperature requirement:

$kT = 6.0$ eV $(Q/1\ \mu C)\ (1\ \mathrm{mm}/r_{\mathrm{spot}})^2\ (dE/dX/2\ \mathrm{MeV\ cm^2/mg})\ (A_{\mathrm{targ}}/27)$

Using $r^2_{spot\_opt} = 2\eta\varepsilon f \dfrac{\Delta v_{tilt}}{v}$ (together with a peaking factor) yields:

$kT = 2.26$ eV $(Q/\ 1\ \mu C)\ (1\mathrm{mm\text{-}mrad}/\varepsilon_n)(1/\Delta v/v|_{\mathrm{tilt}})(f_{\mathrm{peak}}/1)$ x
x $(.29/\eta)(B/15\ \mathrm{T})(7/A_{\mathrm{ion}})(A_{\mathrm{targ}}/27)(dEdX/2\ \mathrm{MeVcm^2/mg})$

(here $f_{\mathrm{peak}} = F/(QV/\pi r_{\mathrm{spot\_opt}}{}^2)$= peaking in J/cm^2 due to peaking of overlapping gaussians = 2.4 to 3.6 (typically 2.8)

Also, $\eta \Delta t|_{\mathrm{targ}} \Delta v/v_{|\mathrm{tilt}} = \Delta t|_{\mathrm{bc}} <\delta p^2/p^2>_{\mathrm{bc}}{}^{1/2}$

kT=1.84 eV $(Q/\ 0.1\ \mu C)\ (1\mathrm{mm\text{-}mrad}/\varepsilon_n)(100\mathrm{ns}/\Delta t|_{\mathrm{bc}})(10^{-3}/\delta p/p_{\mathrm{rms\_bc}})(f_{\mathrm{peak}}/2.8)$ x
x $(B/15\ \mathrm{T})(7/A_{\mathrm{ion}})(A_{\mathrm{targ}}/27)(dEdX/2\ \mathrm{MeVcm^2/mg})(\Delta t_{\mathrm{targ}}/1\mathrm{ns})$

==> $(Q/\ 0.1\ \mu C)\ (1\ \mathrm{mm\text{-}mrad}/\varepsilon_n)(100\mathrm{ns}/\Delta t|_{\mathrm{bc}})(10^{-3}/\delta p/p_{rms\_bc}) >\ 1.12$ for kT > 2 eV

Requirement for potassium is analogous to lithium

$dE/dX =$ 9 MeV cm^2/mg @ 3 MeV,
7 MeV cm^2/mg @ 2 MeV, and
5 MeV cm^2/mg @ 1 MeV

$\Rightarrow \Delta z =$ 1 micron (for 2/3 energy deposition) but $\Delta T/T \sim 4/7 \sim 57\%$
(for 5% variation need $\Delta z \sim 0.1$ micron).

For 1 $\Delta z = 1$ micron, hydro time is ~ 0.2 ns.

kT= 2.0 eV (F/ 9 J/cm^2)(3 MeV/E$_{entrance}$)(dE/dX/7 MeV cm^2/mg) (A$_{targ}$/27)

kT=1.15 eV $(Q/\ 0.1\ \mu\text{C})\ (1\ \text{mm-mrad}/\varepsilon_n)(100\text{ns}/\Delta t|_{bc})(10^{-3}/\delta p/p_{rms\_bc})(f_{peak}/2.8)$ x
x $(B/15\ \text{T})(39/A_{ion})(A_{targ}/27)(dEdX/7\ \text{MeVcm}^2/\text{mg})(\Delta t_{targ}/1\text{ns})$

Implication for final spot radius

From Enrique's simulations:

Before drift compression (z=4.97 m)

v_z = 0.955 x 10^7 β=0.032

$<\delta v/v_{rms}>$=2.4 x 10^{-4}, $kT_{||} = 2\ E <(\delta v_z/v_z)^2>$= 0.32 eV

ε_n = 0.4 mm-mrad

I_{max}=0.7 A

Q = 0.03 μC

λ_{max}= 0.073 μC/m

ℓ_b = 0.57 m

Δt = 33 ns (FWHM)

= 60 ns (FWFM approximate parabolic pulse) -> 1 ns

For 15 T Solenoidal B-field and final pulse duration (FWZM= 1ns), fpeak = 2.5:

r_{spot} (mm) (2 rms)	$\Delta v/v$	kT (eV)	Sp. Fluence J/cm^2	Max ε_n mm-mrad	Max $\delta p/p\_rms$ (x 10^{-4})	l_{drift} (m)	Max r_{drift} (cm)
0.23	0.066	5.8	84.3	0.40	2.4	8	1.3
0.39	0.066	2.0	29.0	1.20	2.4	8	2.2
0.39	0.224	2.0	29.0	0.40	7.2	2.7	0.6
0.39	0.163	2.0	29.0	0.48	6.0	3.2	0.7

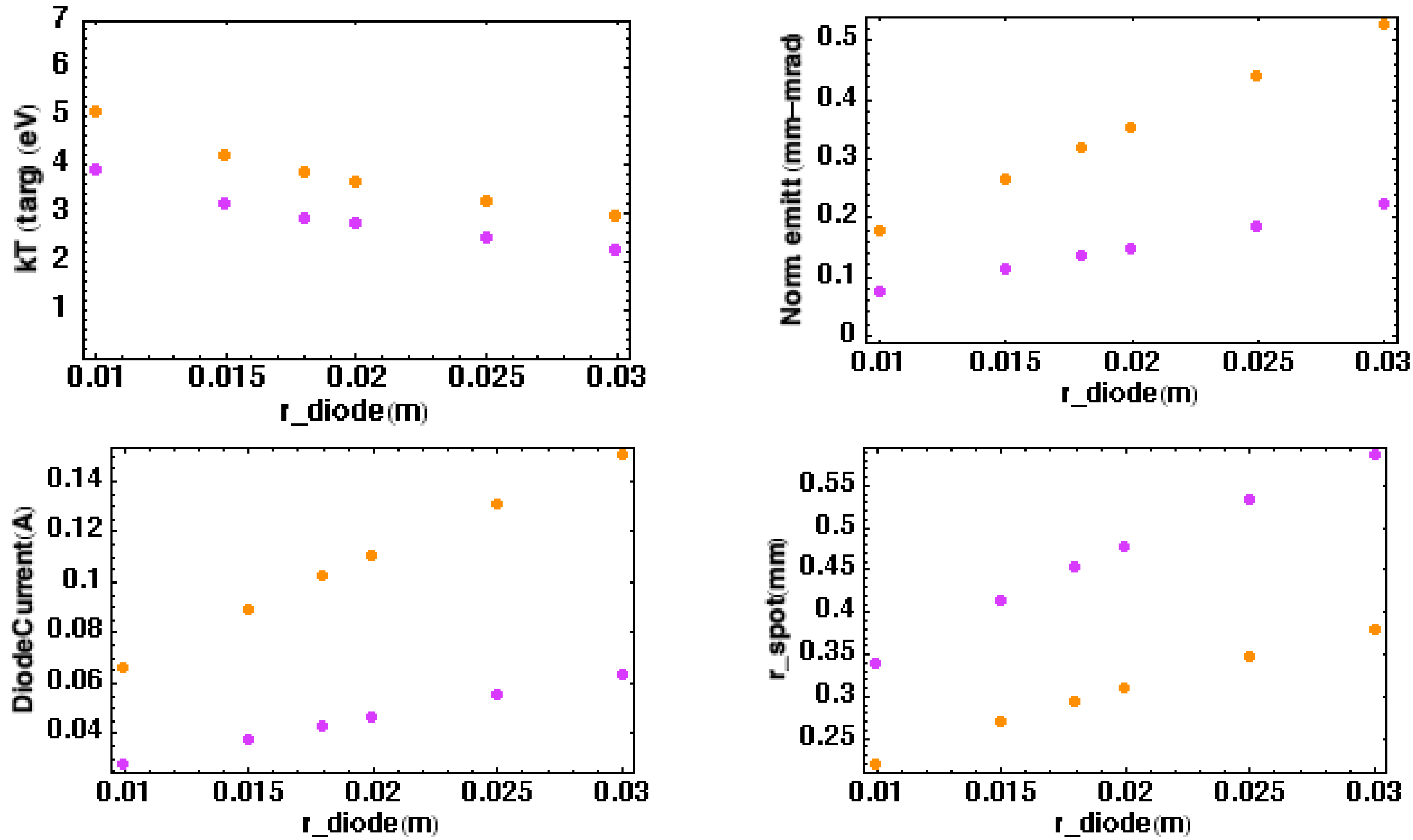
Large Δv allows relaxed $\delta p/p\_rms$ requirement and smaller drift length but stricter requirement on ε_n. The final ion density for the case of a 0.42 mm radius spot is 7.4 x 10^{13} ions/cm^3.

Application to systems model

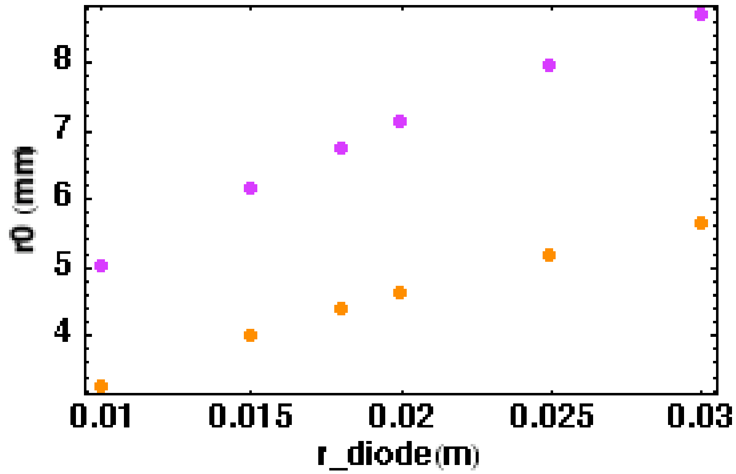
Systems model assumes certain input values (which are adjustable):

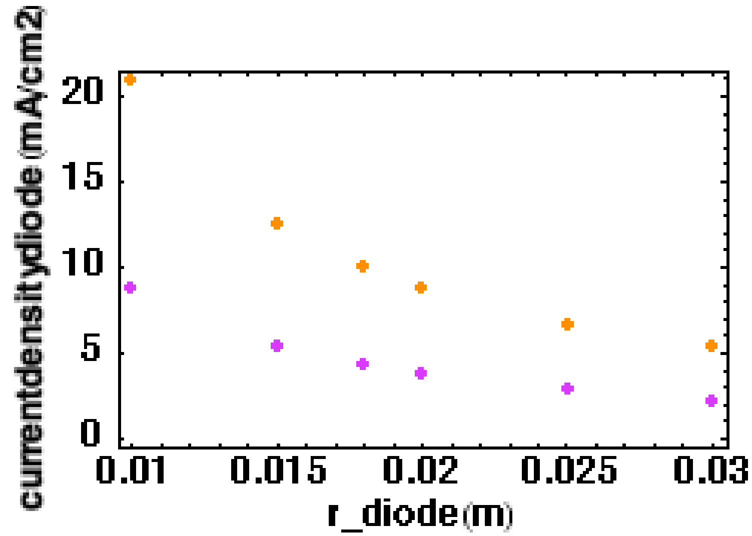
1. **Child-langmuir emission**
2. **Diode gap voltage limited by breakdown voltage:**
   $V \sim \{d^{1/2}, d\}$ for $d\{>, <\}$ 1 cm
3. **Diode gap = Δ x diode radius; (Δ = 18) (to match Enrique's current and current density)**
4. **Normalized emittance ~ diode radius x (0.5 eV)$^{1/2}$**
5. **Longitudinal rms momentum spread = 5 x 10^{-4} (bc)**
6. **Solenoid field = 15 T**
7. **Voltage tilt core = 750 kV**
8. **Peaking factor =2.5 (radius at which beam enters solenoid $r_0 = r_{0opt}$); Higher peaking possible**
9. **Li and K beam, final energy = 2.82 MeV**

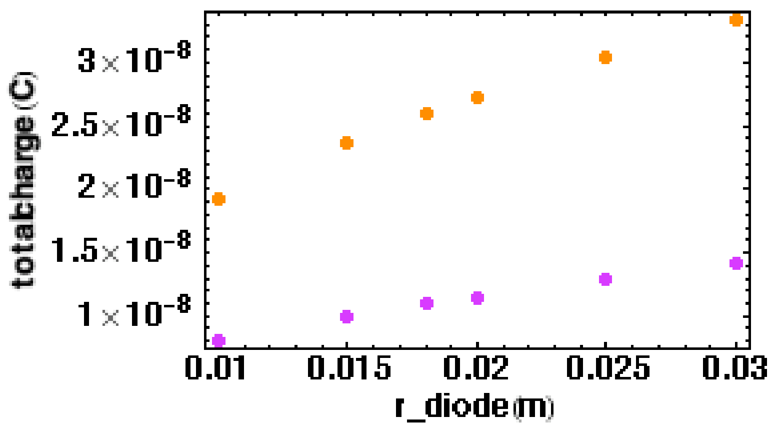
Examples: Varying the diode radius (for Li, and K)

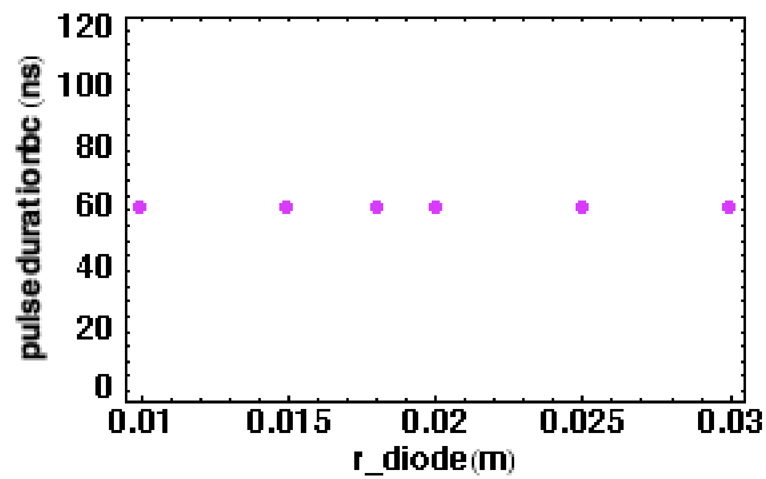


Example continued (for Li, and K)





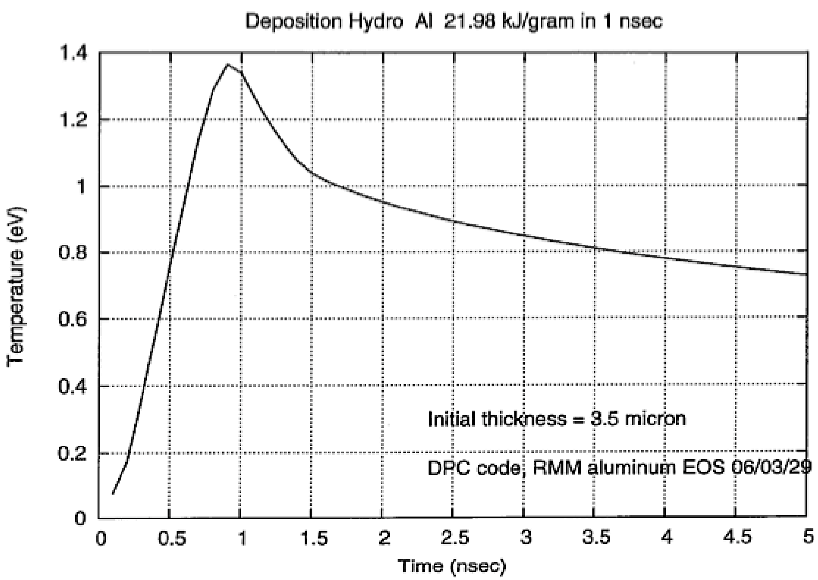




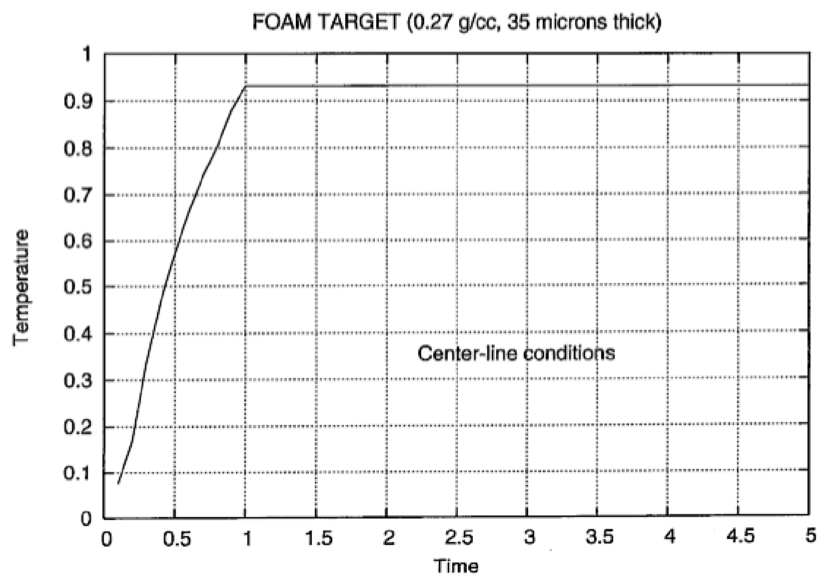
Hydro simulations show that for a 1 ns pulse, target reaches temperature of ~ 1.4 eV (rather than 2 without hydro)

Simulations by R. More using advanced EOS of uniformly heated foil

3.5 μ, solid density



35 μ, 10% density foam



HYDRA simulations with the same energy deposition give similar peak temperature

II. Hydrodynamic experiment requirements

Currently there are two broad classes of "hydro experiments" proposed for NDCX II

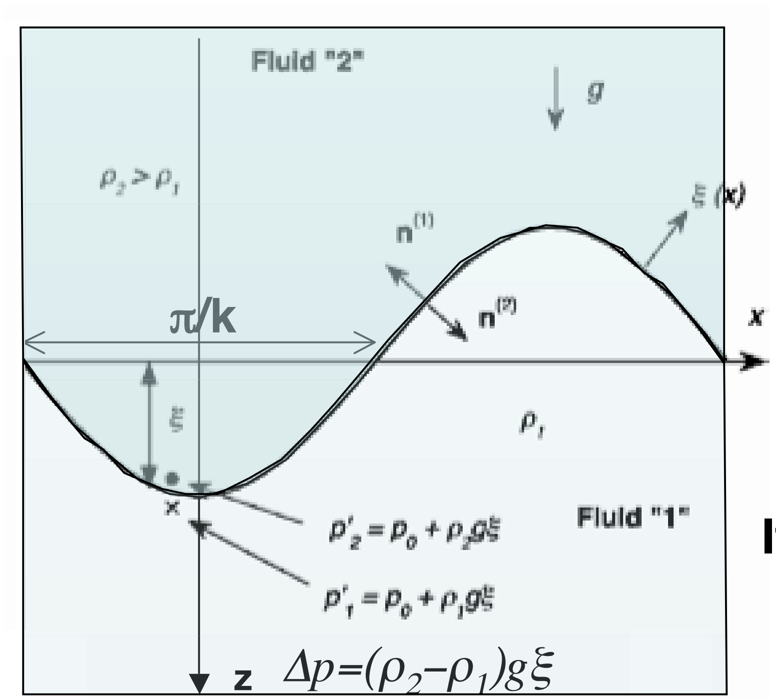
1. **Stopping experiments**
   - **Outflowing material cause ion beams to penetrate less deeply over course of pulse.**
   - **Can coupling be optimized by proper choice of intensity and energy variations over space and time?**
2. **Stability experiments**
   - **Volumetric stopping affects growth of Rayleigh-Taylor instability differently than surface energy deposition. Can we study this instability on NDCX II?**

Classical Rayleigh Taylor -- A review

(see A. R. Piriz, O. D. Cortázar, J. J. López Cela, and N. A. Tahir, "The Rayleigh Taylor Instability," American Journal of Physics, 74, 1095-1098 (2006))

Heuristic derivation:

$\xi = \xi_0 \cos kx$



$m\, d^2\xi_0/dt^2 = \Delta p\, A$ *(Newton's equation)*

$m = (\rho_2 + \rho_1)\, V = (\rho_2 + \rho_1)\, A\, \Delta z$

$\Delta z = \sim 1/k$ (not ξ_0 !!!)

$\Rightarrow \quad d^2\xi_0/dt^2 = ((\rho_2 - \rho_1)/(\rho_2 + \rho_1) gk\, \xi_0$

If $\xi_0 \sim e^{i\omega t}$ **then** $-\omega^2 = ((\rho_2 - \rho_1)/(\rho_1 + \rho_2))gk$

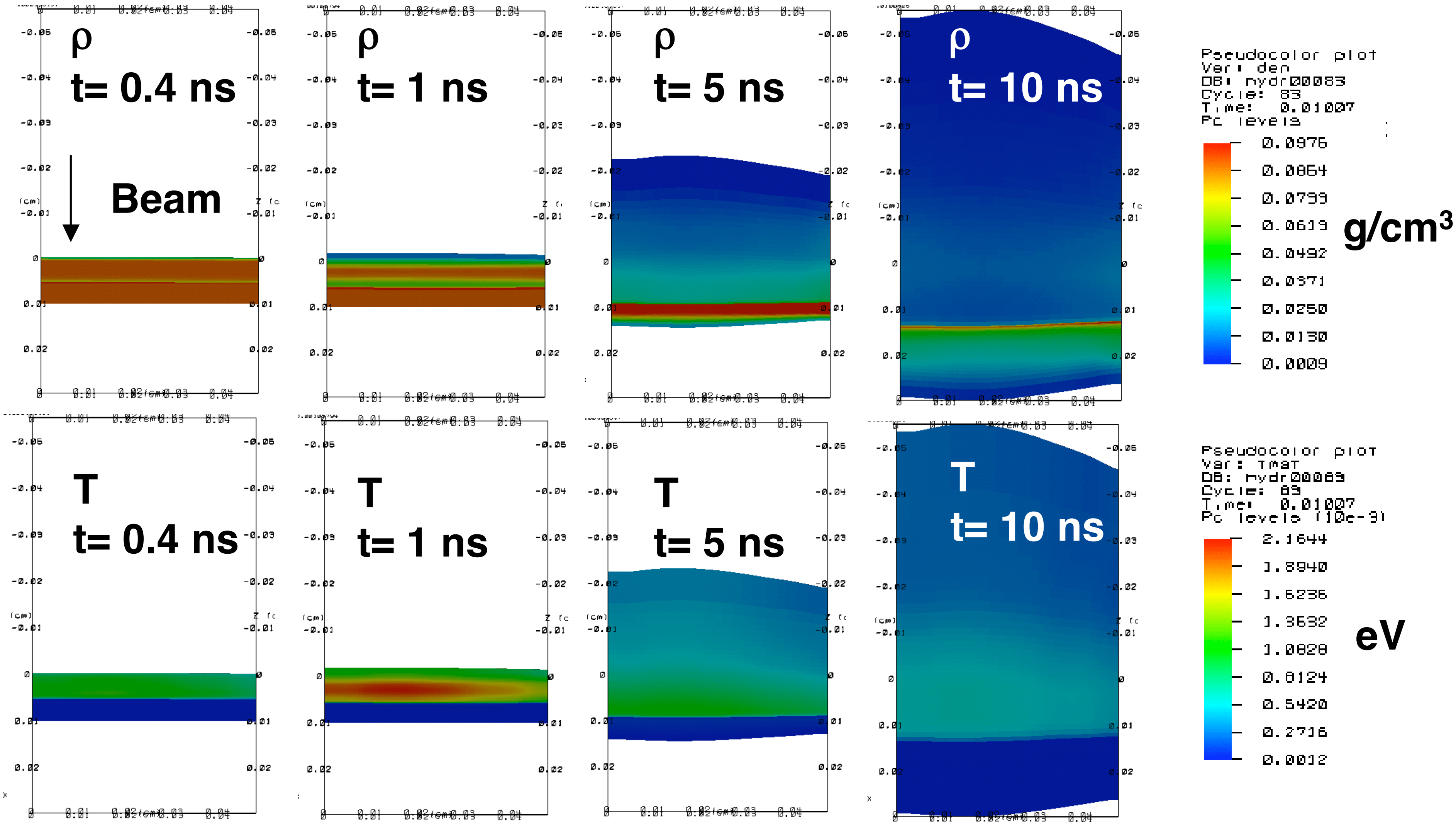
$\boxed{\textbf{If } \rho_2 >> \rho_1 \textbf{ then } \Gamma = -i\omega =\sim (gk)^{1/2}}$

In the case of ablation driven targets g is replaced by a, the acceleration rate.

Two different NDCX II beam parameters have been simulated

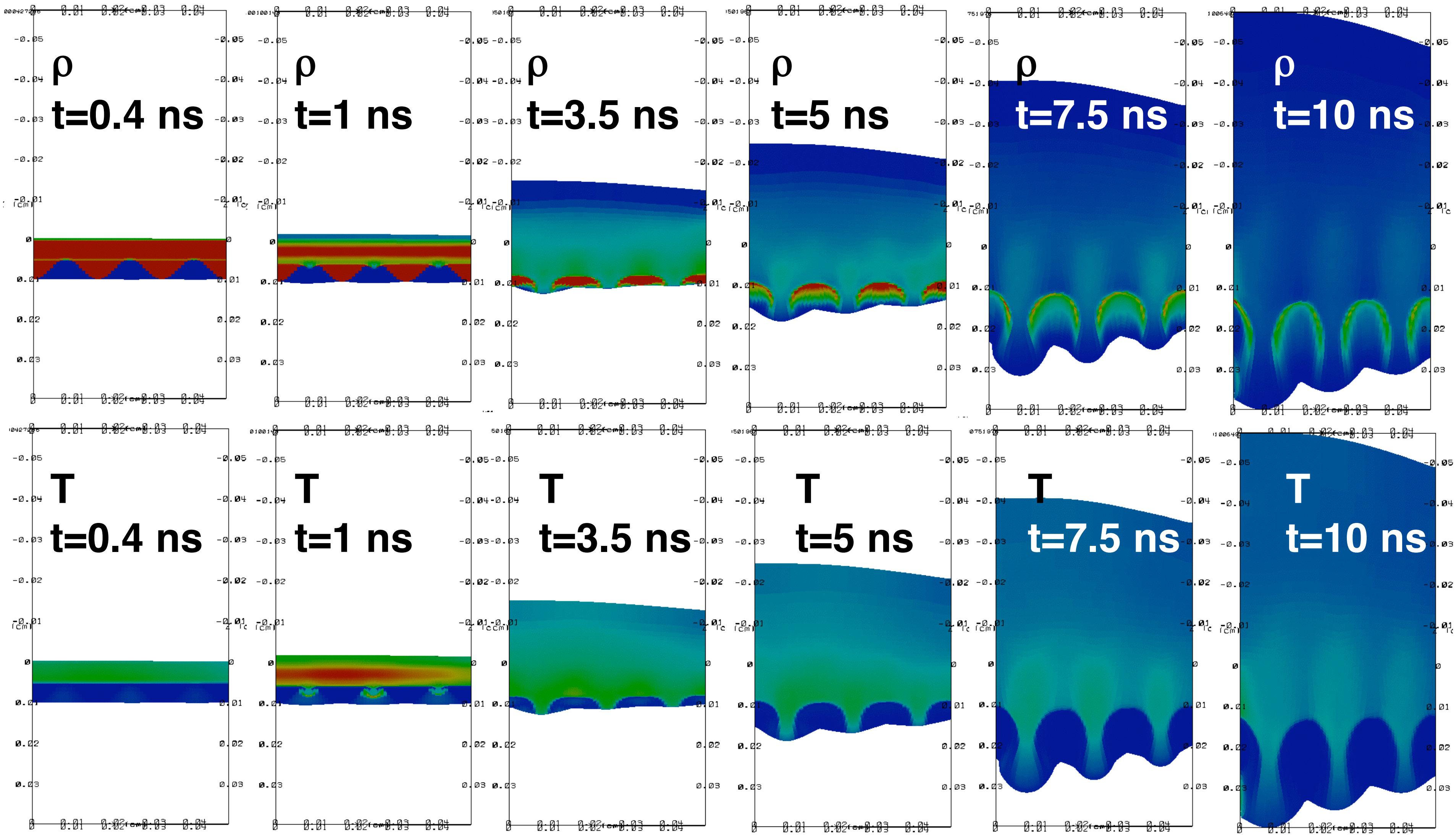
Ion energy	23 MeV	2.8 MeV
Ion species	Ne (A=20.18)	Li (A=6.97)
Total charge	0.1 μC	0.03 μC
Pulse duration (full width)	1 ns	1 ns
Beam radius	0.5 mm	0.5 mm
Range in solid hydrogen (according to Hydra)	50 μ	30 μ
Energy density in hydrogen	5.9×10^{10} J/m^3	3.6×10^{9} J/m^3
Max kT (estimated)	2.3 eV	0.14 eV

We have begun using Hydra to explore accelerator requirements to study beam driven Rayleigh Taylor instability

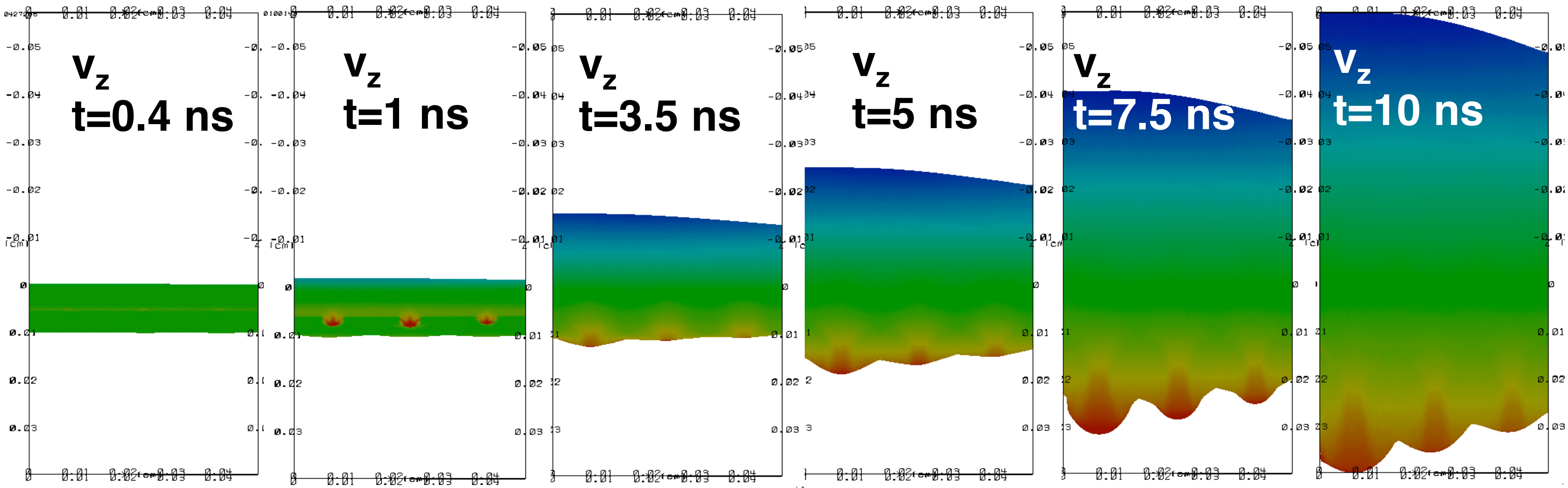


23 MeV Ne, 0.1 μC, 1 ns pulse (NDCX II) impinges on 100 μ thick solid H, T=0.0012eV, ρ =0.088 g/cm3; No density ripple on surface, blowoff accelerates slab

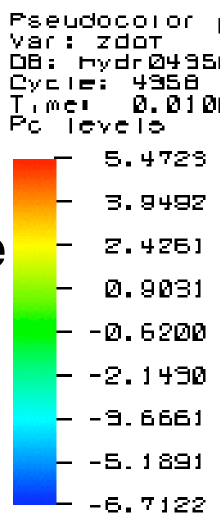
When initial surface ripple is applied, evidence for Rayleigh Taylor instability is suggestive



When initial surface ripple is applied, evidence for Rayleigh Taylor instability is suggestive (-- continued)

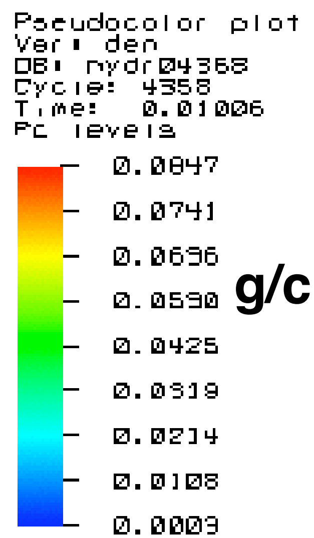


Scale for above figures (v_z):

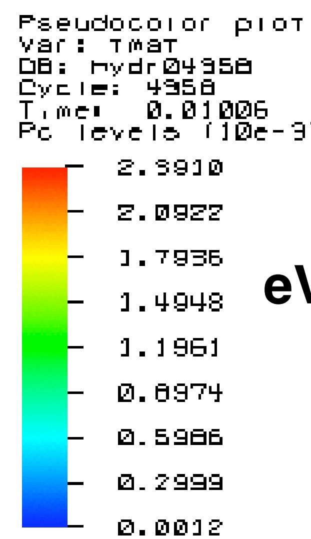


cm/μs

Scales from previous page (ρ and T):

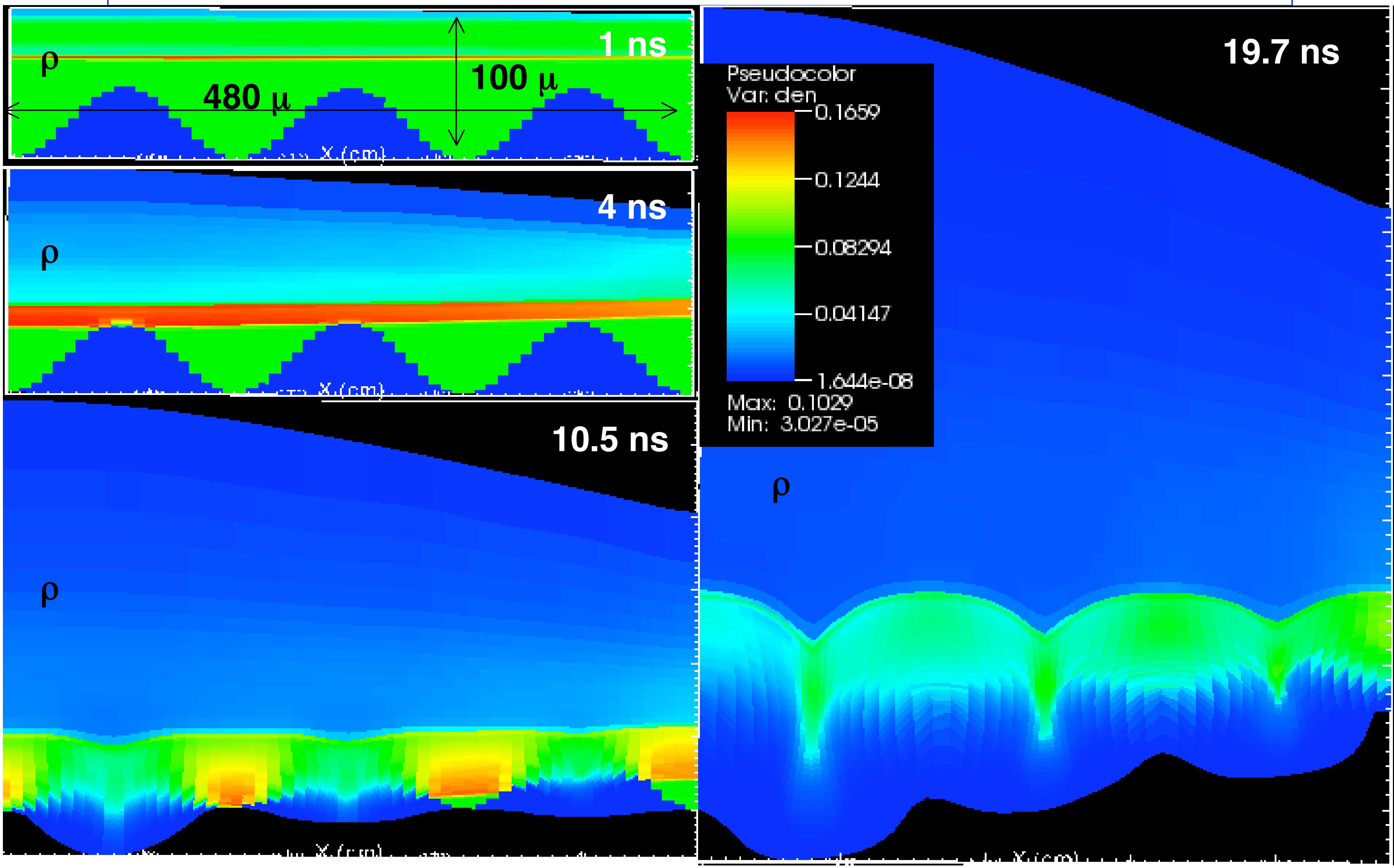


g/cm3

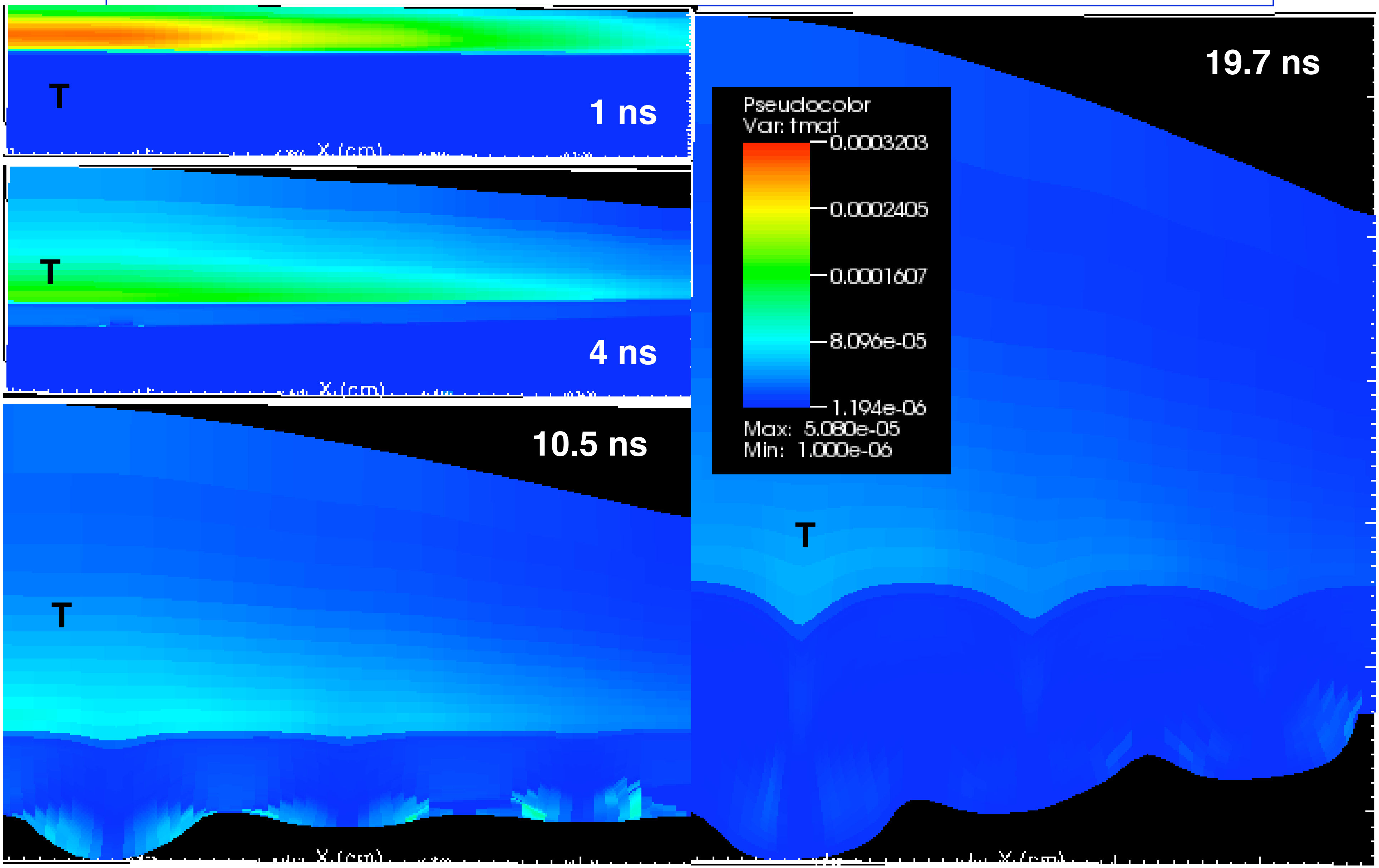


eV

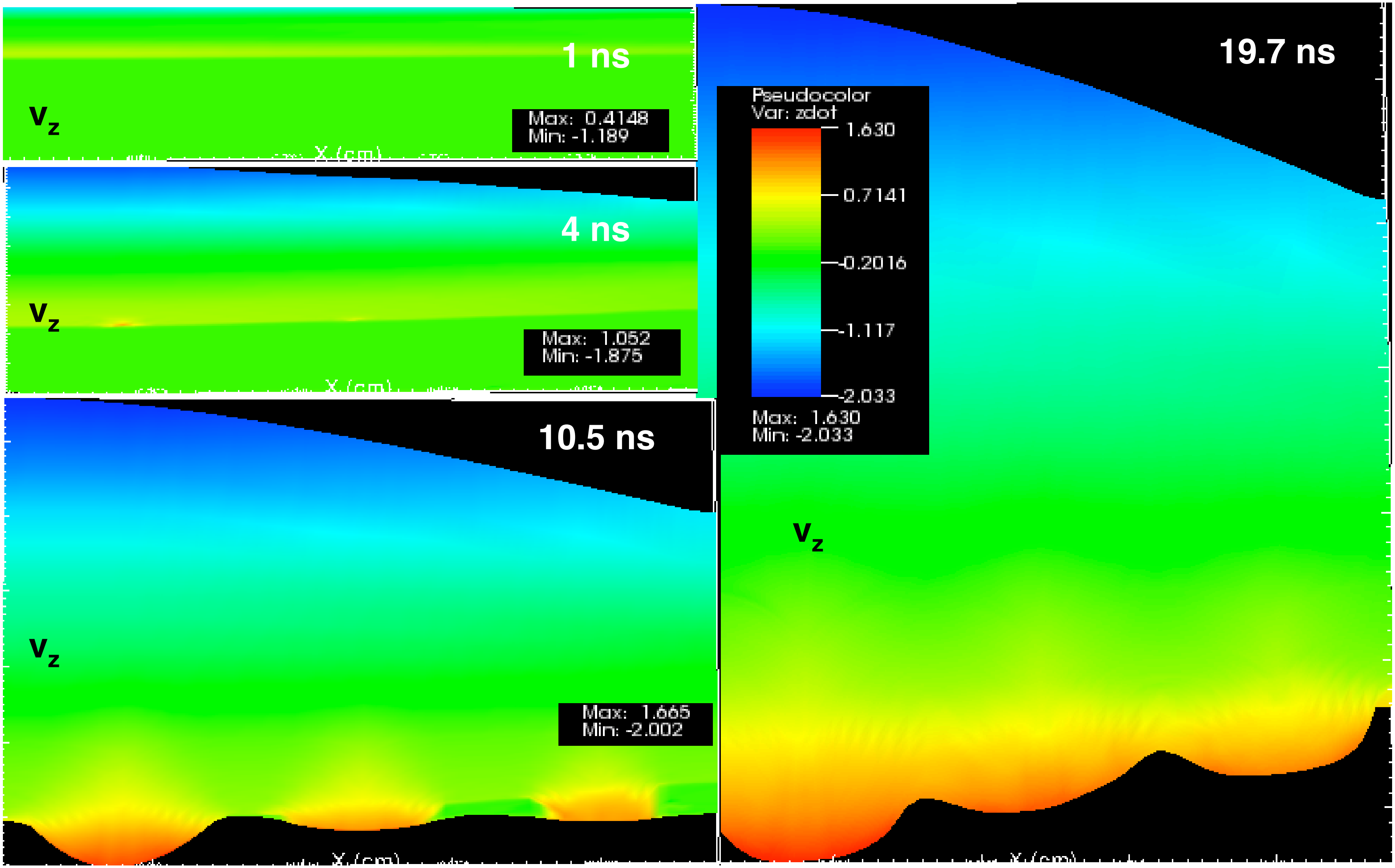
Density evolution -- Li - based NDCX II



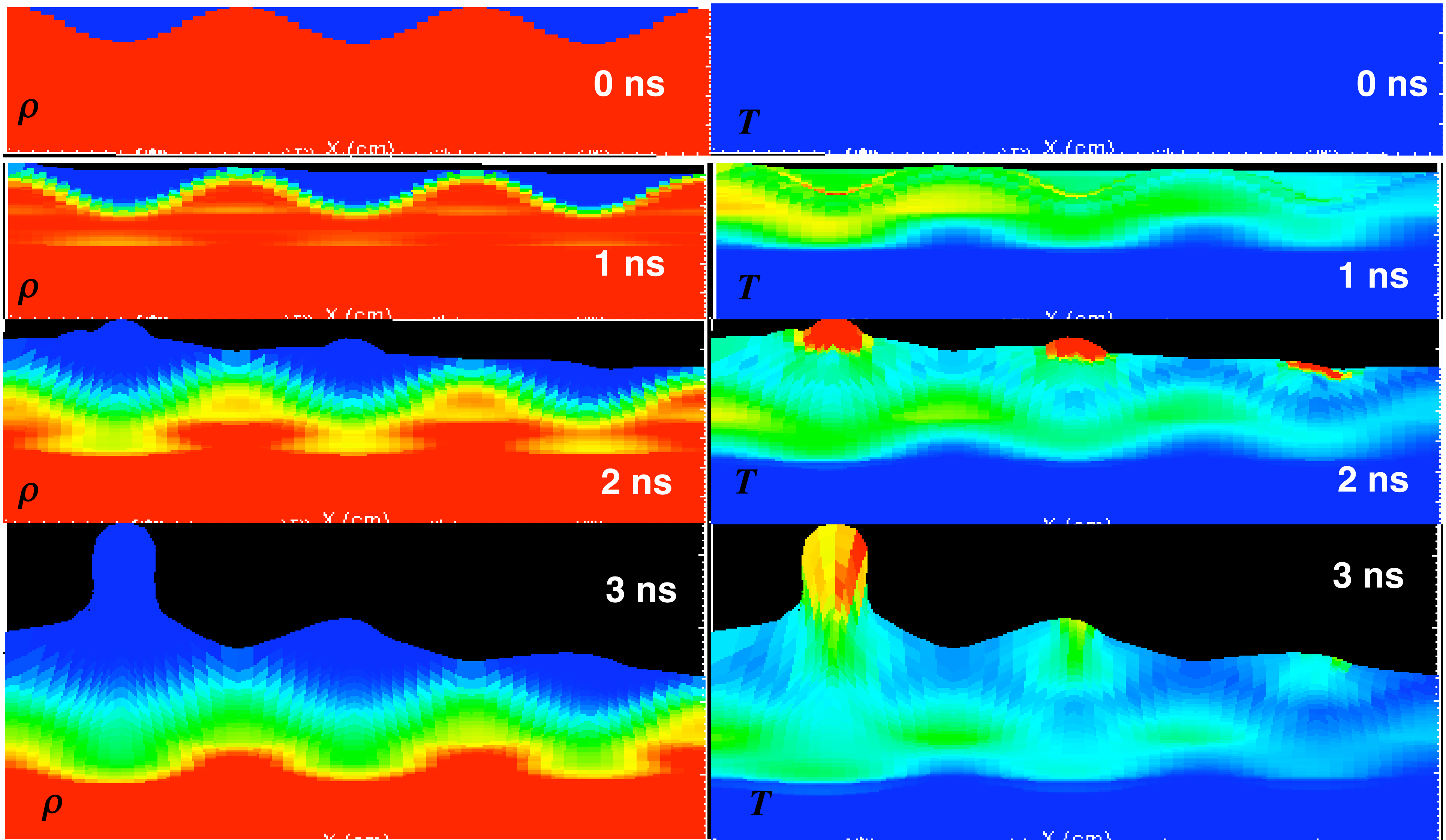
Temperature -- Li - based NDCX II



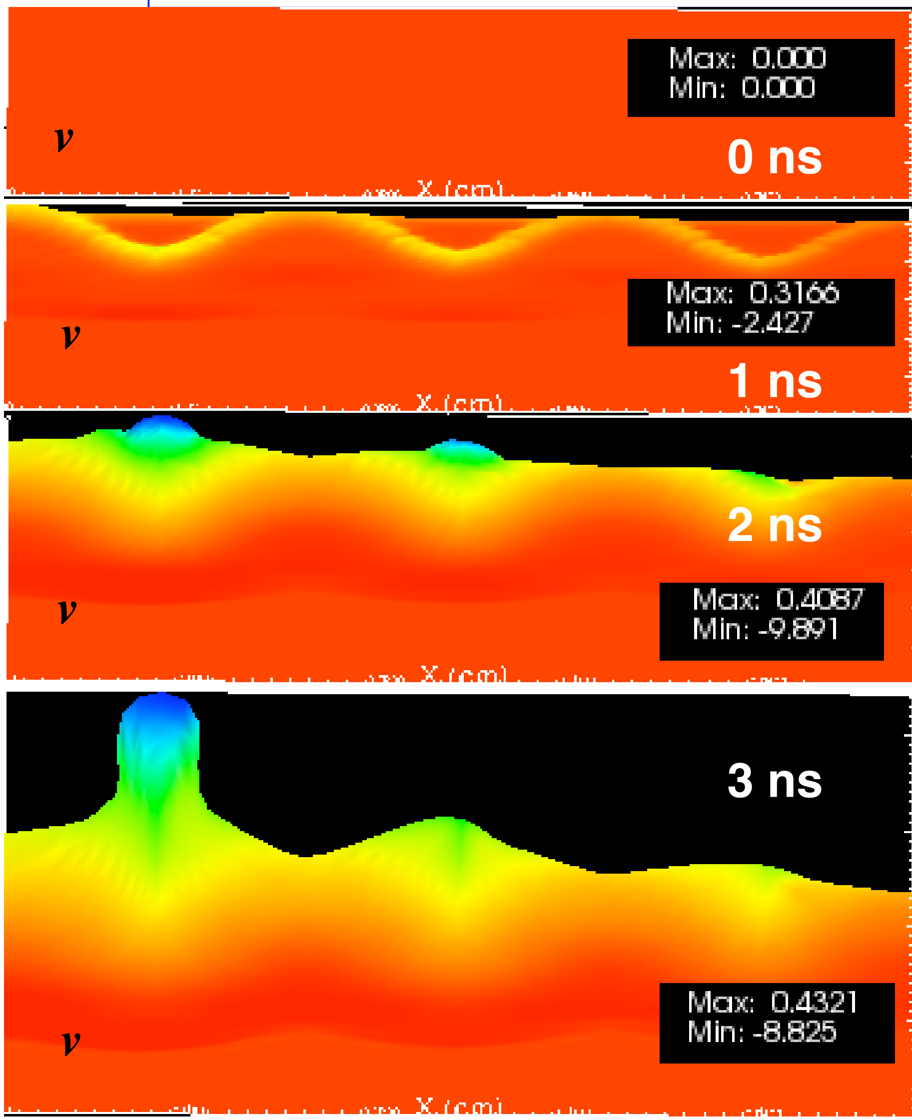
Velocity -- Li - based NDCX II

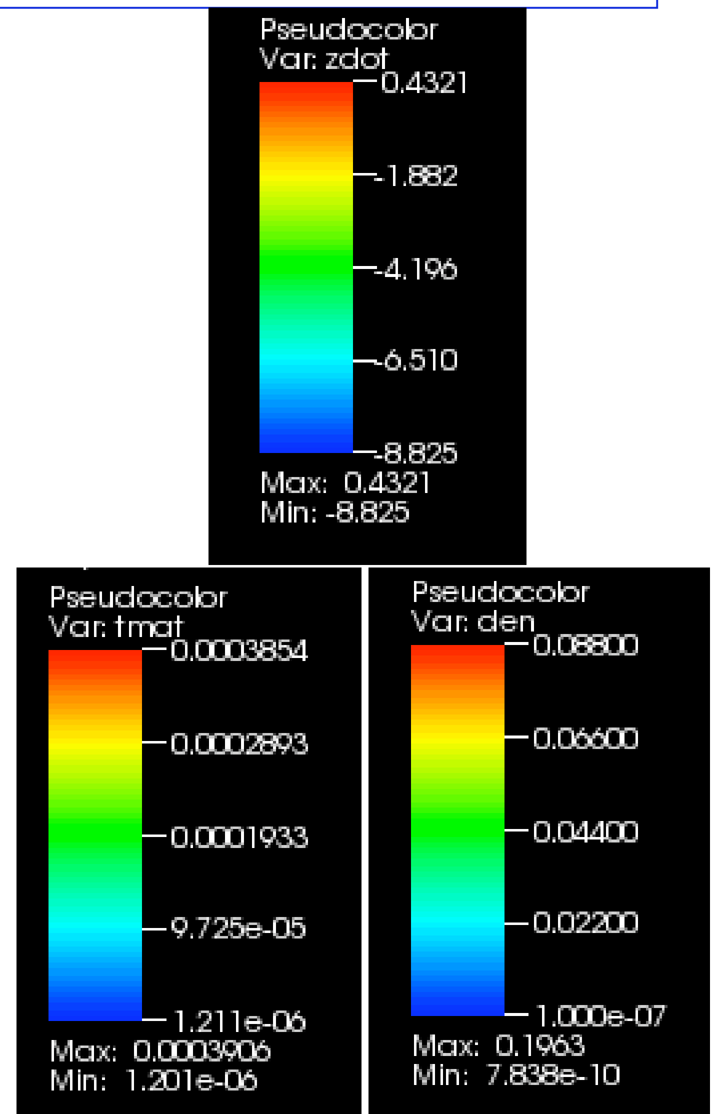


Density and Temperature -- Ablator-side ripple -- Li NDCX II



Velocity -- Ablator-side ripple -- Li NDCX II





Conclusion

Current Li accelerator design will create interesting temperatures and densities for WDM experiments.

If Li source fails, then K source experiments can also create an interesting temperature regime for WDM experiments, although uniformity will not be as originally envisioned.

RT experiments require rapid foil acceleration. Current Li design may not have sufficient flux for H slab acceleration.

Experiments involving interaction of beam with blowoff could be carried out on NDCX II (but needs definition).