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Simple Particle Kinematics in the "Snowplow Mode" of a Pulseline Ion Accelerator

1. Introduction

In the normal operating mode of the Pulseline Ion Accelerator (i.e., in the acceleration sections following the load and fire injector) the ions are injected during the traveling wave voltage ramp with a velocity close to synchronism with the circuit speed. The ion bunch length is less than the voltage ramp length so the ion bunch can "slide up and down" the voltage ramp to maximize the energy gain in that section (see for example the untapered pulseline discussion in my viewgraphs of 8-12-04, "Update on Pulseline Concept for HEDP").

The "snowplow mode" of a helix pulseline has been proposed as an alternative to the resistive column in a load and fire injector. In a load and fire injector, ions from an accel-decel diode are loaded into a solenoid transport system to get as high an initial line charge density as possible. In the resistive column version, an axially uniform accelerating electric field is turned on after the ion bunch is loaded, in a timescale short compared to the ion transit time through the system. In addition to the mean energy gain, a velocity "tilt" is also imparted to the ion bunch because the head of the bunch travels a shorter distance through the resistively-graded accelerating column than the tail.

In the traveling wave "snowplow" mode of a pulseline, a voltage ramp with the shape shown in Fig. 1 is applied at the input of the helical line after the ion bunch has been loaded into the helix. This ramp waveform then propagates through the loaded (and slowly moving) ion bunch. In contrast to the "normal operating mode", here the ions slide up the moving voltage pulse and onto the flat top (where the acceleration drops to zero). Choosing a circuit velocity much faster than the initial ion velocity can ensure that this happens. The reason we chose to avoid "trapping" the ions in this load and fire scenario is that orbit overtaking and mixing would result from the trapping and this would likely have a deleterious effect on the longitudinal emittance.

As we will show, the moving voltage ramp accelerates the ions and it also acts to compress the bunch length. Note that there is no requirement in the snowplow mode for the bunch length to be less than the voltage ramp length. Note also that with a long enough flat top on the drive voltage waveform, the ions will <u>all</u> pick up an additional energy of +Vo at the exit where the pulseline is terminated in its characteristic impedance.

In this note, simple (zero space charge) trajectories of the ions with the idealized voltage waveform shown in Fig. 1 are derived. The results are useful as zero-order design equations for a snowplow injector, and as analytic checks on more complete computer simulations.

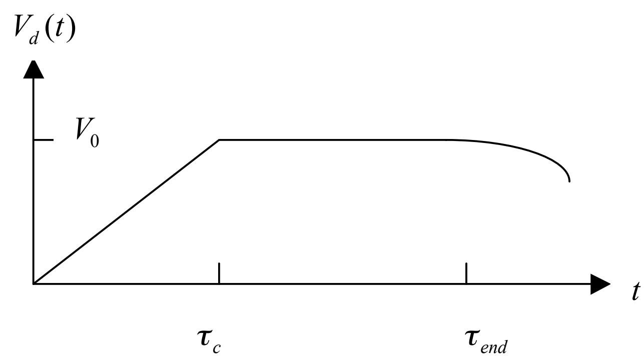

Figure 1. Drive voltage waveform applied at the input of the helix.
(Ion bunch has been loaded prior to t = 0.)

2. Ion Kinematics

With the voltage waveform in Fig. 1 applied at the input end, the traveling wave propagating down the helix has a constant axial electric field during the ramp portion equal to

$$E_z = V_0 / l_c \tag{1}$$

where the ramp length l_c is the ramp duration τ_c times the circuit propagation velocity v_c. The trajectory of a "typical" ion located initially at $z = z_o$ and moving with an initial velocity v_{bi} is illustrated in Figure 2. In the figure, the axial coordinate is normalized as $Z = z / l_c$ and time is normalized as $T = t / \tau_c$, so the propagation of the circuit wave has a slope of unity.

Neglecting the space charge fields, the solution for the particle trajectory in Lagrangian coordinates is elementary. Up to the time t_1 when the leading edge of the accelerating field reaches the ion, it moves with the constant initial velocity,

$$z(z_0, t) = z_0 + v_{bi} t \tag{2}$$

While the ion is in the acceleration region, it has constant acceleration so

$$z(z_0,t) = z_0 + v_{bi}t + \frac{qE_z}{2m}(t-t_1)^2 \qquad (3)$$

Beyond the point where the ion drops behind the propagating acceleration region (at $t = t_2$), it again moves with a constant velocity v_{bf} that can be calculated from Eq. (3).

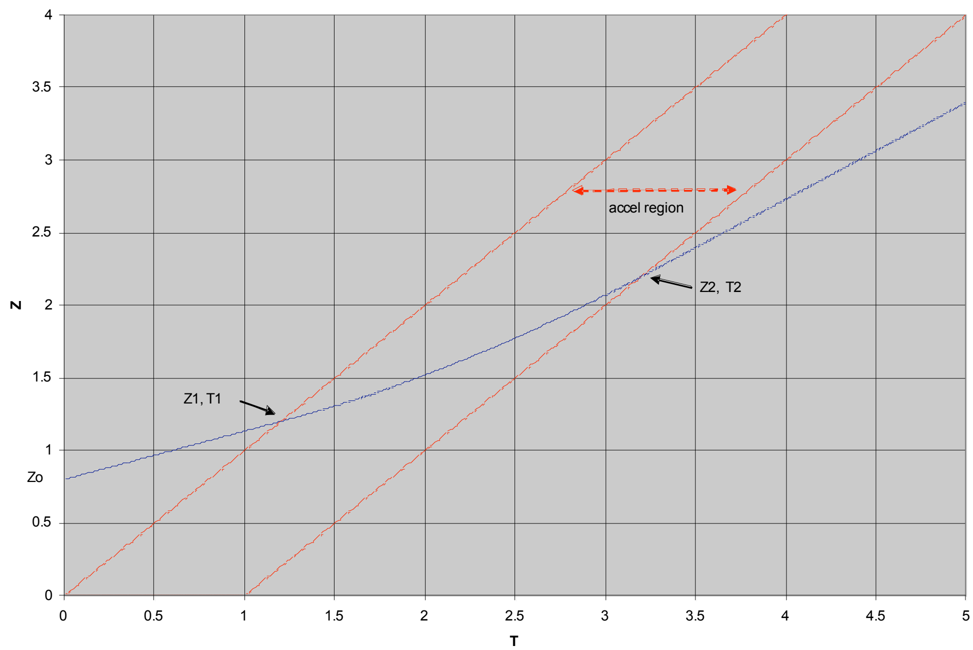

Figure 2 Trajectory of a typical ion loaded into the helix.

The algebra remaining is to calculate the intersection points with the propagating wave (z_1, t_1 and z_2, t_2) in terms of the initial position and velocity of the ion. Readers can fill in these details for themselves; the results are summarized below.

The following are a useful set of normalized parameters:

$$u_0 = \frac{2qV_0}{m(v_c - v_{bi})^2} \qquad (4)$$

$$\alpha = 1 - v_{bi}/v_c \qquad (5)$$

The physical interpretation of the normalized wave voltage amplitude u_0 is that $u_0 = 1$ is the threshold voltage amplitude where an ion with the initial velocity v_{bi} becomes "trapped" and doesn't pass through the accelerating wave.

In terms of these parameters, the intersection points are

$$z_1 = v_c t_1 = z_0 / \alpha \tag{6}$$

$$t_2 - t_1 = \frac{2\tau_c}{\alpha u_0}(1 - \sqrt{1-u_0}\,) \tag{7}$$

$$z_2 = \frac{z_0}{\alpha} + l_c \left\{ \frac{2}{\alpha u_0}(1 - \sqrt{1-u_0}\,) - 1 \right\} \tag{8}$$

The final velocity of the ion after passing through the acceleration region is

$$v_{bf} = v_c (1 - \alpha \sqrt{1-u_0}\,) \tag{9}$$

This velocity is independent of the initial location of the ion, as it should be. All ions drifting with the same initial velocity that pass completely through the acceleration region will therefore exit the helix with the same energy.

The ion bunch will be compressed because the ions located nearest the helix input begin their acceleration before the ions further downstream. To evaluate this bunching in the zero space charge limit, we use the fact that the ions located between z_0 and $z_0 + \Delta z_0$ at $t = 0$ end up located between z and $z + \Delta z$ at a later (fixed) time t. The compression can therefore be calculated from Eq. (3) as

$$\frac{\partial z(z_0, t)}{\partial z_0} = 1 - \frac{\alpha u_0}{2\tau_c}(t - t_1) \tag{10}$$

where we used the relation $t_1 = z_0 / \alpha v_c$. The line charge density in the vicinity of our "typical ion" therefore increases in time as it is accelerated, as

$$\frac{\lambda_b(t)}{\lambda_{bi}} = \left(\frac{\partial z}{\partial z_0}\right)^{-1} \tag{11}$$

The timescale for this compression is of order $2\tau_c / \alpha u_0$. For analyses of the radial dynamics, a comparison of this bunching timescale with the ion cyclotron period is useful. With typical "snowplow injector" parameters ($\alpha u_0 \quad 0.5$, $\tau_c \quad 0.25 \mu\,\mathrm{sec}$) the bunching timescale is of order one microsecond, compared to a cyclotron period of 0.5 microseconds in a 5 T solenoid field. The adjustment of the radial equilibrium will therefore typically be closer to adiabatic than a step function change.

Using Eq. (7) in Eq. (10), the final compression of the line charge density (in the absence of any expansion from the space charge forces) is

$$\frac{\lambda_{bf}}{\lambda_{bi}} = \frac{1}{\sqrt{1-u_0}} \tag{12}$$

A plot of this bunching factor vs. $u_0 \quad V_0 / V_{sync}$ is shown in Fig. 3.

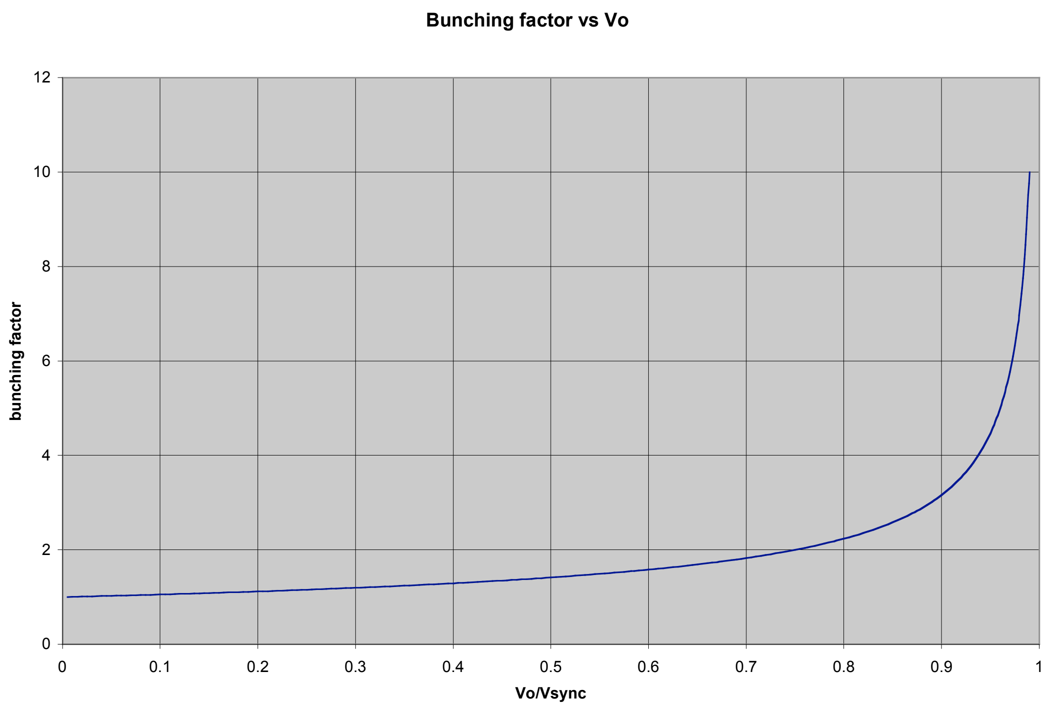

Fig. 3 Final Bunching Factor vs. u_0.

3. Design Considerations

A very attractive feature of the helix traveling wave structure as the first stage in a "load and fire" experiment like the NDCX-1(c) is its flexibility to cover a wide parameter range and a variety of operating modes. We will illustrate this with a couple of examples;

(a) the "full snow-plow mode" where <u>all</u> ions pass through the acceleration region by the end of the system, and

(b) a "gentle gradient mode" where the output ion bunch has a significant velocity tilt similar to the resistive column.

(a) The "Full Snow-plow Mode"

Ion trajectories for a "full snow-plow mode" are illustrated in Fig. 4. The axial coordinate and time are normalized to the circuit ramp length and duration, respectively, as before. The case presented has the dimensionless parameters u_0 = ¾ (final bunching factor of 2) and α = 2/3 (initial ion velocity = 1/3 the circuit velocity).

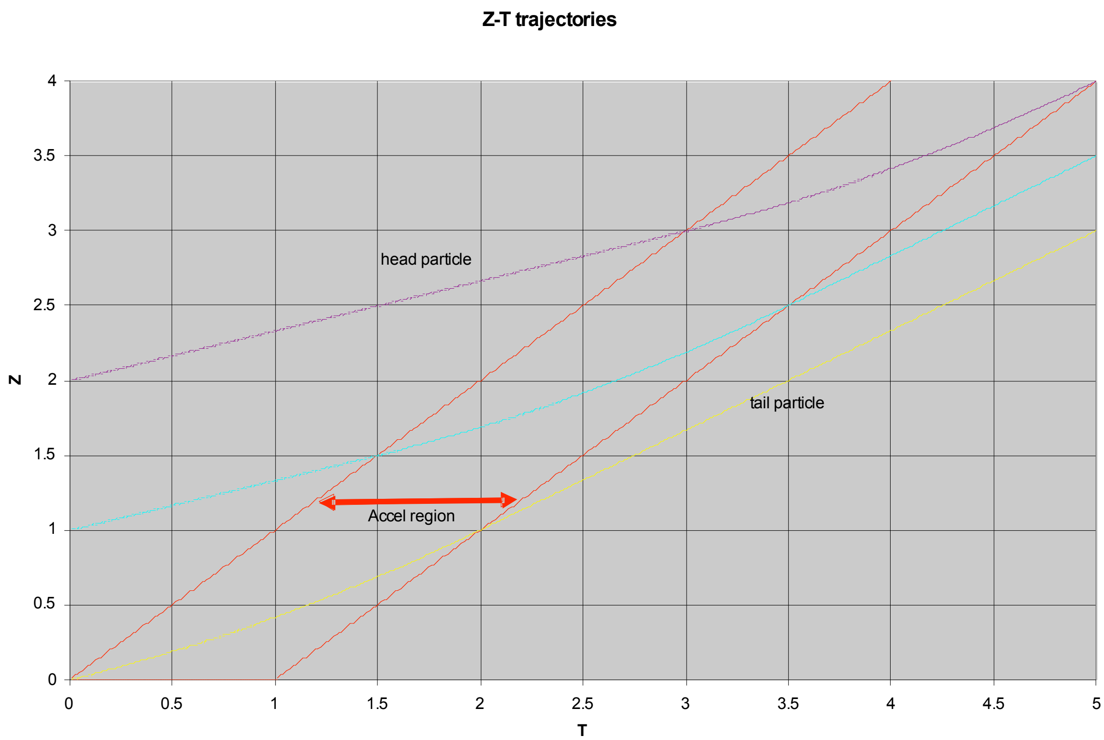

Fig. 4 Ion Trajectories in a "Full Snow-Plow Mode"

The ion bunch initially extends from Z = 0 to Z = 2, where the "head particle" is located. If we chose the end of the propagating region of the helix to be at Z = 4, the head particle will just finish passing through the accelerating (ramp) region at the end. (The helical termination resistor is just beyond Z = 4). Note that the ion bunch length has been compressed by a factor of 2 at the point the head leaves the helix, as predicted by Eq.(12). Note also that at the end of the helix, all the ions exit with the same velocity, namely 2/3 the circuit velocity, as predicted by Eq. (9) for our parameters.

The general condition for the head ion to pass completely through the acceleration region by the end of the helix can be obtained from Eq. (8). The helix propagation length must exceed

$$l_{helix} = \frac{l_{bi}}{\alpha} + l_c \left\{ \frac{2}{\alpha u_0} (1 - \sqrt{1 - u_0}\,) - 1 \right\} \tag{13}$$

where l_{bi} is the initial bunch length.

Going through the termination resistor region, all the ions will gain an additional energy Vo as long as the flat part of the drive voltage is long enough (see Fig 1). The criteria on the minimum duration of the flat part can be derived from the condition that the tail particle reaches the end by

$$\tau_{end} = \tau_c + \left(\frac{\sqrt{1-u_0}}{1-\alpha\sqrt{1-u_0}} \right) \frac{l_{bi}}{v_c} \tag{14}$$

As a specific numerical example that might be applicable to NDCX-1(c), consider $v_c = 1.8x10^6\ m/\sec$, $v_{bi} = 0.6x10^6\ m/\sec$, $V_{sync} = 300\ keV$, $V_0 = 225\ keV$ (75 keV initial ion energy). Picking l_c = 25 cm, the initial ion bunch length is 50 cm and the helix propagation length is a meter (input transformer coupler and output termination resistor adds to this length). The ramp duration τ_c = 140 ns, and the required flat part is $\tau_{end} - \tau_c$ = 210 ns.

At the end of the helix, the ion energy is 300 keV, which increases to 525 keV as the ions cross the termination resistor region. Note also that as the ions speed up, the pulse expands axially by a factor of 1.32 (to ~ 33 cm).

(b) A "Gentle Gradient Mode"

The "full snow-plow mode" with significant bunching ratios will undoubtedly have its limits in emittance preservation, and it would be nice if we could experimentally explore parameter ranges with more gentle acceleration and compression, similar to a resistive "load and fire" column.

As an illustration, consider the same parameters as before with the *only change* being the *drive waveform*, namely a ramp of one meter length (~ 556 ns duration) to the same peak voltage Vo = 225 keV. (As we will see, the flat portion of the drive waveform following the peak is not needed in this case).

Normalized trajectories are shown in Fig. 5, where now the normalized helix length and the ramp length are = 1, and the initial ion pulse extends from Z = 0 to Z = 0.5.

Trajectories with a resistive column that erects a linear voltage profile at T = 0 going from + Vo at Z = 0 to zero at Z = 1 (meter) are also shown. Note that the trajectory of the "tail particle" is the same in both cases, since it sees the same voltage ramp (Ez field) as it moves through either one meter long system. The other particles are not accelerated as much in the traveling wave (TW) system, however, since the traveling wave has to propagate some distance before the acceleration ramp reaches them.

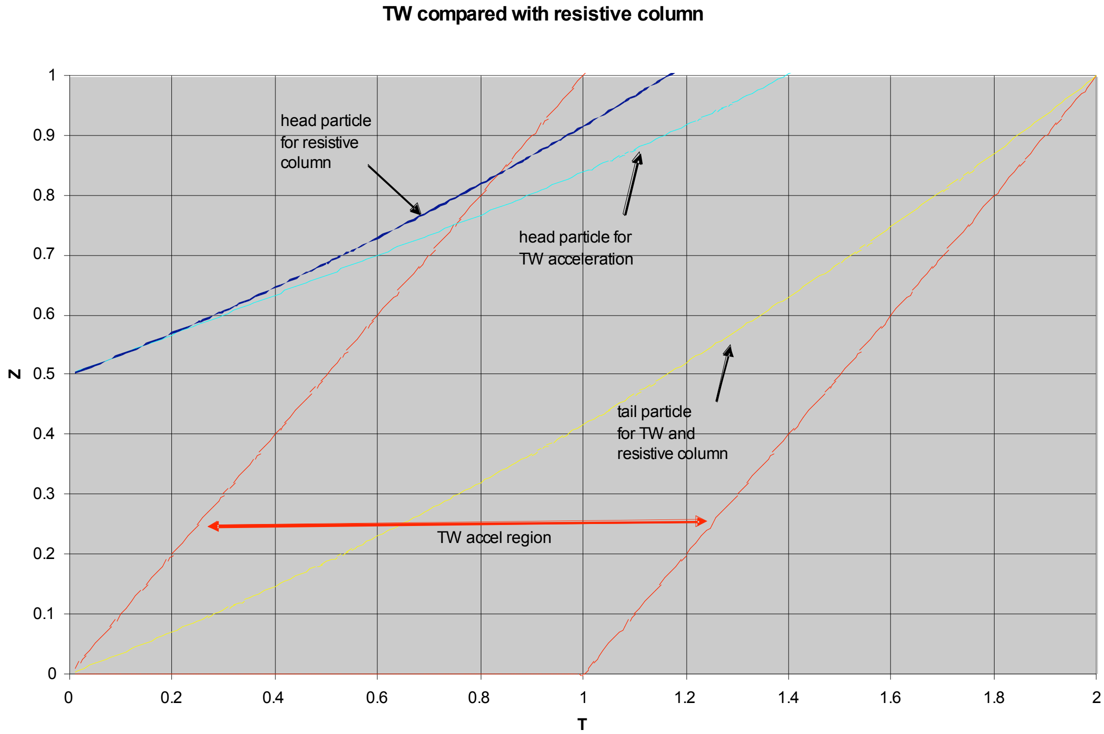

Fig. 5 Ion Trajectories in a "Gentle Gradient Mode"

With the TW system, at the point where the head particle is at the end of the helix, the ion pulse has been compressed by a factor of about 1.35 (to a length of about 37 cm). As the ions pass through the termination resistor region, they are accelerated further since the voltage traveling wave has already arrived at that end (and reached its maximum amplitude by the time the tail particle arrives).

The energies attained by the head and tail particles at the end of the helix, and after the matched termination region, are shown in the table below (the values for the head are estimates assuming ~ 5 cm transit region). The same quantities are given for the resistive column with the input end pulsed to the same voltage as the helix.

	TW - End of helix	TW- through termination region	Resistive column
Head particle	~ 135 keV	~ 360 keV	75 + 113 = 188 keV
Tail particle	300 keV	525 keV	75 + 225 = 300 keV