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An implicit "drift-Lorentz" mover for plasma and beam simulations

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In order to efficiently perform particle simulations in systems with widely varying magnetization, we developed a drift-Lorentz mover, which interpolates between full particle dynamics and drift kinetics in such a way as to preserve a physically correct gyroradius and particle drifts for both large and small ratios of the timestep to the cyclotron period. In order to extend applicability of the mover to systems with plasma frequency exceeding the cyclotron frequency such as one may have with fully neutralized drift compression of a heavy-ion beam we have developed an implicit version of the mover. A first step in this direction, in which the polarization charge was added to the field solver, was described previously. Here we describe a fully implicit algorithm (which is analogous to the direct-implicit method for conventional particle-in-cell simulation), summarize a stability analysis of it, and describe several tests of the resultant code.

1. Introduction

There are a number of physical systems for which simulation requires the ability to follow charged particles through regions of both strong and weak (or no) magnetic fields. Examples include electron clouds in positive-charged particle accelerators with localized focussing magnets and plasmas in cusp and multipole magnetic fields. To efficiently describe these systems, the drift-Lorentz mover [1] was developed; this mover interpolates between a full-ion (Boris) particle push and drift kinetics in such a way that proper drifts, parallel dynamics and gyroradius are maintained for timesteps both large and small compared to the gyro period.

The mover as originally described was limited to explicit implementation, which, to avoid numerical instability, required that the plasma period (inverse of the plasma frequency) be longer than the simulation timestep for species being simulated. For simulation of a heavy-ion beam passing through a neutralizing plasma, this can be an unacceptable restriction. The same can be said for application to almost any high-density plasma. A first step toward removal of this limitation was described in Ref. [2], where the polarization response was added to the Poisson field solve, thereby introducing partial implicitness into the perpendicular (to the magnetic field) dynamics.

Here we summarize a fully implicit extension of the drift-Lorentz mover and its stability analysis, as well as verification tests and a first application to a problem of interest to heavy-ion fusion science, namely simulation of a fast Faradat cup (FFC) diagnostic to measure beam current in the presence of a neutralizing background. More detail on the algorithm and its implementation and testing can be found in a separate publication [3].

2. Algorithm

The algorithm developed is analogous to that previously developed for direct-implicit particle simulation with full particle dynamics [4]. In the conventional scheme (restricted here to electrostatics), one updates velocities replacing the present-time electric field by an average of a retarded and an (a priori unknown) advanced electric field. A predictor step is taken in which particles are advanced with the advanced electric field set to zero; then a position increment due to the advanced field, linearized in that field, is calculated, and its charge density inferred. The resulting term, linear in the advanced potential, is moved to the left-hand side of the Poisson (field) equation, where it contributes to an effective susceptibility. The resulting equation is solved for the advanced electric field, and that is then used to construct corrected particle positions.

In the drift-Lorentz mover, particles are advanced with an effective velocity that is an interpolation between the full particle velocity and the drift velocity. The drift velocity that enters in

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advancing particles from step n to step $n+1$ is that at step $n+\frac{1}{2}$, which is proportional to the electric field at that half step; we represent this as an average of the fields at the *current* and advanced steps, and as in the direct-implicit method, calculate predicted positions neglecting the advanced field but then include their contribution through an effective susceptibility in the field equation. The susceptibility now includes the charge resulting from the (interpolated portion of the) drifts at the advanced timestep. As with direct implicit, we calculate corrected particle positions once the (approximate) advanced field is known.

The resulting algorithm is, clearly, implicit in the drifts as well as the Lorentz part. Curiously it in general includes a contribution to the electric drift (in contrast to the case with pure drift kinetics), because the interpolated fraction of the electric drift in general differs for electrons and ions. The method only requires one field solve per timestep, and so is faster than the partially implicit method reported in Ref. [2], where it was found that two corrector steps and three field solves per timestep were required for stability. As with the direct-implicit method, the presence of a large susceptibility suppresses noise in the field solve, and hence leads to reduced requirements on particle number.

A stability analysis for the partially implicit and new schemes, done in the cold plasma limit for uniform plasma in a uniform magnetic field, predicts that the partially implicit scheme should be unstable with a single corrector step (as observed), while the new, fully implicit, scheme is predicted to be stable.

3. Tests

The fully implicit algorithm has been implemented in the WARP code, and several verification tests have been performed. First, the predictor–corrector particle mover (with the relevant combinations of current, advanced, and retarded positions) was tested following orbits in a quadrupole magnetic field, as was done for the original drift-Lorentz mover as described in Ref. [1]. Excellent agreement was found for non-chaotic orbits, and chaotic orbits were correctly identified, much as in the earlier work.

Second, the full implicit scheme was tested by simulating the Buneman instability (two-stream instability of electrons passing through ions) with a magnetic field added in the direction of the relative velocity. The simulation is done in $x-z$ geometry (z is the direction of the relative velocity), with periodic boundary conditions in z and either periodic or bounded (reflecting particles, zero potential) in x. For doubly periodic boundary conditions one can obtain a textbook (one-dimensional) instability. We choose a timestep δt large compared to the inverse of the electron plasma frequency ω_{pe} (40×) and electron cyclotron frequency ω_{ce} (10×). The simulations show an initial growth rate in excellent agreement with the analytically derived result, followed by a phase of faster growth (in which multiple spatial harmonics participate, suggesting a nonlinear instability). For the transversely bounded case there is not a simple analytic theory, but the results appear to be converged with respect to resolution and particle number. We attempted to compare the drift-Lorentz scheme with the full-orbit direct-implicit scheme (with the Boris mover), ignoring the large value of $\omega_{ce}\delta t$, as there are instances where the latter can be expected to give results with adequate accuracy (particularly if $v_t\delta t/\delta x$ is less than or of order unity, where v_t is the thermal speed and δx is the grid spacing transverse to the magnetic field). However, for our parameters, including ones for which $v_t\delta t/\delta x\sim 1$, we found the full-orbit implicit scheme to be unstable (even with no relative velocity between electrons and ions).

Our third test—one of interest for heavy-ion fusion science applications—is the simulation of the plasma at the entrance to a multi-pinhole fast Faradat cup detector. In this detector, two electrode plates are placed in front of the collector plate; each electrode has a set of small holes; the holes in the mid-plate are aligned with and larger than those in the front-plate. The front-plate is grounded; the mid-plate is negatively biased, and the collector is positively biased. The purpose of the two plates is to shield the collector from the presence of plasma. We have simulated this problem in $x-z$ geometry (the holes are replaced by slits in the $x-y$ plane) with the implicit drift-Lorentz and full-orbit implicit schemes, and also with an explicit simulation with 20-times smaller timestep (to resolve the plasma and cyclotron frequencies). For parameters typical of the FFC device used on the NDCX experiment, we find that all three simulations agree well in distribution of plasma electrons, plasma ions, and secondary electrons, and (apart from initial transients) in the current collected on the front-plate; in particular, all predict that the plates are effective in holding the plasma electrons and ions away from the collector. There are some subtle differences with the explicit simulation which are reduced by increasing the number of particles in the latter; hence we believe that the differences are primarily the result of the increased noise in the explicit simulation. We have also performed an implicit simulation with no magnetic field, and find only small differences in the particle distributions and currents. While in this case the drift-Lorentz and full-dynamics implicit simulations agree and both appear to be stable, the parameters are only moderately different from those used for the Buneman tests where the full-dynamics implicit simulation was found to be unstable.

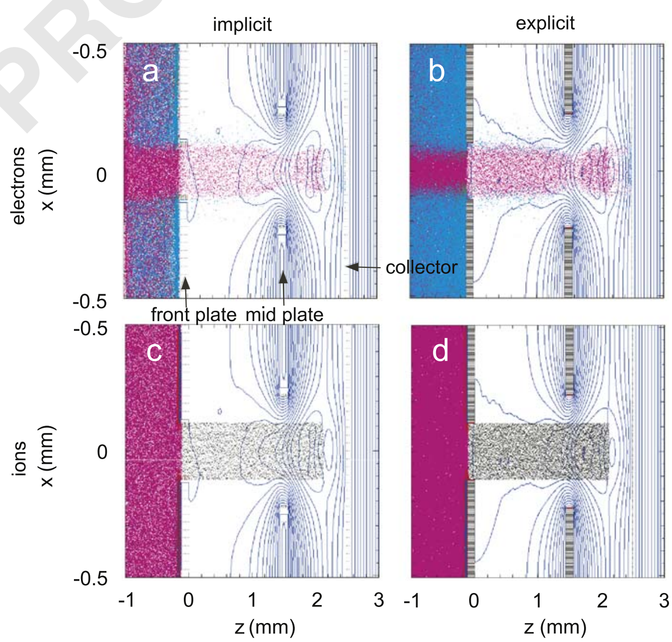

Fig. 1. Simulations of the fast Faradat cup in x–z geometry. The beam and plasma enter from the left. The front-plate (grounded) and its slit are centered at $z=-0.0625$ mm; the mid-plate (at -200 V) is centered at $z=1.5$ mm; the collector (at $z=200$ V) is centered at $z=2.5$ mm. The implicit simulation has a timestep $\omega_{pe}\delta t=6$; the explicit simulation has $\omega_{pe}\delta t=0.45$ and four times as many particles. The implicit electron and ion simulations are labeled as (a) and (c), respectively; the explicit simulations are labeled as (b) and (d), respectively. The color coding is as follows: magenta, plasma electrons and ions; black, ion beam; red, primary emitted electrons and ionized hydrogen; cyan, secondary electrons; green, ionized electrons (mostly hidden behind secondaries); blue, hydrogen gas. The blue lines in all plots are electrostatic potential contours. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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We have also applied the implicit drift-Lorentz code to the simulation of the FFC detector with full physics: beam propagation, gas and primary ion desorption and propagation, and ionization of the gas. For times of the order of the ion beam transit time, comparison of implicit simulation and explicit simulation with 4 times more particles and 13 times smaller timestep show quite good agreement for both the electron and ion distributions, as shown in Fig. 1, including the occurrence of a pinching of the electron column in the vicinity of the mid-plate. The increased number of particles is required in the explicit simulation to clearly identify the pinching. Not shown is an explicit simulation with timestep as large as for the implicit simulation; as expected, it quickly develops numerical instability, leading to large potentials which rapidly purge most of the electrons.

If we run these simulations for longer times, long compared to the beam transit time, substantial differences between explicit and implicit simulations appear in both electron and ion distributions, but only if the simulations include ionization of emitted gas; good agreement is obtained if we include electron emission from walls but not gas. We also observe substantial differences between explicit simulations with gas emission that do and do not resolve the Debye sheath near the collector. The implicit simulations do not (and cannot) resolve the sheath, and our speculation is that this is the source of the differences, a point we will explore in the future via incorporation of an analytic sheath model.

We conclude that the implicit drift-Lorentz mover is a useful approach for simulating self-consistent dynamics of high-density plasmas and beams in high-density plasmas. Initial testing indicates that the method is accurate (except where unresolved sheath effects are likely important) and can have advantages in terms of accuracy, stability, or both compared to an implicit full-ion-dynamics scheme in the limit where both $\omega_c \delta t$ and $\omega_p \delta t$ exceed unity. Further tests and applications will be undertaken.

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References

[1] R.H. Cohen, A. Friedman, M. Kreeff Covo, et al., Phys. Plasmas 12 (2005) 056708.

[2] R.H. Cohen, A. Friedman, D.P. Grote, J.-L. Vay, Nucl. Instr. and Meth. A 577 (2007) 52.

[3] R.H. Cohen, A. Friedman, D.P. Grote, J.-L. Vay, J. Comput. Phys., to be submitted.

[4] A. Friedman, A.B. Langdon, B.I. Cohen, Comments Plasma Phys. Cont. Fusion 6 (1981) 225.

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