Recent Advances in the Physics of Collective Excitations and Transverse Compression Dynamics in the Paul Trap Simulator Experiment*

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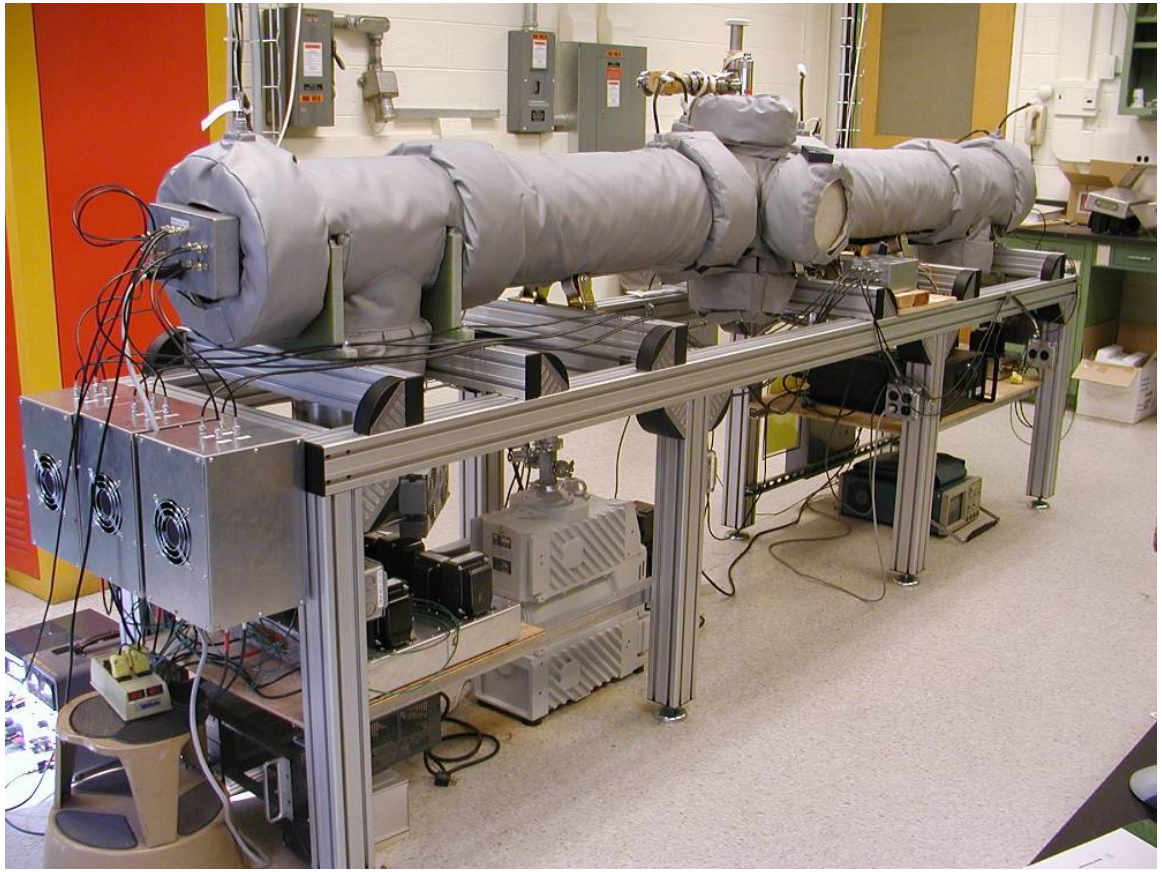
Outline

- Introduction/Motivation
- The Paul Trap Simulator Experiment (PTSX)
- Collective Modes
- Transverse Compression
- Summary

PTSX Simulates Nonlinear Beam Dynamics in Magnetic Alternating-Gradient Systems

- PTSX simulates, in a compact experiment, the transverse nonlinear dynamics of intense beam propagation over large distances through magnetic alternating-gradient transport systems.
- PTSX is a beam-frame experiment.
- The equations of motion for transverse particle dynamics have the same form.



Other Intense-Beam Studies, Paul-trap-based and otherwise:

Okamoto and Tanaka

Drewsen et al.

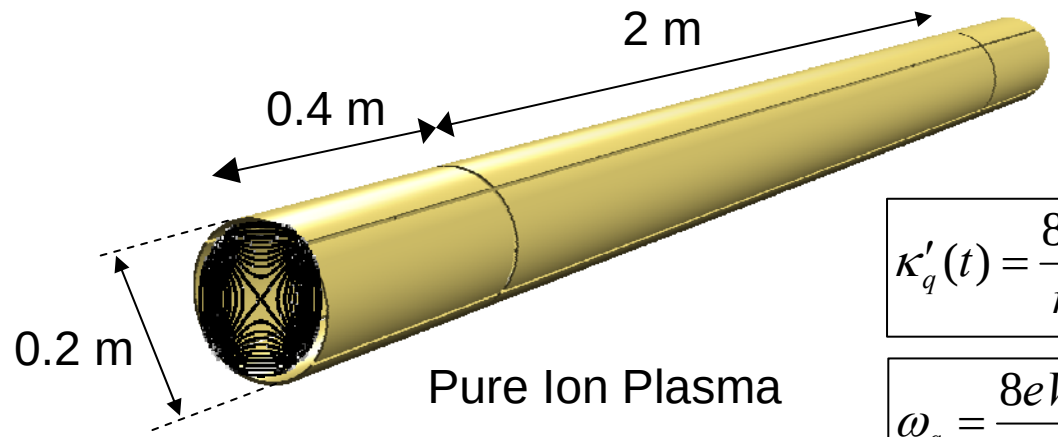
Kishek et al.

Scientific Motivation

As self-field effects become important, it is important to develop an understanding of:

- Beam mismatch and envelope instabilities;
- Collective wave excitations;
- Chaotic particle dynamics and production of halo particles;
- Mechanisms for emittance growth;
- Compression techniques; and
- Effects of distribution function on stability properties.

PTSX Configuration – A Cylindrical Paul Trap



$$e\phi_{ap}(x,y,t) = \frac{1}{2}\kappa_q'(t)(x^2 - y^2)$$

$$\kappa_q'(t) = \frac{8eV_0(t)}{m\pi r_w^2}$$

sinusoidal in this work

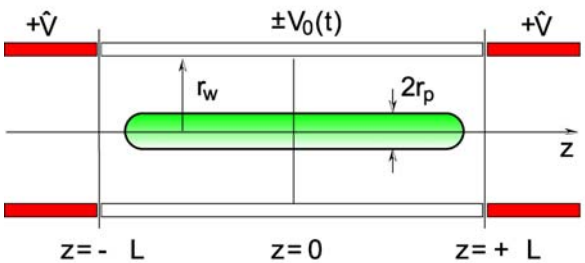
$$\xi = \frac{1}{2\sqrt{2}\pi}$$

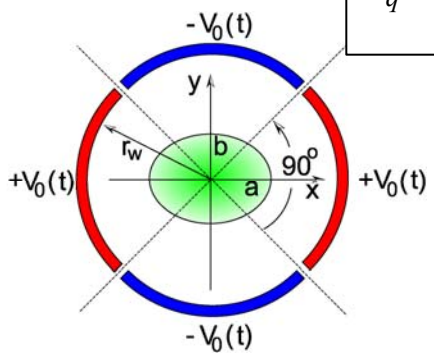
for a sinusoidal waveform $V(t)$.

$$\omega_q = \frac{8eV_{0\,\max}}{m\pi r_w^2 f}\xi$$

$$\xi = \frac{\eta\sqrt{3-2\eta}}{4\sqrt{3}}$$

for a periodic step function waveform $V(t)$ with fill factor η.





$$\sigma_v^{sf} = \frac{\omega_q}{f} \propto \frac{V_{0\ \max}}{f^2}$$

Plasma length	2 m	Maximum wall voltage	~ 400 V
Wall radius	10 cm	End electrode voltage	< 150 V
Plasma radius	~ 1 cm	Frequency	< 100 kHz
Cesium ion mass	133 amu	Pressure	5×10^{-9} Torr
Ion source grid voltages	< 10 V	Density	10^5-10^6 cm^{-3}

Transverse Dynamics are the Same Including Self-Field Effects

$\boldsymbol{B}_q^{foc}(\boldsymbol{x}) = B_q'(z)\left(y\hat{\boldsymbol{e}}_x + x\hat{\boldsymbol{e}}_y\right)$ $\boldsymbol{F}_{foc}(\boldsymbol{x}) = -\kappa_q(z)\left(x\hat{\boldsymbol{e}}_x - y\hat{\boldsymbol{e}}_y\right)$ $\kappa_q(z) = \dfrac{ZeB_q'(z)}{\gamma m\beta c^2}$	Quadrupolar Focusing	$e\phi_{ap}(x,y,t) = \dfrac{1}{2}\kappa_q'(t)(x^2 - y^2)$ $\kappa_q'(t) = \dfrac{8eV_0(t)}{m\pi r_w^2}$
$\psi = \dfrac{Ze}{\gamma m\beta^2 c^2}\left[\phi(x,y,s) - \beta A_z(x,y,s)\right]$	Self-Forces	usual $\phi_{\text{self}}(x,y,t)$
$\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)\psi = -\dfrac{2\pi K}{N}\int dx'dy'\, f_b$	Field Equations	Poisson's Equation

Vlasov Equation

$$\left\{\frac{\partial}{\partial s} + x'\frac{\partial}{\partial x} + y'\frac{\partial}{\partial y} - \left(\kappa_q(s)x + \frac{\partial\psi}{\partial x}\right)\frac{\partial}{\partial x'} - \left(-\kappa_q(s)y + \frac{\partial\psi}{\partial y}\right)\frac{\partial}{\partial y'}\right\}f_b = 0$$

Transverse Dynamics are the Same Including Self-Field Effects

If…

- Long coasting beams
- Beam radius << lattice period
- Motion in beam frame is nonrelativistic

Then, when in the beam frame, both systems have…

- Quadrupolar external forces
- Self-forces governed by a Poisson-like equation
- Distributions evolve according to nonlinear Vlasov-Maxwell equation

→ Ions in PTSX have the same transverse equations of motion as ions in an alternating-gradient system ***in the beam frame***.

Force Balance and Normalized Intensity *s*

If $p = n\,kT$, then the statement of local force balance on a fluid element can be integrated over a radial density distribution such as,

$$n(r) = n(0)\exp\left[-\frac{m\omega_q^2 r^2 + 2q\phi^s(r)}{2kT}\right]$$

to give the global force balance equation,

$$m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}$$

$$s \equiv \frac{\omega_p^2}{2\omega_q^2} < 1$$

$$\frac{\upsilon}{\upsilon_0} = (1-s)^{1/2}$$

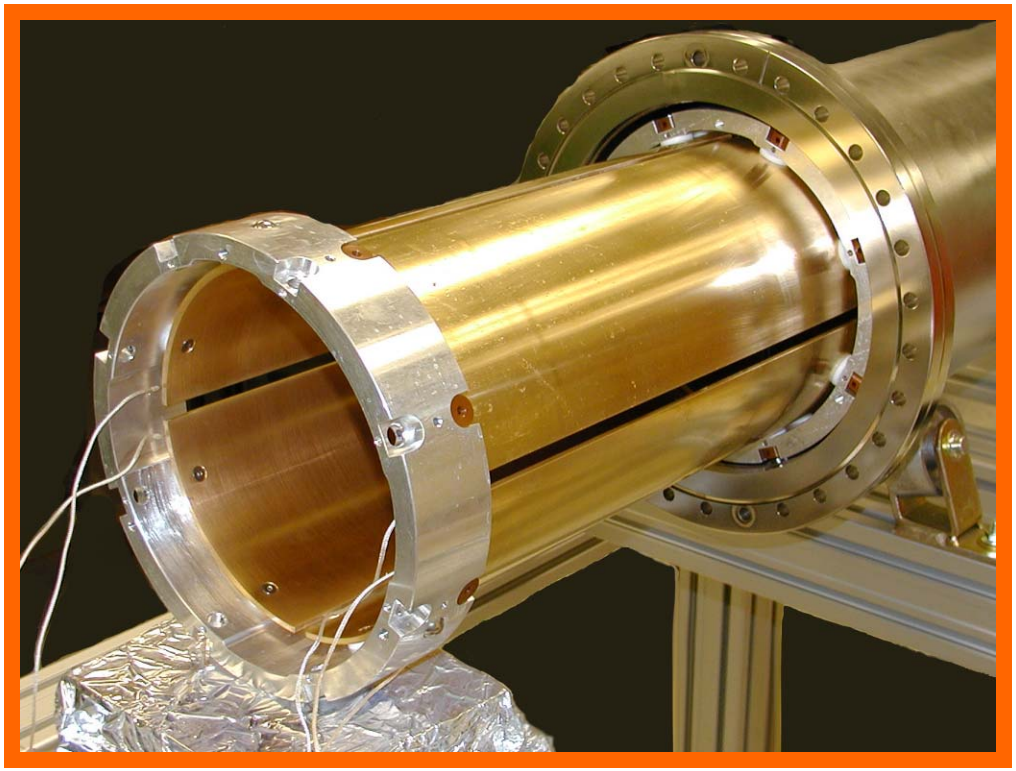
for a flat-top radial density distribution

s	v/v_0
0.1	0.95
0.2	0.90
0.3	0.84
0.4	0.77
0.5	0.71
0.6	0.63
0.7	0.55
0.8	0.45
0.9	0.32
0.99	0.10
0.999	0.03

PTSX-accessible

Electrodes, Ion Source, and Collector

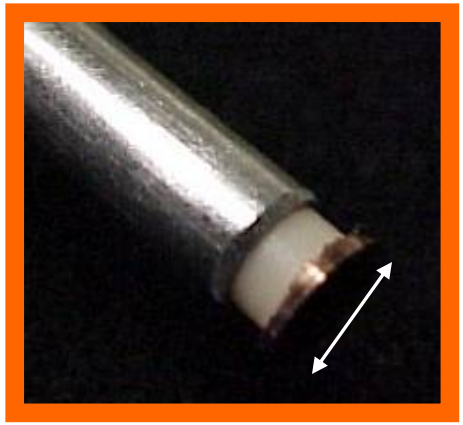
Broad flexibility in applying V(t) to electrodes with arbitrary function generator.



Increasing source current creates plasmas with intense space-charge.

1.25 in

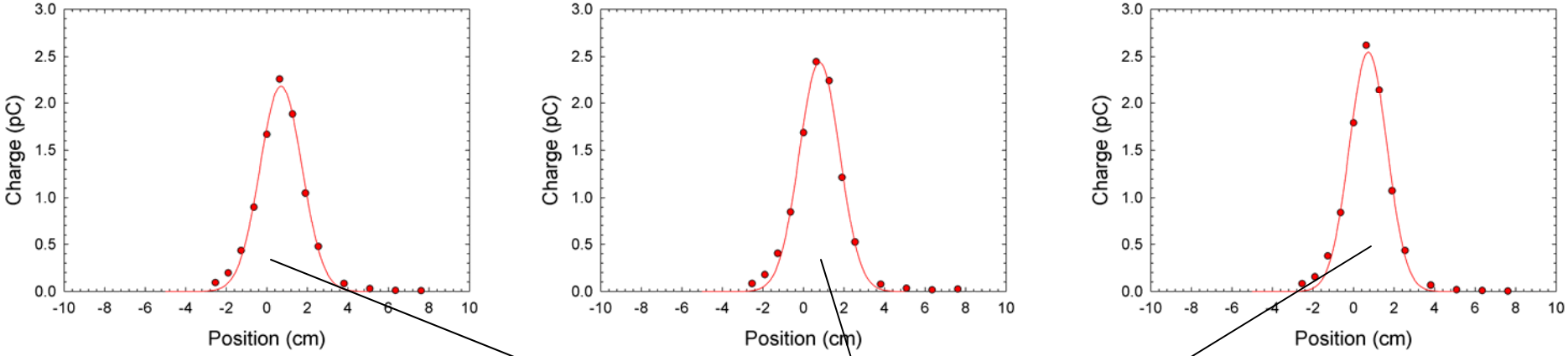
Large dynamic range using sensitive electrometer.

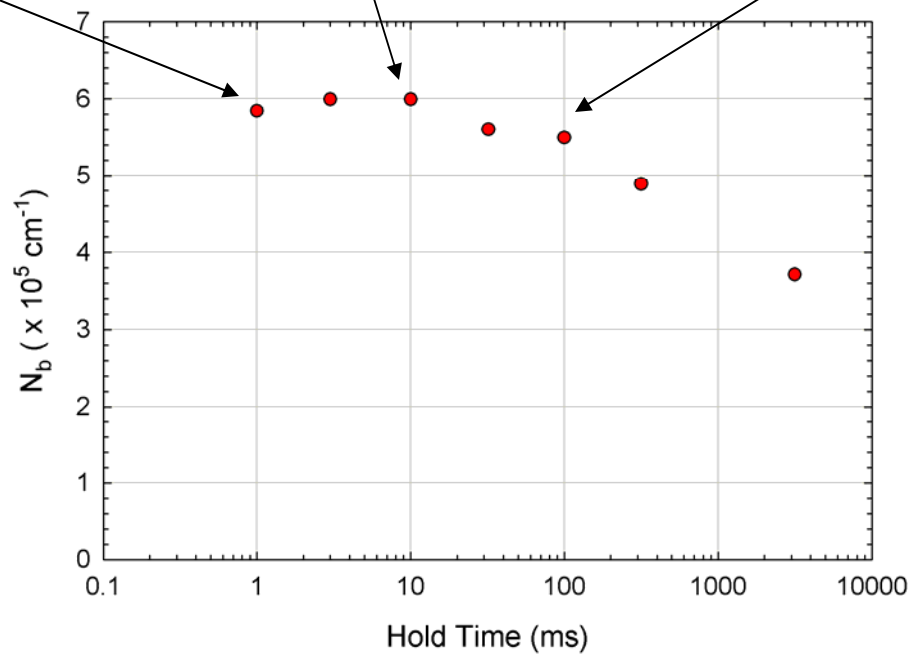


5 mm

Measures average Q(r).

PTSX Simulates Equivalent Propagation Distances of 7.5 km





- At f = 75 kHz, a lifetime of 100 ms corresponds to **7,500 lattice periods**.
- If S is 1 m, the PTSX simulation experiment would correspond to a **7.5 km beamline**.

- $s = \omega_p^2/2\omega_q^2 = 0.18$.
- V_0 = 235 V
  f = 75 kHz
  $\sigma_v = 49^o$

Collective Modes

Can we use measurements of the spectrum of collective modes to infer plasma properties?

m = 0 and *m* = 2 Collective Mode Frequencies can be Derived from KV Envelope Model

$s = \beta ct$

$$\frac{d^2}{ds^2}a(s) + \kappa_x(s)a(s) - \frac{2K_b}{a(s)+b(s)} = \frac{\epsilon_x^2}{a^3(s)}$$

$$\frac{d^2}{ds^2}b(s) + \kappa_y(s)b(s) - \frac{2K_b}{a(s)+b(s)} = \frac{\epsilon_y^2}{b^3(s)}$$

Now take $\kappa_{x,y}(s) \rightarrow \kappa_{sf}$ and $\varepsilon_x = \varepsilon_y = \varepsilon$

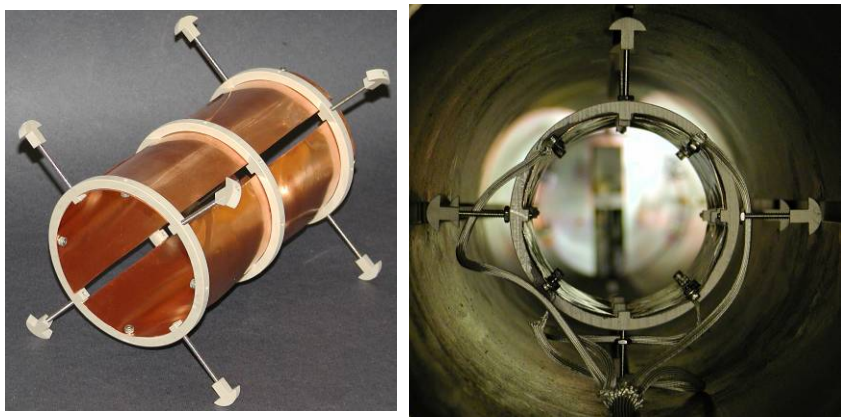
The equilibrium radius r_b is determined by: $\kappa_{sf} r_b - \frac{K_b}{r_b} = \frac{\epsilon^2}{r_b^3}.$

If $\delta a(s) = \delta b(s)$ $$\frac{d^2}{ds^2}\delta a(s) + \left(\kappa_{sf} + \frac{K_b}{r_b^2} + \frac{3\epsilon^2}{r_b^4}\right)\delta a(s)$$

Then $$\omega_{m=0}^2 = \omega_q^2(4 - 2\hat{s})$$

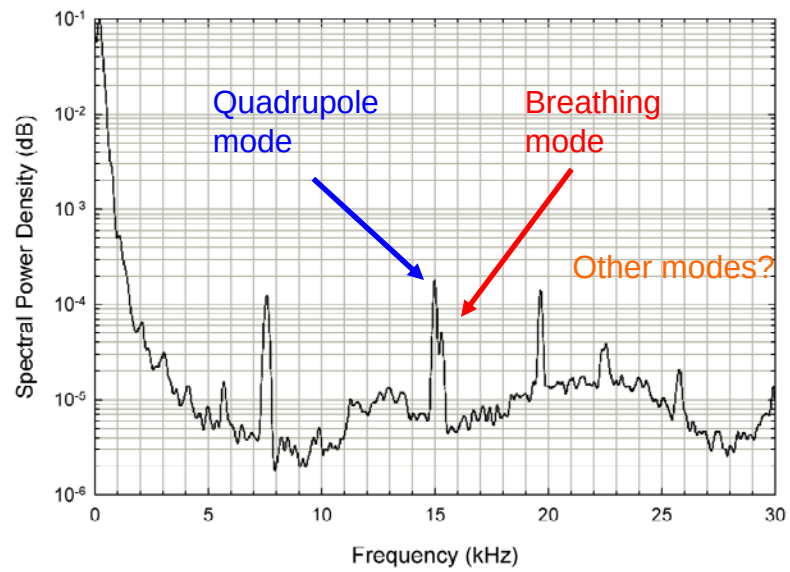
Simlarly, if $\delta a(s) = -\delta b(s)$ $$\omega_{m=2}^2 = \omega_q^2(4 - 3\hat{s})$$

Measuring Beam Oscillations and Inferring Beam Normalized Intensity *s*



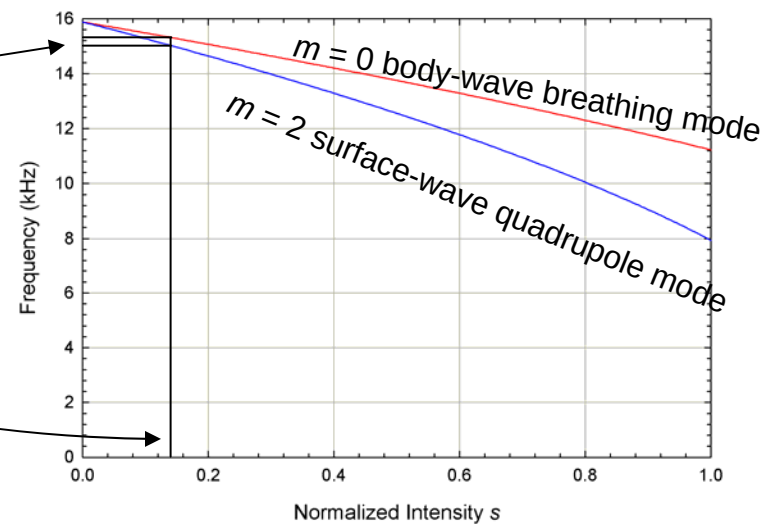
Segmented collective-mode capacitive pick-up diagnostic is sensitive to beam collective-mode oscillations.

Spectrum of measured signal



Measured frequencies…

… determine unique "normalized intensity."



$$\omega = \omega_q \left(4 - 2s\right)^{1/2}$$

$$\omega = \omega_q \left(4 - 3s\right)^{1/2}$$

Transverse Bunch Compression by Increasing ω_q

$$m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}$$

If line density N is constant and kT doesn't change too much, then increasing ω_q decreases R, and the bunch is compressed.

$$\omega_q = \frac{8eV_{0\,\max}}{m\pi r_w^2 f}\xi$$

Either

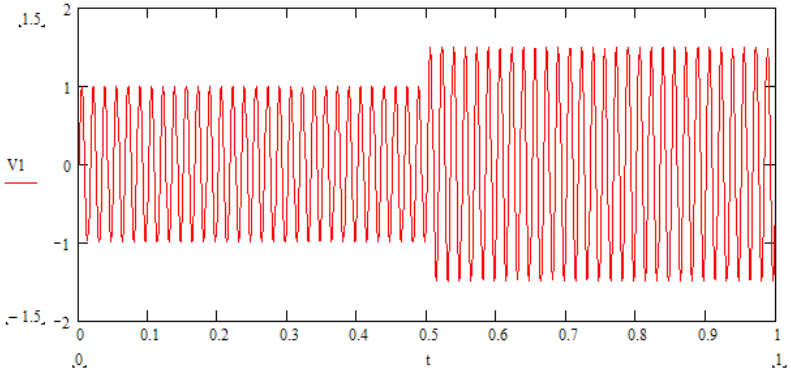
1.) increasing $V_{0\,\max}$ (increasing magnetic field strength) or

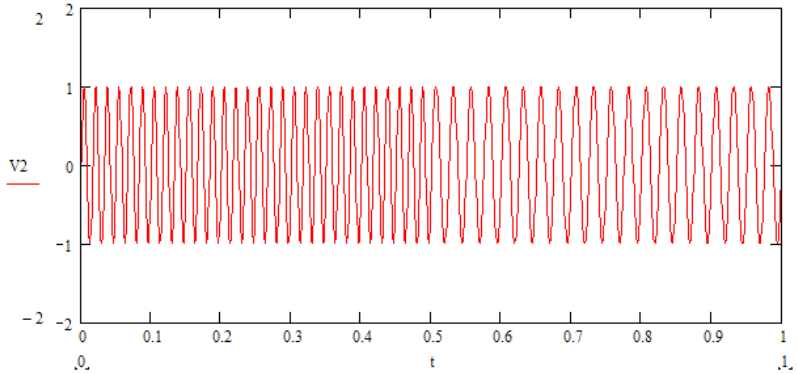
2.) decreasing f (increasing the magnet spacing)

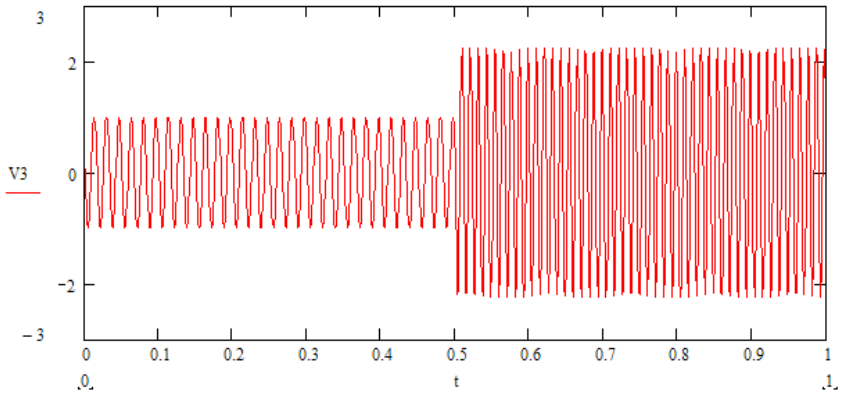
increases ω_q

Instantaneous Increase in Average Transverse Focusing Frequency ω_q

$$\omega_q = \frac{8eV_{0\,\max}}{m\pi r_w^2 f}\xi \qquad \omega_q \propto \frac{V_{0\ \max}}{f} \qquad \sigma_v^{sf} \propto \frac{V_{0\ \max}}{f^2}$$



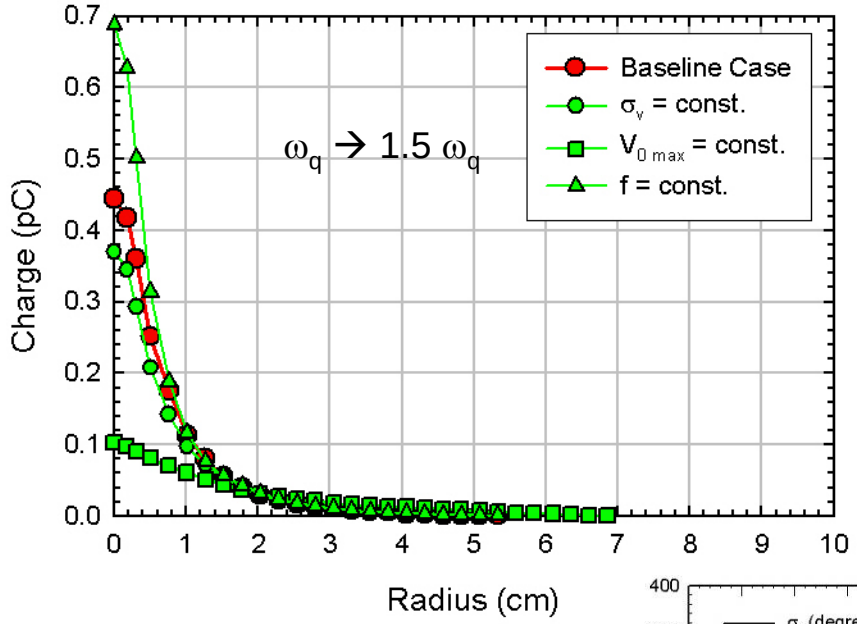


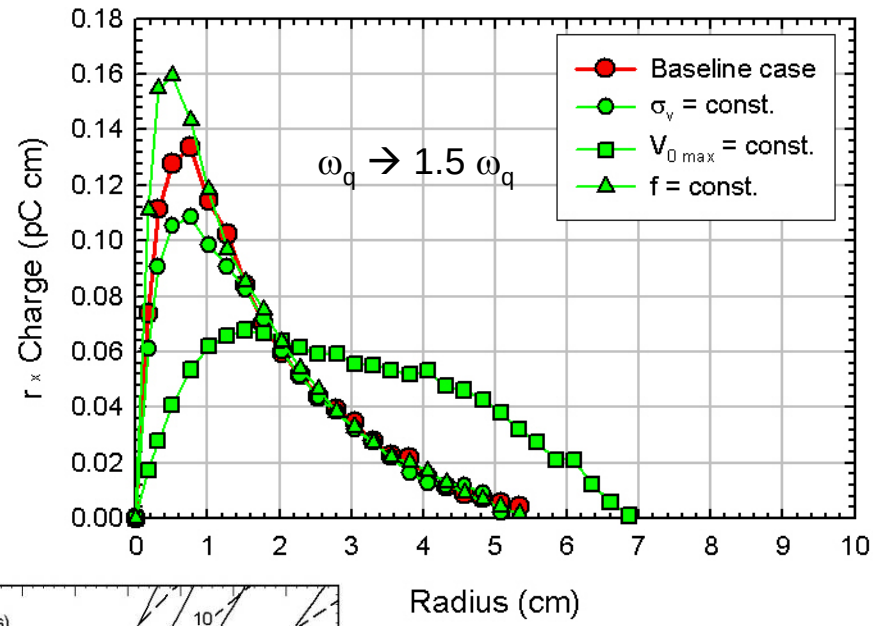


Changing $V_{0\,\max}$, f, or both.

Instantaneously.

Compression is Possible but Increasing Phase Advance Degrades Transverse Confinement

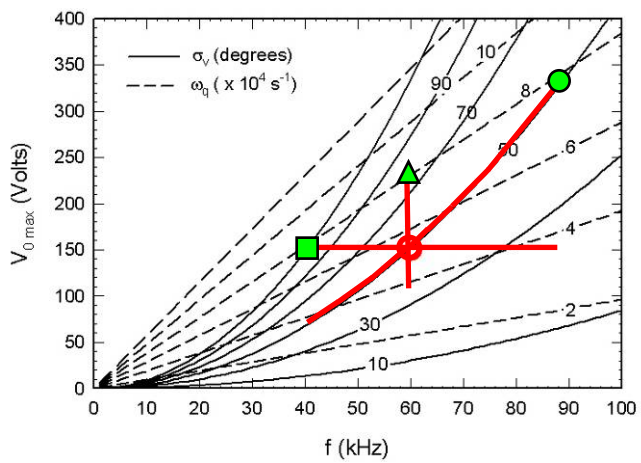




N ~ constant

- ● s = 0.08, kT = 1.6 eV
- ▲ s = 0.14, kT = 1.4 eV
- ■ s = 0.02, kT = 4.7 eV

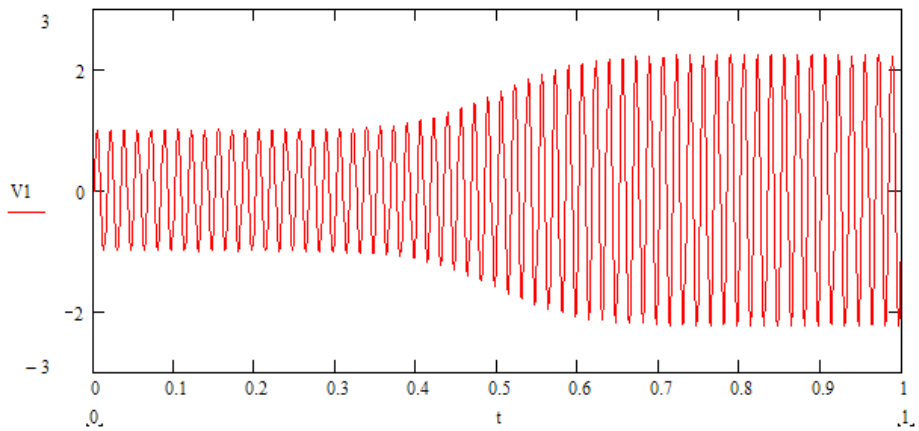
$$m\omega_q^2 R^2 = 2kT + \frac{Nq^2}{4\pi\varepsilon_o}$$



Emittance increases

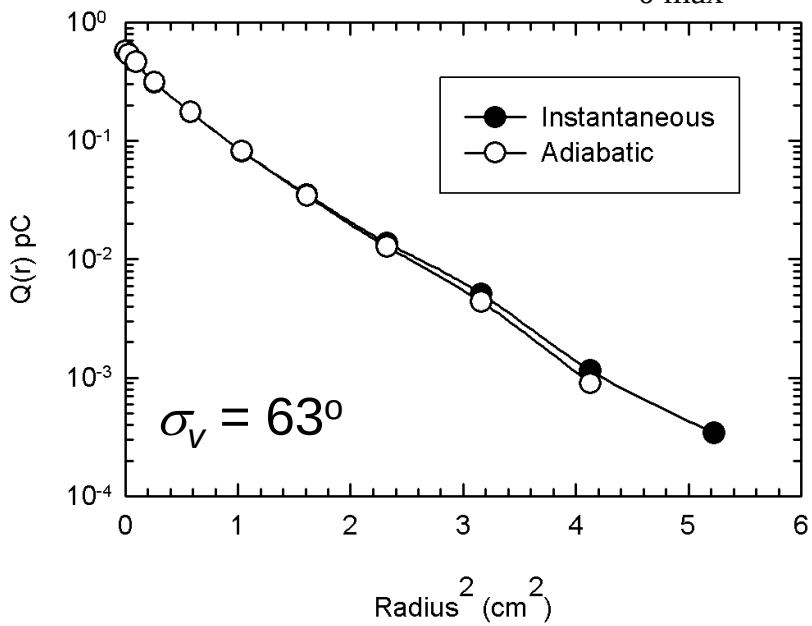
- ● σ_v = 50º **60%**
- ▲ σ_v = 75º **40%**
- ■ σ_v = 112º **450%**

Adiabatic Increase in Average Transverse Focusing Frequency ω_q

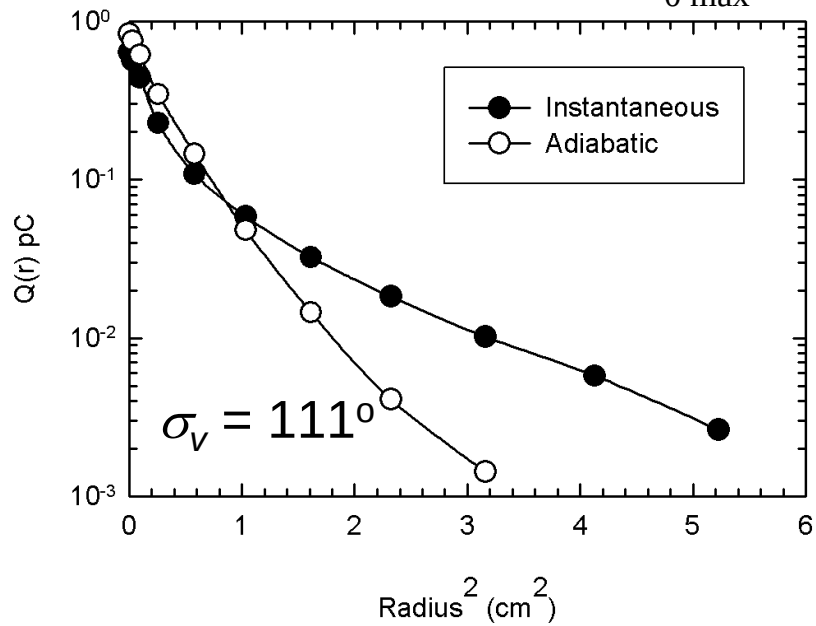


Adiabatic Amplitude Increases Transversely Compress the Bunch Better

20% increase in $V_{0\,\max}$



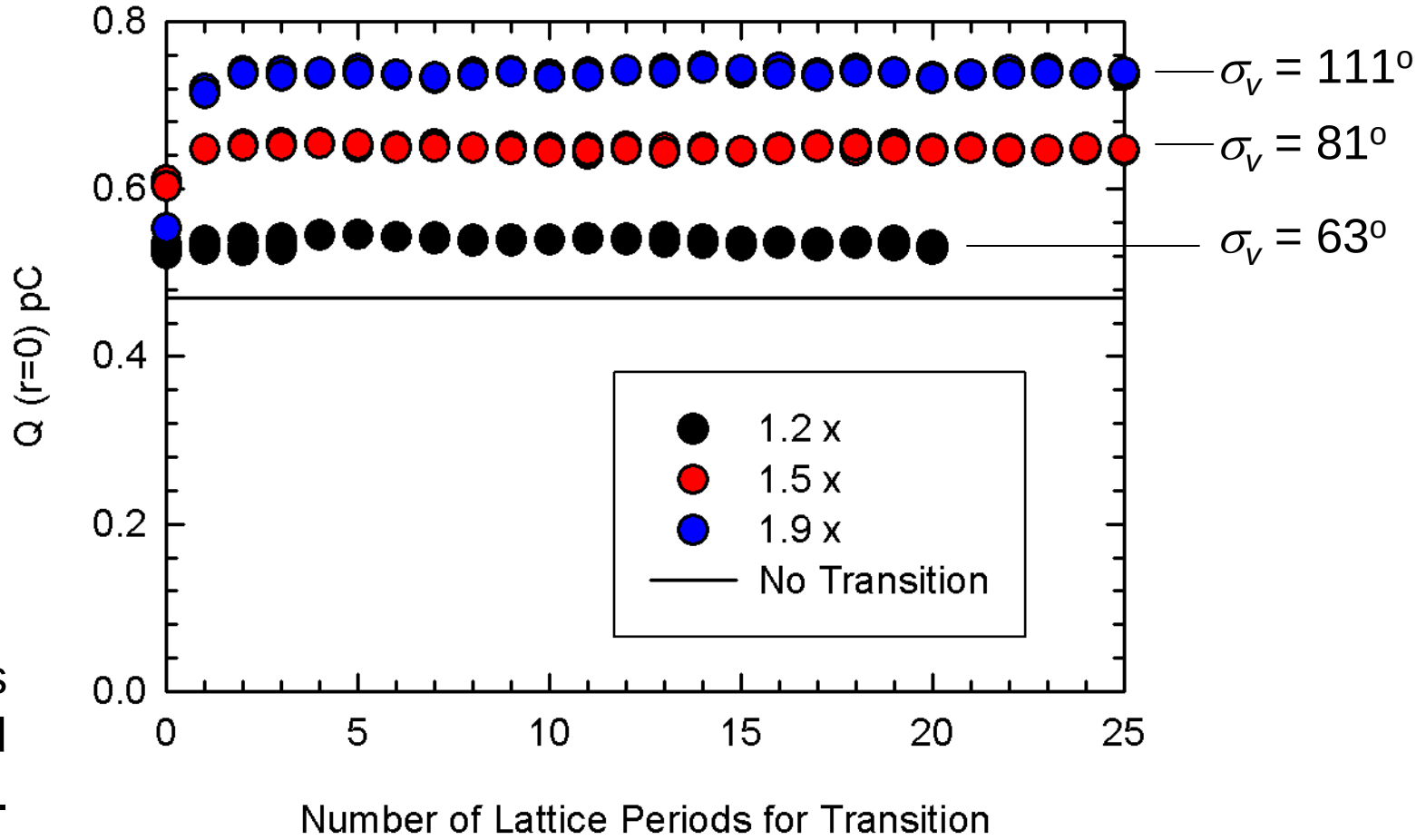
90% increase in $V_{0\,\max}$



Baseline		Adiabatic	Instantaneous
R = 0.83 cm	R = 0.79 cm	R = 0.63 cm	R = 0.93 cm
kT = 0.12 eV	kT = 0.16 eV	kT = 0.26 eV	kT = 0.58 eV
s = 0.20	s = 0.18	s = 0.10	s = 0.08
$\varepsilon \sim R\sqrt{kT} \rightarrow \Delta\varepsilon$ = 10%		$\Delta\varepsilon$ = 10%	$\Delta\varepsilon$ = 140%

- $s = \omega_p^2/2\omega_q^2 = 0.20$
- $\nu/\nu_0 = 0.88$
- $V_{0\,\max}$ = 150 V
- f = 60 kHz
- σ_v = 49°

Less Than Four Lattice Periods are Needed to Make the Transition Adiabatic

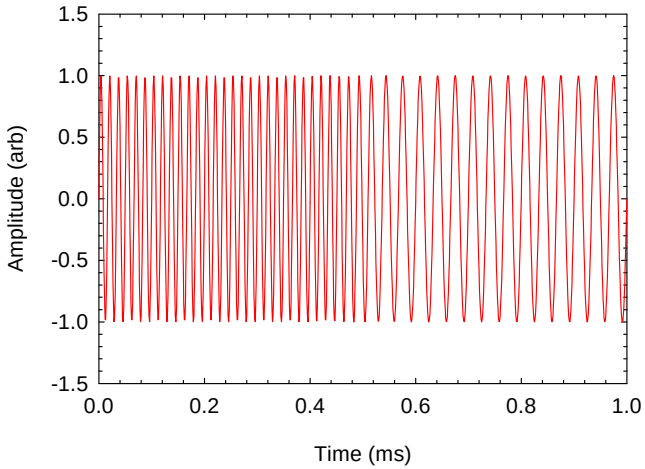


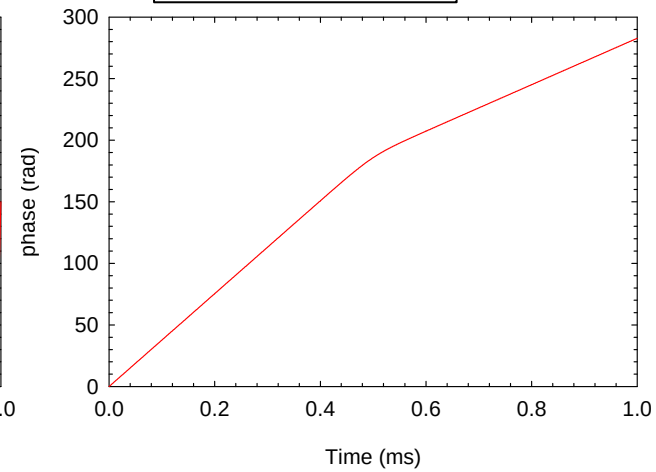
WARP 2D simulations are in good agreement.

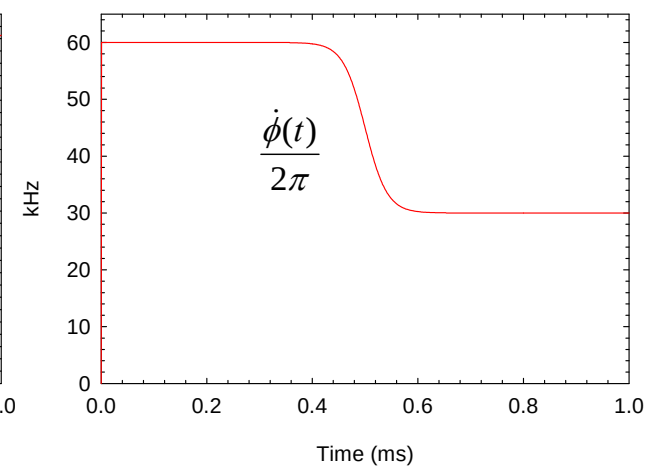
- $s = \omega_p^2/2\omega_q^2 = 0.20$
- $\nu/\nu_0 = 0.88$
- $V_{0\,max} = 150$ V
- $f = 60$ kHz
- $\sigma_v = 49°$

Increasing ω_q by Adiabatically Decreasing *f*

$$\omega_q = \frac{8eV_{0\,\max}}{m\pi r_w^2 f}\xi$$





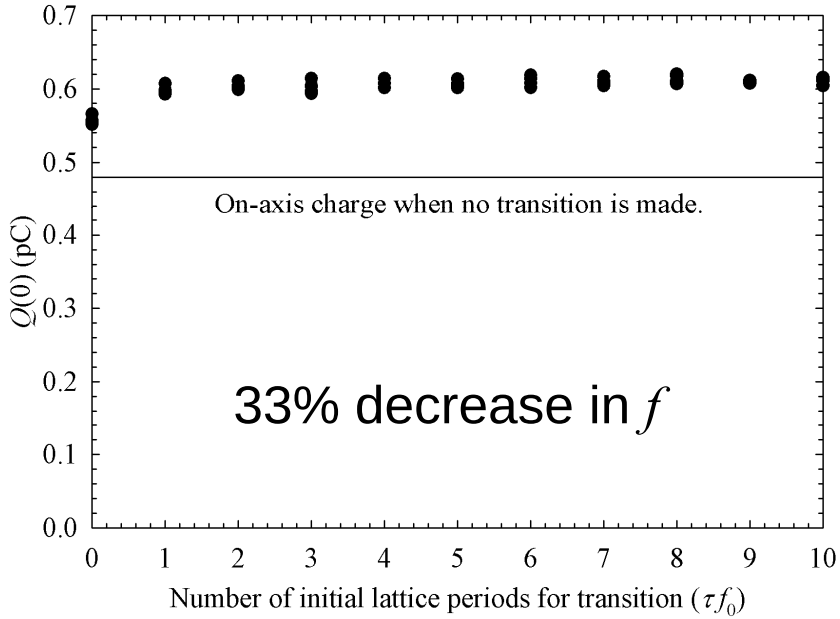


$$\frac{\phi(t)}{2\pi} = \frac{f_i + f_f}{2}t + \frac{f_f - f_i}{4}\tau \ln\left[\cosh\frac{-\left(t - t_{1/2}\right)}{\tau/2}\right] - f_0$$

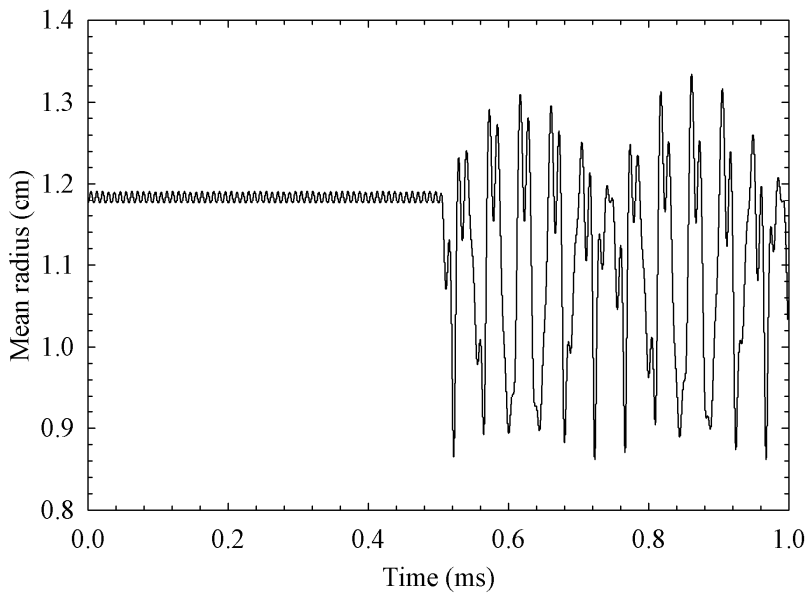
$$V(t) = V_{0\,\max}\sin\phi(t)$$

Adiabatically Decreasing *f* Compresses the Bunch

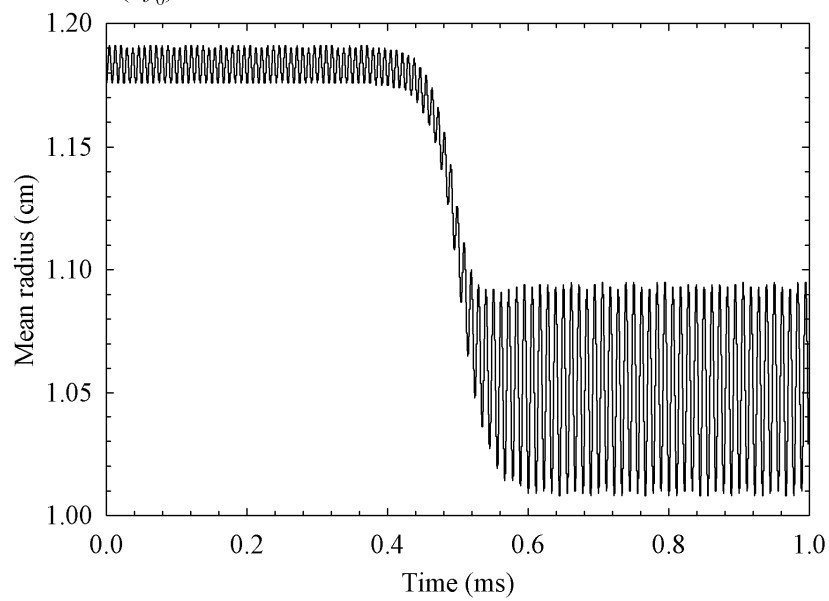
- $s = \omega_p^2/2\omega_q^2 = 0.2$.
- $\nu/\nu_0 = 0.88$
- $V_{0\,max} = 150$ V
  $f = 60$ kHz
  $\sigma_v = 49^o$



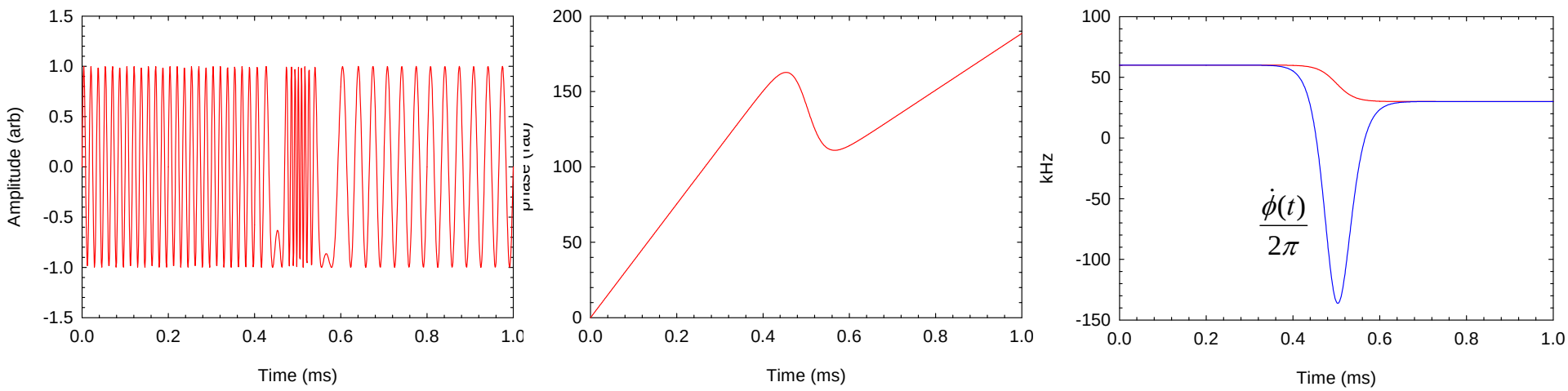
$$\omega_q = \frac{8eV_{0\,max}}{m\pi r_w^2 f}\xi$$



Good agreement with KV-equivalent beam envelope solutions.



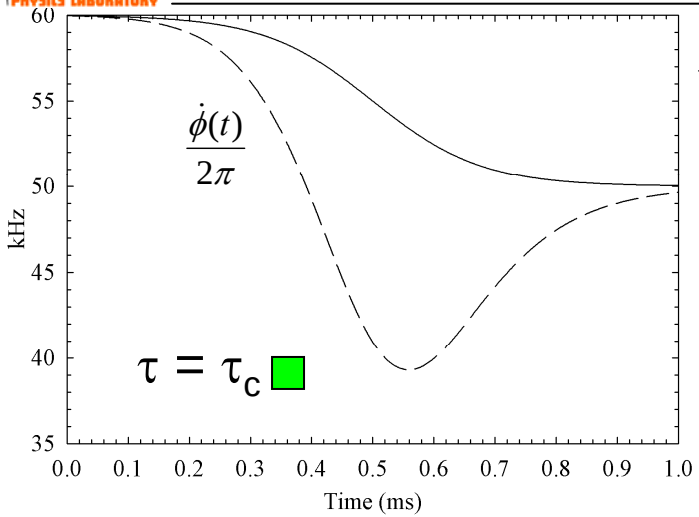
Arbitrary Function Generator Allows Non-Monotonic Frequency Changes to be Considered



$$\frac{\phi(t)}{2\pi} = f_f t + \frac{f_i - f_f}{2} t \left[\tanh \frac{-\left(t - t_{1/2}\right)}{\tau/2} + 1 \right]$$

$$V(t) = V_{0\,\max} \sin \phi(t)$$

Transverse Confinement is Lost When Single-Particle Orbits are Unstable

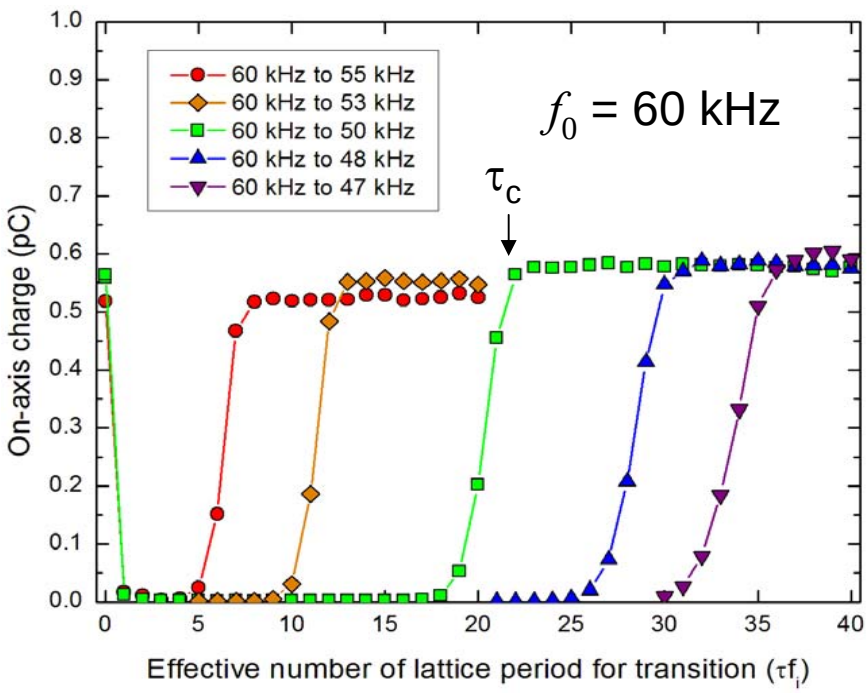


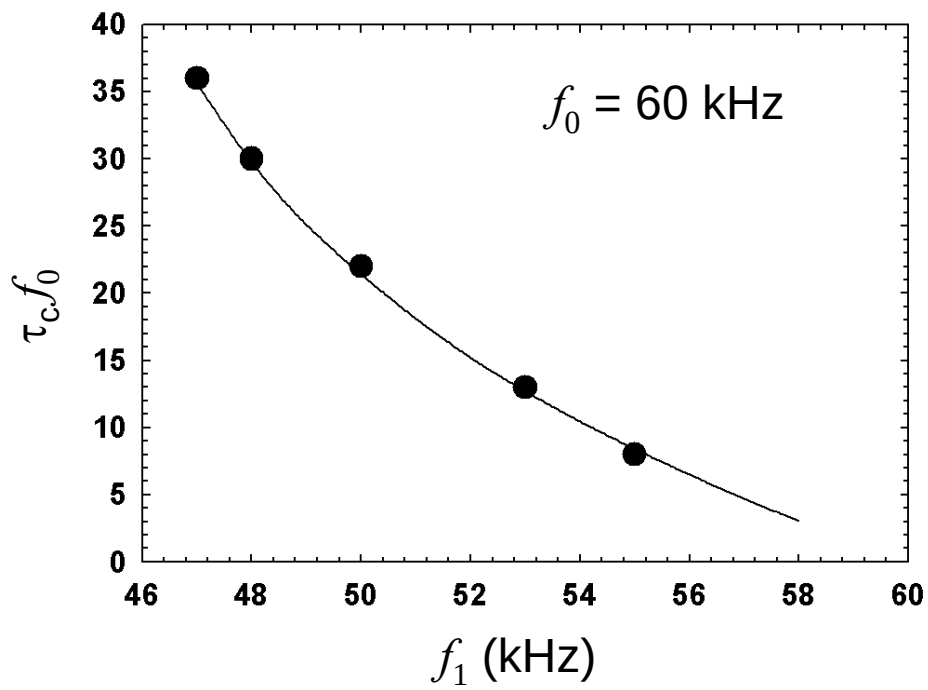
$$\frac{\phi(t)}{2\pi} = f_f t + \frac{f_i - f_f}{2} t \left[\tanh \frac{-\left(t - t_{1/2}\right)}{\tau/2} + 1 \right]$$

$$\sigma_v^{sf} = \frac{\omega_q}{f} \propto \frac{1}{f^2}$$

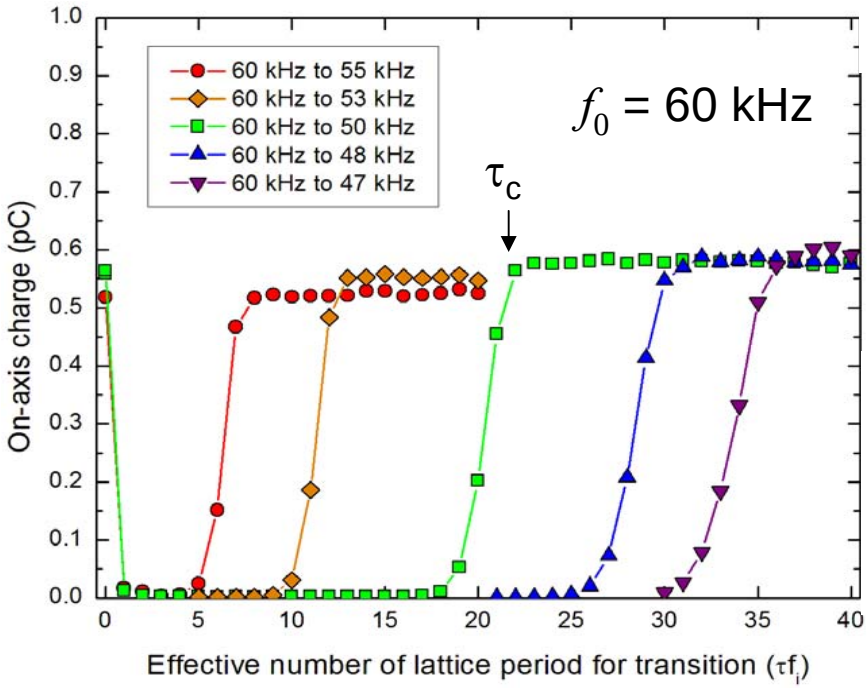
Measured τ_c (dots)

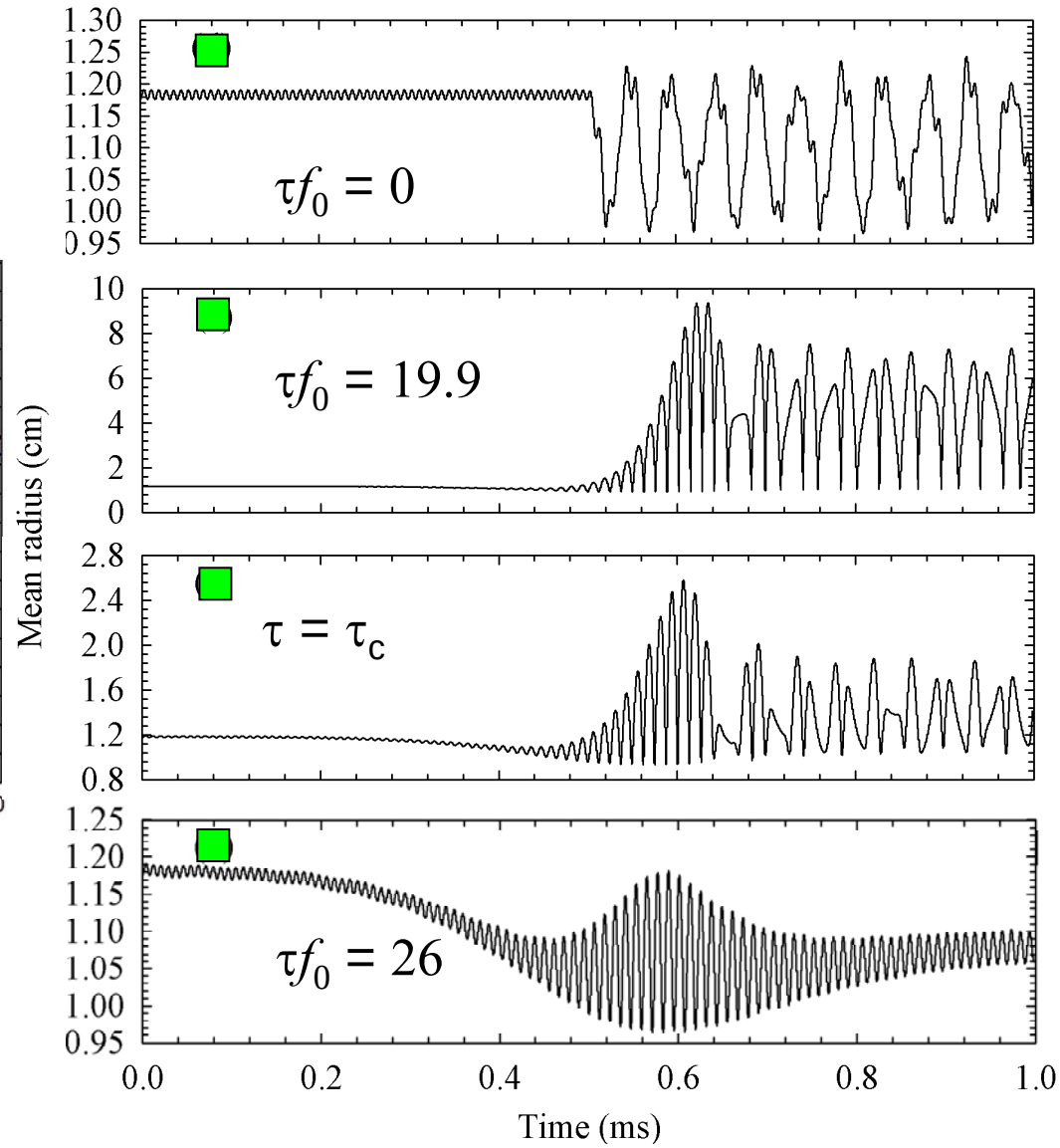
Set $\sigma_{v\ max}$ = 180° and solve for τ_c (line)





Good Agreement Between Data and KV-Equivalent Beam Envelope Solutions





PTSX is a Compact Experiment for Studying the Propagation of Beams Over Large Distances

PTSX is a versatile research facility in which to simulate collective processes and the transverse dynamics of intense charged particle beam propagation over large distances through an alternating-gradient magnetic quadrupole focusing system using a compact laboratory Paul trap.

- Measured frequency spectrum is consistent with the observation of the $m = 0$ and $m = 2$ collective-modes, but more work needs to be done.
- Transverse compression can be achieved with low emittance growth through either adiabatic lattice amplitude increases or lattice period decreases.