Survey of Collective Instabilities and Beam-Plasma Interaction Processes in Intense Heavy Ion Beams

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Outline of Presentation

- Introduction

- Nonlinear stability theorem for collective interactions.

- Collective interactions and instabilities:

  - One-component intense ion beam (Harris, Weibel).
  - Partially neutralized intense ion beam (electron cloud instability).
  - Intense ion beam propagating through background plasma (multispecies Weibel, multispecies two-stream).
  - Effects of velocity tilt on two-stream instability.

- Use of neutral plasmas for focusing/compressing high-intensity charge bunches - Examples of dynamic beam equilibria.

Nonlinear Stability Theorem

- Consider general perturbations about a quasi-steady equilibrium distribution function $f_{eq}(\mathbf{x}', \mathbf{p}')$. For $f_{eq} = f_{eq}(H')$, using the global conservation constraints, it can be shown that

$$\frac{\partial}{\partial H'} f_{eq}(H') \leq 0$$

is a sufficient condition for linear and nonlinear stability.

- Here, H' is the single-particle Hamiltonian defined by

$$H' = \frac{1}{2m_b} \mathbf{p}'^2 + \Psi'_{sf}(\mathbf{x}') + q_b \phi'(\mathbf{x}'),$$

where $\phi'(\mathbf{x}')$ is the equilibrium space-charge potential.

Nonlinear Kinetic Stability Theorem

Therefore a necessary condition for instability is that the beam distribution function have some nonthermal feature such as:

- An inverted population in phase space.
- Or a strong energy anisotropy.
- Or that the beam have directed kinetic energy relative to background charge components.
- Or dissipation mechanisms (e.g., finite wall resistivity).

Collective Instabilities in Intense Charged Particle Beams

One-Component Beams

- Electrostatic Harris instability

$$T_{\perp b} >> T_{\parallel b}$$

- Electromagnetic Weibel instability

$$T_{\perp b} >> T_{\parallel b}$$

- Resistive wall instability

Propagation Through Background Electrons

- Electron-ion two-stream (Electron Cloud) instability

Propagation Through Background Plasma

- Resistive hose instability

- Multispecies Weibel instability

- Multispecies two-stream instability

Harris Instability in Intense One-Component Beams

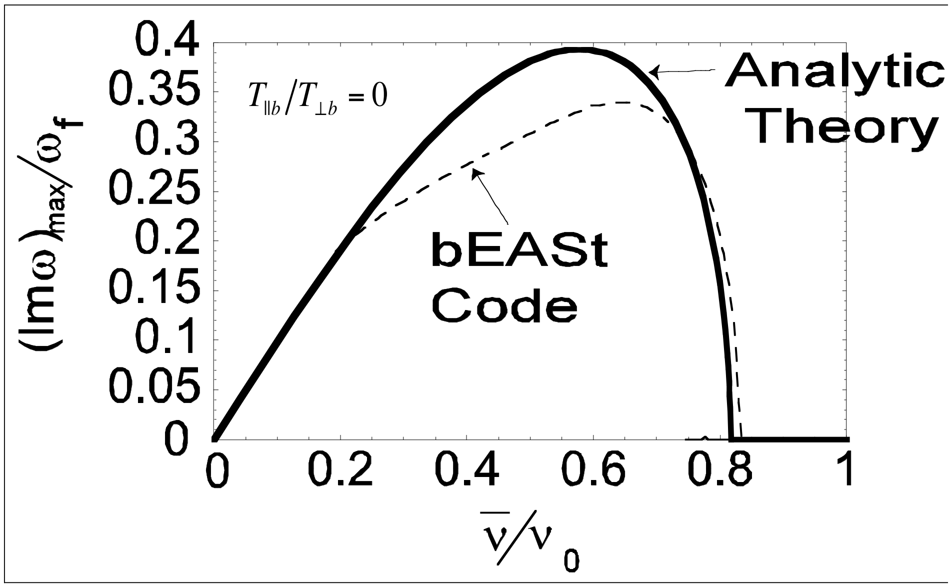
- Electrostatic Harris instability* can play an important role in multispecies plasmas with very strong temperature anisotropy $T_{\parallel j} < T_{\perp j}$.
- Harris instability is inherently three-dimensional and involves a coupling of the longitudinal and transverse particle dynamics.
- Harris-like instability** also exists in intense one-component beams*** provided the anisotropy is sufficiently large and the beam intensity is sufficiently large.

* E. G. Harris, Phys. Rev. Lett. **2**, 34 (1959).

** I. Haber et al., Phys Plasmas **6,** 2254 (1999).

*** E. A. Startsev, R. C. Davidson and H. Qin, Phys. Plasmas 14, 056705 (2007); H. Qin, R. C. Davidson and E. A. Startsev, Phys. Rev. ST Accelerators and Beams; E. A. Startsev, R. C. Davidson and H. Qin, Phys. Rev. ST Accelerators and Beams **8**, 124201(2005); Phys. Plasmas **9**, 3138 (2002); Laser and Particle Beams **20**, 585 (2002); Phys. Rev. ST Accelerators and Beams **6**, 084401 (2003).

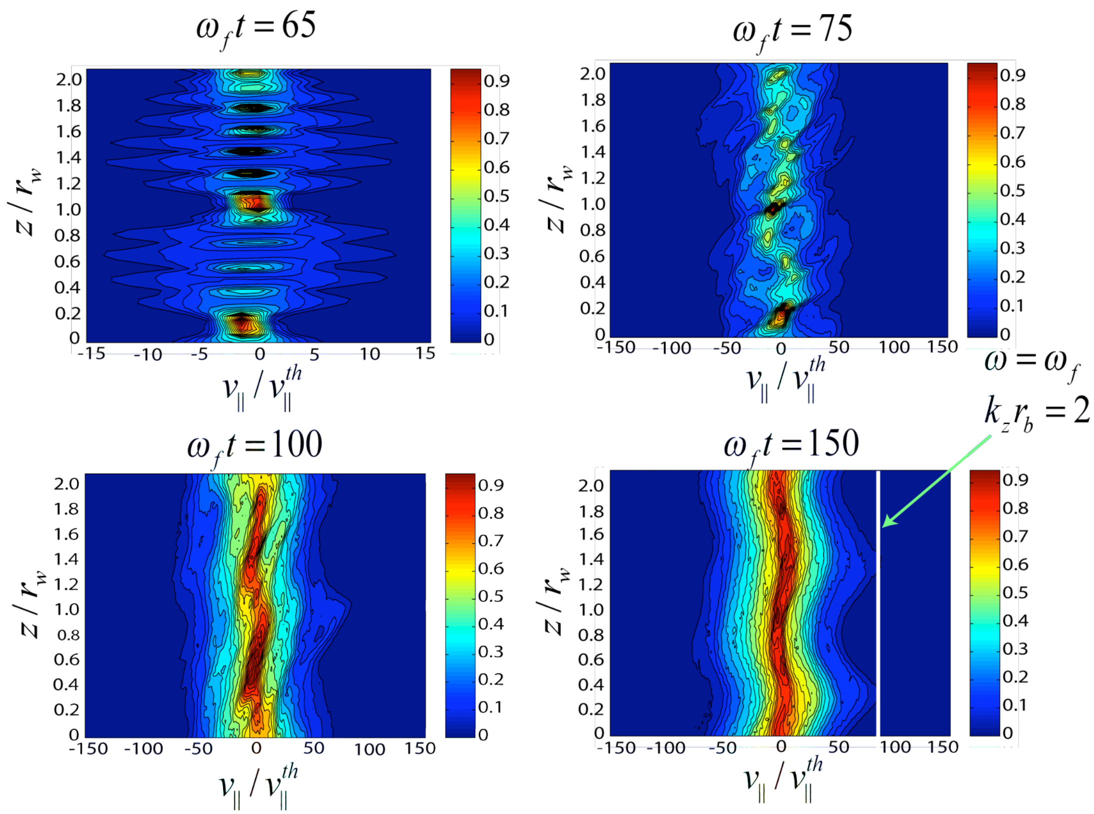
Large temperature anisotropies develop naturally in intense charged particle beams and can provide the free energy to drive the Harris Instability at high beam intensity

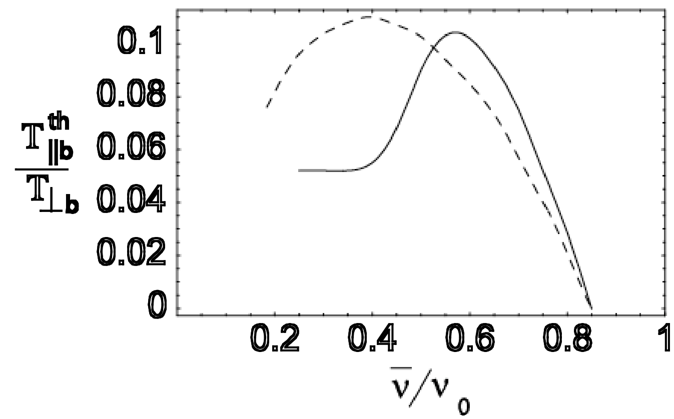


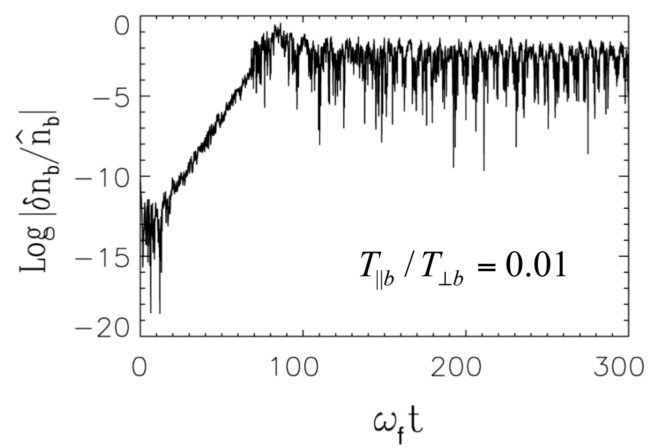
Plots of normalized growth rate versus normalized tune for azimuthal mode number m=1 (dashed curve). Results have been obtained using the eigenmode code bEASt The solid curve corresponds to a simple analytical estimate.

Strong Harris Instability for Beams with Large Temperature Anisotropy

- Moderate intensity → largest threshold temperature anisotropy.
- BEST simulations show nonlinear saturation by particle trapping → tail formation.

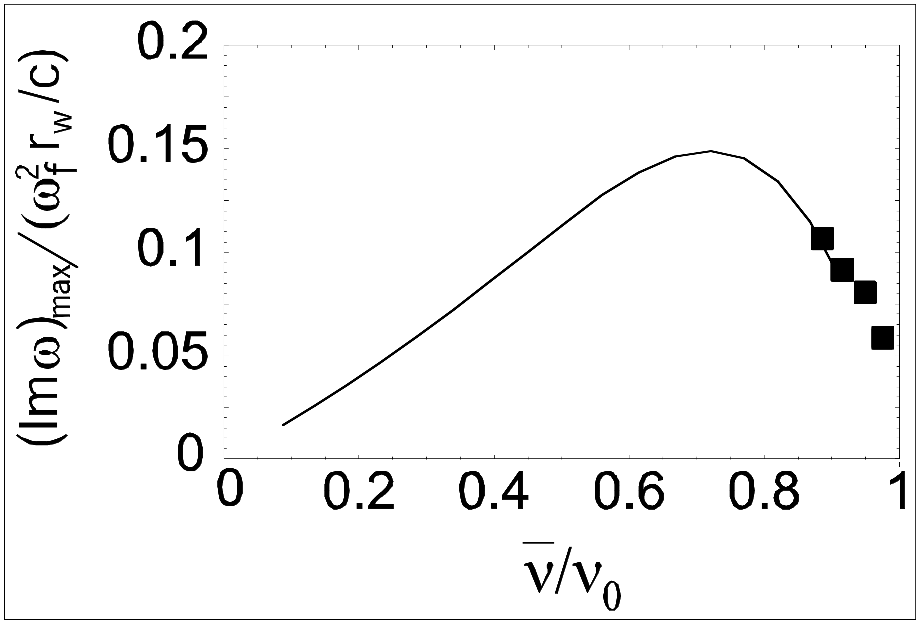






E. A. Startsev, R. C. Davidson and H. Qin, Phys. Plasmas 14, 056705 (2007).

Large temperature anisotropies can also provide the free energy to drive the electromagnetic Weibel Instability with growth rate smaller than the electrostatic Harris instability



The maximum growth rate of the Weibel instability is plotted versus normalized tune, and is given approximately by

$$\frac{(\mathrm{Im}\,\omega)_{\max}}{\omega_f} = \frac{1}{\sqrt{2}} \frac{\bar{\omega}_{pb}}{\omega_f} \frac{v_{\perp b}^{th}}{c}$$

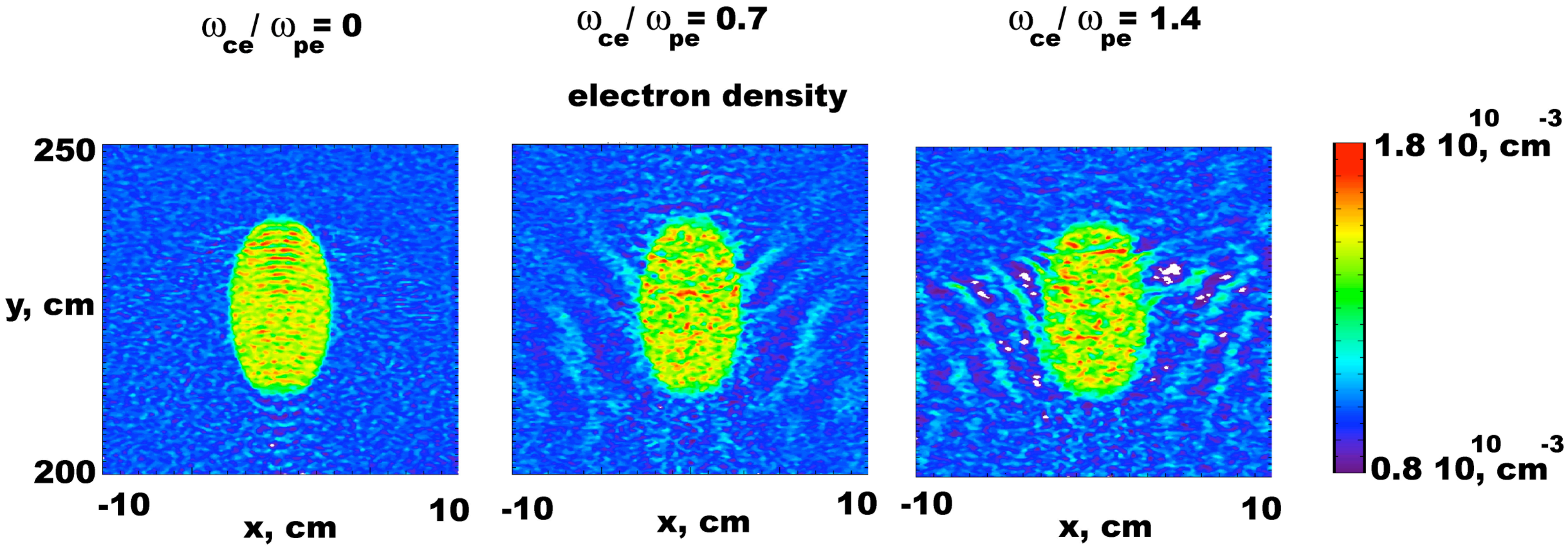
Background plasma can be used to focus and compress intense ion beam pulses

The electrons in a large-volume background plasma can be used to charge neutralize and current neutralize an intense ion charge bunch, thereby greatly facilitating the transverse focusing and longitudinal compression of the charge bunch to a small spot size.

- A. B. Sefkow and R. C. Davidson, Phys. Rev. ST Accel. & Beams **10,** 100101 (2007).
- R. C. Davidson and H. Qin, Phys. Rev. ST Accel. & Beams **8,** 064201 (2005).
- I.D. Kaganovich, E. A. Startsev, A. B. Sefkow and R. C. Davidson, Phys. Rev. Lett. **99,** 235002 (2007).
- E. A. Startsev, R.C. Davidson and M. Dorf, Phys. Plasmas **15,** 062107 (2008).
- P.A. Seidl et al., Nucl. Instr. Meth. Phys. Res. **A577,** 215 (2007).
- D. R. Welch et al., Nucl. Instr. Meth. Phys. Res. **A577,** 231 (2007).
- P. K. Roy, S. S. Yu et al. ,Phys. Rev. Lett. **95,** 234801 (2005).

Analytical studies show that the solenoidal magnetic field influences the neutralization by plasma if $\omega_{ce}>\beta\omega_{pe}$ *

Plots of electron charge density contours in (x,y) space, calculated in 2D slab geometry using the LSP code with parameters: Plasma: $n_p=10^{11}cm^{-3}$; Beam: $V_b=0.2c$, 48.0A, r_b=2.85cm and pulse duration τ_b=4.75 ns. A solenoidal magnetic field of 1014 G corresponds to $\omega_{ce}=\omega_{pe}$.

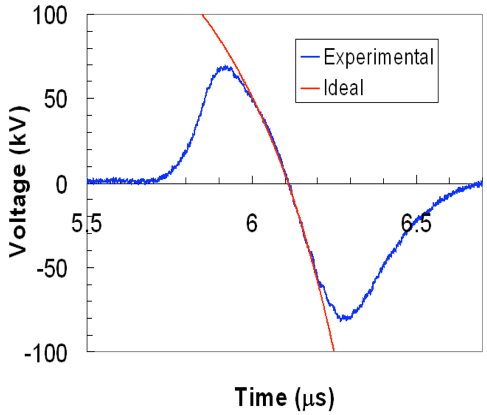


- In the presence of a solenoidal magnetic field, whistler waves are excited, which propagate at an angle with the beam velocity and can perturb the plasma ahead of the beam pulse.

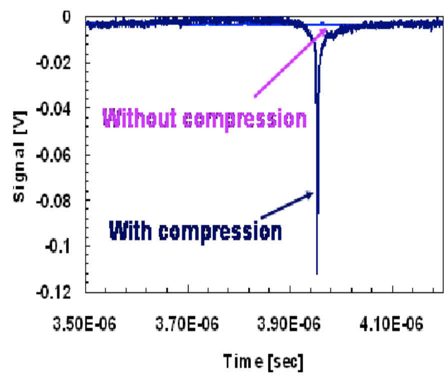
* I.D. Kaganovich et al., Phys. Rev. Lett. 99, 235002 (2007).

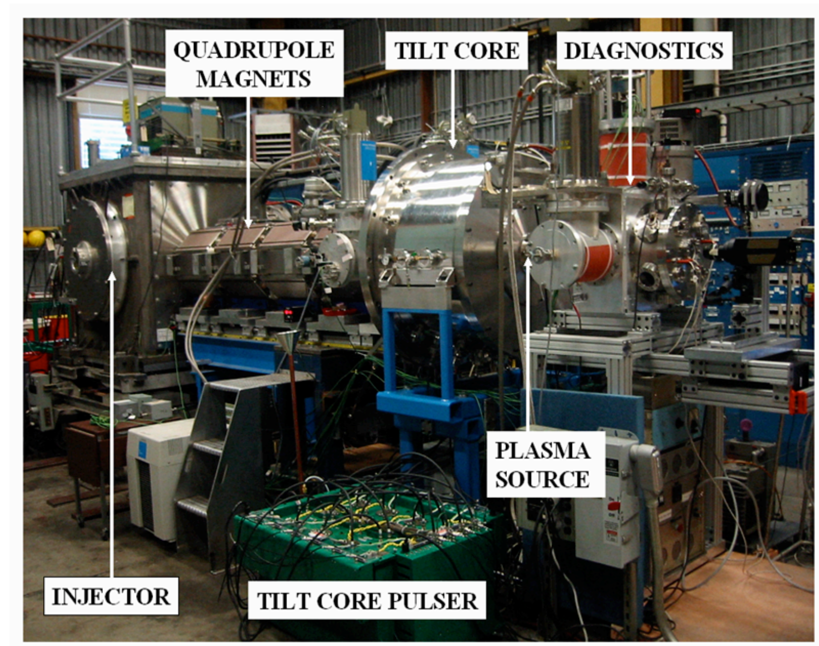
Neutralized Drift Compression Experiment (NDCX) - 300 keV K+ Ions @ 25 mA

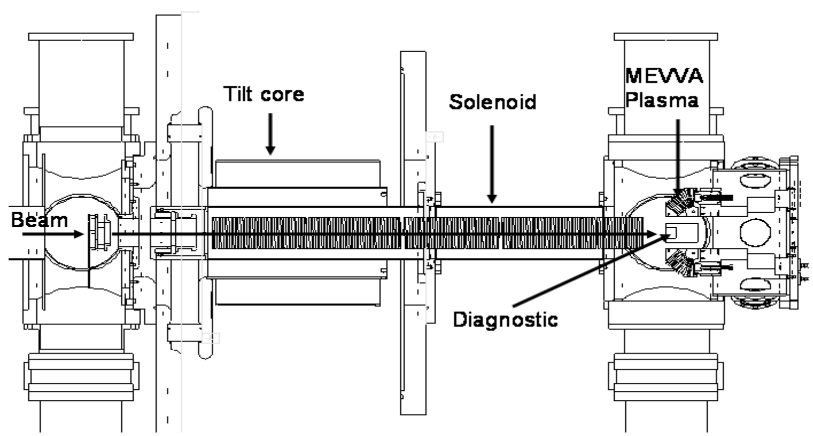
Tiltcore waveform



Beam current diagnostic







Class of Exact Dynamically-Compressing Neutralized Beam Equilibria with Linear Velocity Tilt in a Solenoidal Magnetic Field*

Assuming complete charge and current neutralization ($\underline{\mathbf{E}}^s = 0 = \underline{\mathbf{B}}^s$) for an intense ion beam pulse propagating through background plasma, Vlasov equation supports class of exact, dynamically-compressing solutions

$$f_b(\mathbf{x}, \mathbf{p}, t) = f_b(W_\perp, W_z)$$

where W_z and $W_\perp$ are defined by

$$W_z = \frac{z^2}{z_b^2(t)} + \frac{z_b^2(t)}{z_{b0}^2(t)\mathrm{v}_{T0}^2}\left(\mathrm{v}_z - \frac{z}{z_b(t)}\frac{dz_b(t)}{dt}\right)^2$$

$$W_\perp = \frac{x^2 + y^2}{r_b^2(t)} + \frac{r_b^2(t)}{r_{b0}^2 \mathrm{v}_{\mathrm{T}0}^2}\left[\left(\mathrm{v}_x + \mathring{\Omega}_L y - \frac{x}{r_b(t)}\frac{dr_b(t)}{dt}\right)^2 + \left(\mathrm{v}_y - \mathring{\Omega}_L y - \frac{y}{r_b(t)}\frac{dr_b(t)}{dt}\right)^2\right]$$

and Ω_L is the Larmor frequency

$$\Omega_L = -\frac{e_b}{2m_b c}B_z\left[\gamma_b\left(z + \mathrm{v}_b t\right)\right]$$

* **A. B. Sefkow and R. C. Davidson, Phys. Rev. ST Accel. & Beams 10, 100101 (2007); R. C. Davidson and H. Qin, Phys. Rev. ST Accel. & Beams 8, 064201 (2005).**

Dynamically-Compressing Neutralized Beam Equilibria

Many choices of beam equilibria are possible. As one example, consider

$$f_b(W_\perp, W_z) = const.\sqrt{(1-W_z)}\,e^{-W_\perp}, \text{ where } \quad 0 \le W_z < 1$$

Then, the density profile $n_b(r,z,t)$ is given dynamically by

$$n_b(r,z,t) = \hat{n}_{b0}\left[\frac{r_{b0}^2}{r_b^2(t)}\frac{z_{b0}}{z_b(t)}\right]\left(1-\frac{z^2}{z_b^2(t)}\right)e^{-r^2/r_b^2(t)}, \text{ where } \quad 0 \le z^2 < z_b^2(t)$$

where $r_b(t)$ and $z_b(t)$ solve the envelope equations introduced earlier. For this choice of $f_b(W_\perp, W_z)$ note that the line density is parabolic with

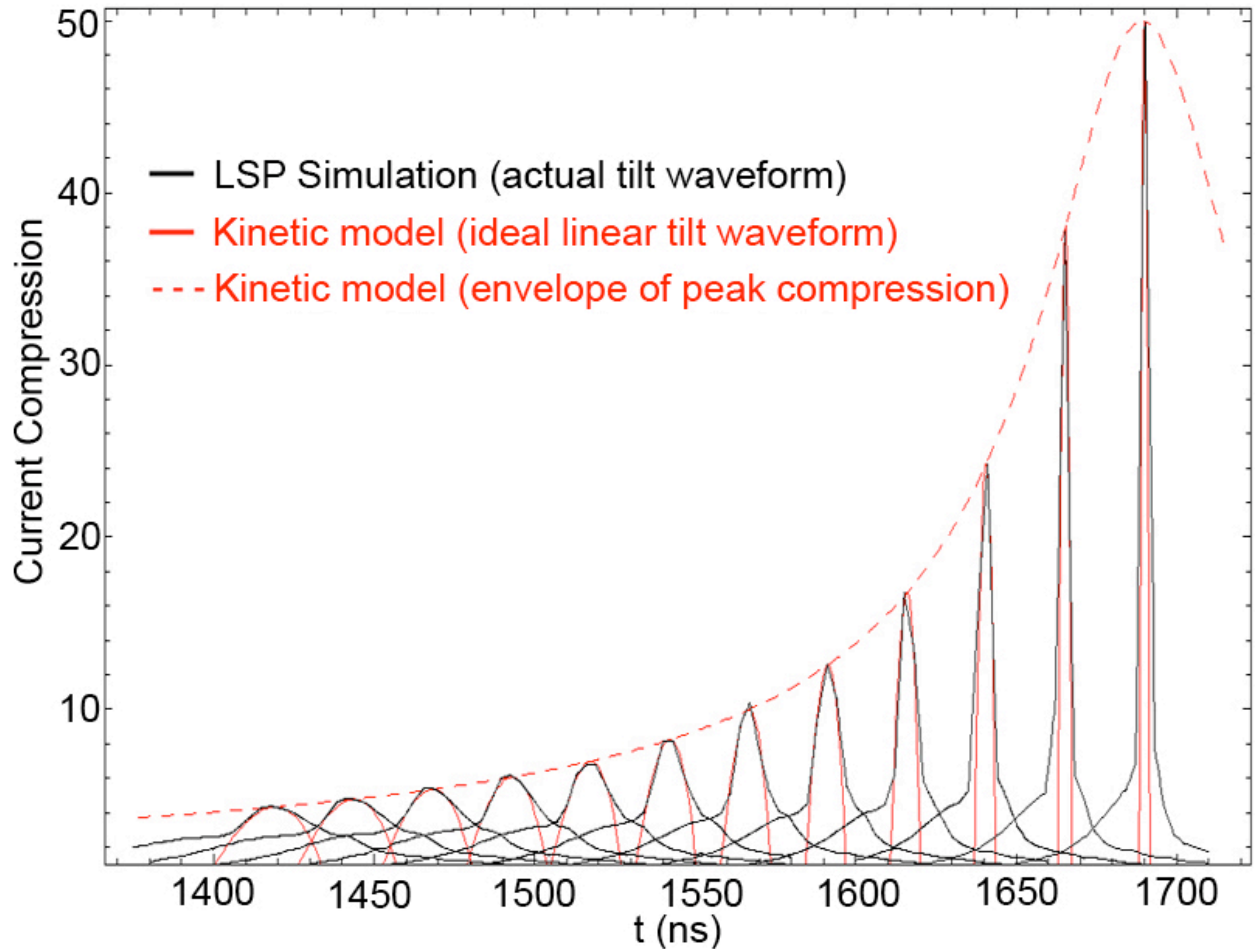
$$\lambda_b(z,t) = \lambda_{b0}\left[\frac{z_{b0}}{z_b(t)}\right]\left(1-\frac{z^2}{z_b^2(t)}\right), \text{ where } \quad 0 \le z^2 < z_b^2(t)$$

Comparison with longitudinal current compression in NDCX*

Parabolic waveform of main compression peak is described well by kinetic model

Deviation from ideal (linear) tilt waveform produce pedestals around main parabolic peaks

- A. B. Sefkow and R.C. Davidson, Phys.Rev. ST Accel. & Beams 10, 100101 (2007).



Multispecies Weibel Instability

In the collisionless regime, the large directed kinetic energy of the beam ions propagating through a background plasma provides the free energy to drive the electromagnetic Weibel instability.

Assumptions

- Charge and current neutralization

$$\sum_{j=b,e,i} n_j^0(r) e_j = 0 \quad \text{and} \quad \sum_{j=b,e,i} n_j^0(r) e_j \beta_j c = 0$$

- Electromagnetic perturbations with $\partial/\partial\theta = 0,\ \partial/\partial z = 0$ and polarization

$$\delta\mathbf{E} = \delta E_r \mathbf{e}_r + \delta E_z \mathbf{e}_z \quad \text{and} \quad \delta\mathbf{B} = \delta B_\theta \mathbf{e}_\theta$$

Multispecies Weibel Instability

Express

$$\delta E_z(r,t) = \delta \hat{E}_z \exp(-i\omega t)$$

Obtain the eigenvalue equation

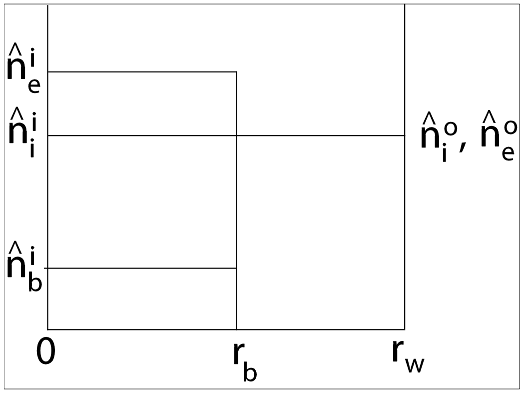
$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(1 + \sum_{j=b,e,i} \frac{\beta_j^2 \omega_{pj}^2(r)}{\omega^2} + \sum_{j=b,e,i} \frac{\left[\beta_j \omega_{pj}^2(r)\right]^2}{\omega^2\left[\omega^2 - \sum_{j=b,e,i}\omega_{pj}^2(r)\right]} \right) \frac{\partial}{\partial r}\delta E_z \right] + \left(\frac{\omega^2}{c^2} - \sum_{j=b,e,i} \frac{\omega_{pj}^2(r)}{\gamma_j^2 c^2} \right)\delta E_z = 0$$

where $\omega_{pj}(r) = \left[4\pi n_j^0(r) e_j^2 / \gamma_j m_j\right]^{1/2}$ and $\gamma_j = \left(1-\beta_j^2\right)^{-1/2}$

Slow-wave Weibel instability driven by the terms proportional to

$$\sum_{j=b,e,i} \beta_j^2 \omega_{pj}^2(r) \quad \text{and} \quad \sum_{j=b,e,i} \beta_j \omega_{pj}^2(r)$$

Multispecies Weibel Instability



An intense ion beam with step-function density profile propagates through background plasma with uniform density.

Treating the plasma ions as stationary $\beta_i = 0$ and assuming local charge and current neutralization gives

$$\hat{n}_e^i = Z_b \hat{n}_b^i + Z_i \hat{n}_i^i$$

$$\beta_e = \frac{\beta_b Z_b \hat{n}_b^i}{Z_b \hat{n}_b^i + Z_i \hat{n}_i^i}$$

Multispecies Weibel Instability

For the case of uniform density profiles the eighenvalue equation can be solved exactly to obtain a closed dispersion relation for the complex frequency ω

For perturbations with short wavelength the characteristic growth rate of the Weibel instability scales as $\mathrm{Im}\,\omega \sim \Gamma_W$ where

$$\Gamma_W^2 = \beta_e^2 \hat{\omega}_{pi}^{i2} + \left(\beta_b - \beta_e\right)^2 \omega_{pb}^{i2}$$

and $\hat{\omega}_{pj}^{i2} = 4\pi \hat{n}_j^i e_j^2 \big/ \gamma_j m_j$

Multispecies Weibel Instability

The full dispersion relation has been solved numerically over a wide range of beam-plasma parameters.

As an illustrative example consider an intense cesium ion beam propagating through background argon plasma $(Z_i = 1)$ with $(Z_b = 1)$

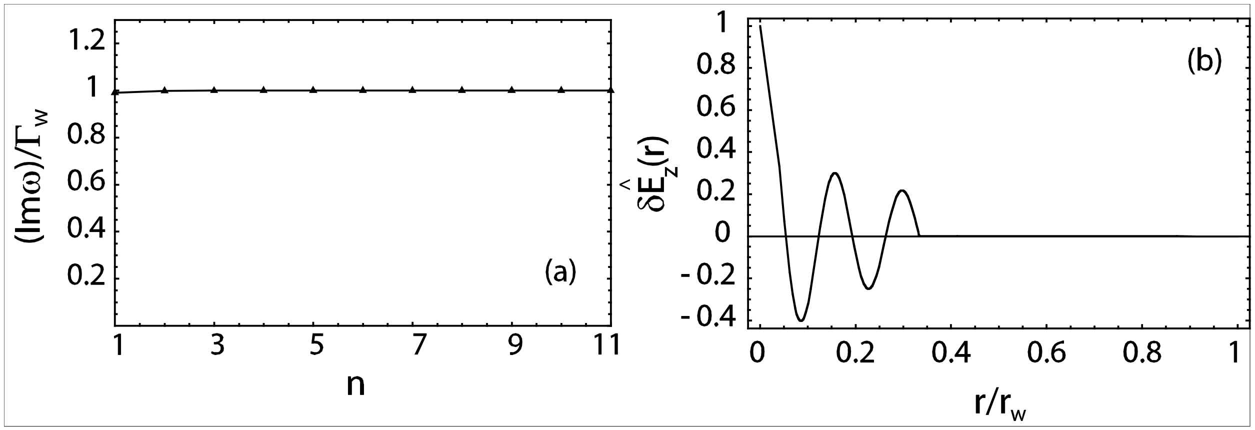
$$\beta_e = 0.1$$

$$\beta_b = 0.2, \quad \beta_i = 0,$$

$$\hat{n}_i^i = \hat{n}_e^i / 2 = \hat{n}_b^i$$

The background plasma provides complete charge and current neutralization.

Multispecies Weibel Instability



Plots of (a) Weibel instability growth rate $\mathrm{Im}\,\omega/\Gamma_W$ versus radial mode number *n*, and (b) eigenfunction $\delta\hat{E}_z(r)$ versus r/r_w for *n*=5. System parameters are $r_b = r_w/3$, $\Omega_p^i r_b/c = 1/3$ and $\hat{n}_i^o = 0$ (vacuum region outside beam).

Multispecies Two-Stream Instability

In the collisionless regime, the large directed kinetic energy of the beam ions relative to the background plasma components also provides the free energy to drive the electrostatic two-stream instability.

Assumptions

- Charge and current neutralization

$$\sum_{j=b,e,i} n_j^0(r) e_j = 0 \quad \text{and} \quad \sum_{j=b,e,i} n_j^0(r) e_j \beta_j c = 0$$

- Longitudinal electrostatic perturbations with $\delta \mathbf{E}^{\mathbf{L}} = -\nabla \delta \phi$ and $\partial / \partial \theta = 0$

$$\delta \mathbf{E}^{\mathbf{L}} = \delta E_r \mathbf{e}_r + \delta E_z \mathbf{e}_z$$

Multispecies Two-Stream Instability

Express

$$\delta\phi(r,z,t) = \delta\hat{\phi}(r)\exp\left[i\left(k_z z - \omega t\right)\right]$$

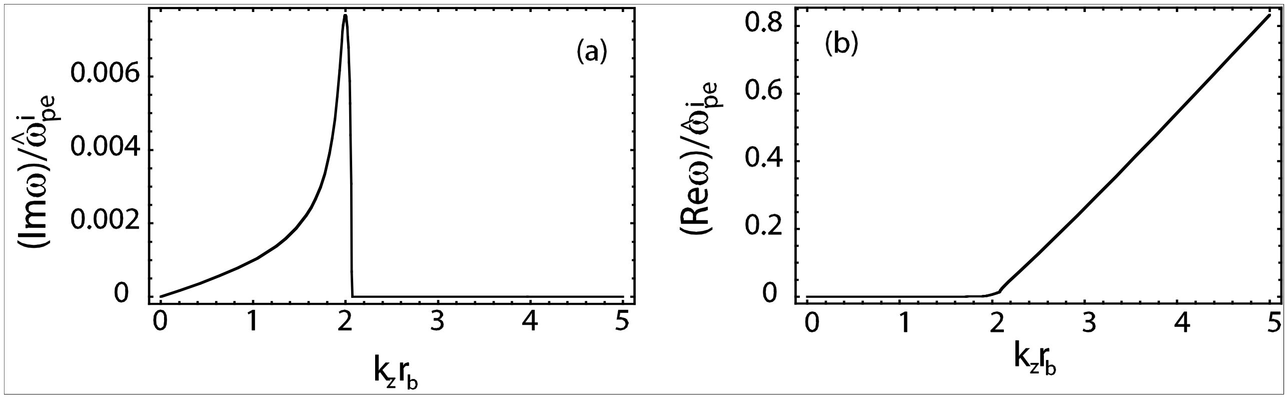
Obtain eigenvalue equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(1-\sum_{j=b,e,i}\frac{\omega_{pj}^2(r)/\gamma_j^2}{\left(\omega - k_z V_{zj}\right)^2}\right)\frac{\partial}{\partial r}\delta\hat{\phi}\right] - k_z^2\left(1-\sum_{j=b,e,i}\frac{\omega_{pj}^2(r)/\gamma_j^2}{\left(\omega - k_z V_{zj}\right)^2}\right)\delta\hat{\phi} = 0$$

Here, $\omega_{pj}(r) = \left[4\pi n_j^0(r) e_j^2/\gamma_j m_j\right]^{1/2}$ Is the relativistic plasma frequency, $V_{zj} = \beta_j c = const$ is the average axial velocity of components *j* (*j*=*b*,*e*,*i*), and $\gamma_j = \left(1-\beta_j^2\right)^{-1/2}$ is the relativistic mass factor.

Two-stream instability is driven by the relative axial motion V_{zj} of the beam-plasma components.

Multispecies Two-Stream Instability



Plots of (a) $(\mathrm{Im}\,\omega)/\hat{\omega}^{i}_{pe}$ and (b) $(\mathrm{Re}\,\omega)/\hat{\omega}^{i}_{pe}$ versus $k_z r_b$

calculated from the two-stream dispersion relation for

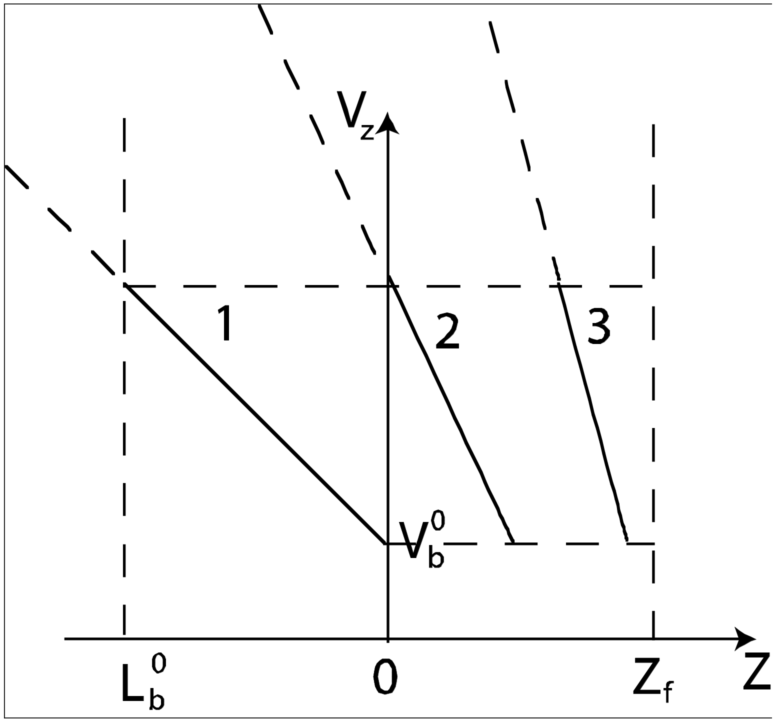
$r_b = r_w/3$, $\beta_b = 0.2$, $\beta_e = 0.1$, and $\hat{\omega}^{i}_{pe} r_b / c = 1/3$

Multispecies Two-Stream Instability

- A small axial momentum spread of the beam ions and the plasma ions leads to a reduction in the growth rate of the two-stream instability.

- Two-stream is likely to lead to a longitudinal heating of the plasma electrons in the nonlinear regime.

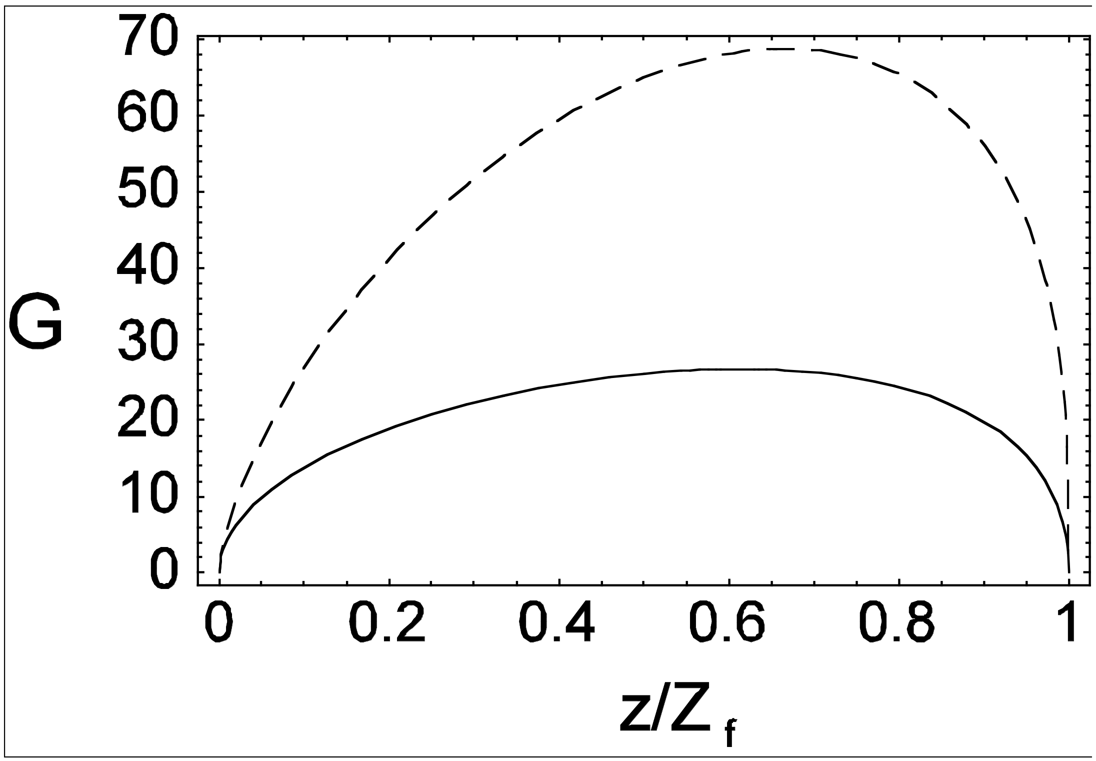
A longitudinal velocity tilt can significantly reduce the growth rate of the electron-ion two-stream instability*



Plot of ion beam phase space at different times during the compression. Line 1 corresponds to t=0.

*E. A. Startsev and R. C. Davidson, Nucl. Instr. Meth. Phys. Res. **A577,** 79 (2007).

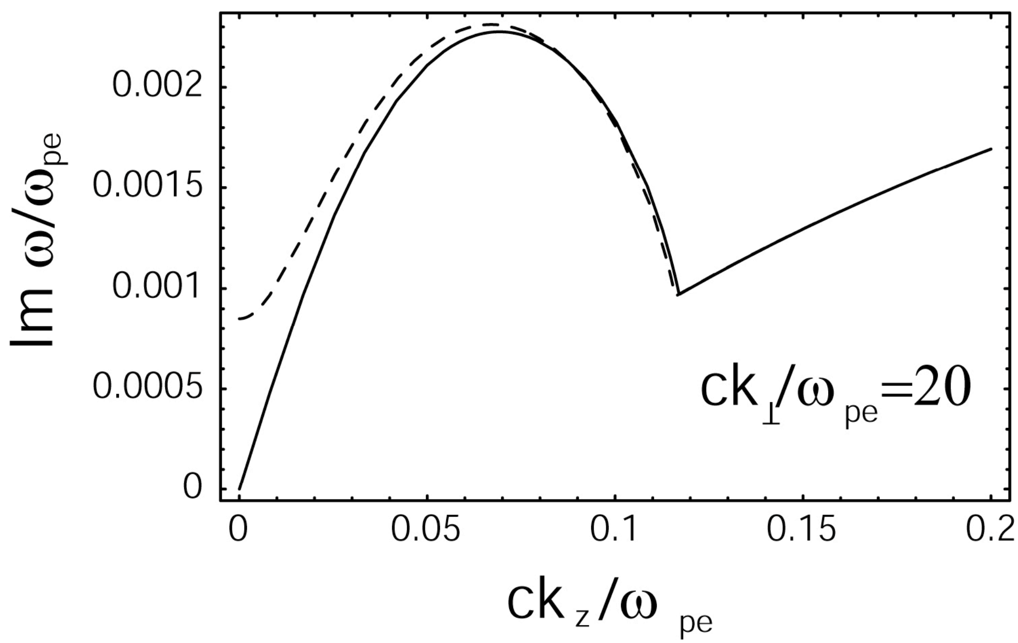
A longitudinal velocity tilt can significantly reduce the growth rate of the electron-ion two-stream instability*



Comparison of the instability gain G plotted as a function of axial distance for ion beam with velocity tilt (solid curve) and without velocity tilt (dashed curve).

*E. A. Startsev and R. C. Davidson, Nucl. Instr. Meth. Phys. Res. **A577,** 79 (2007).

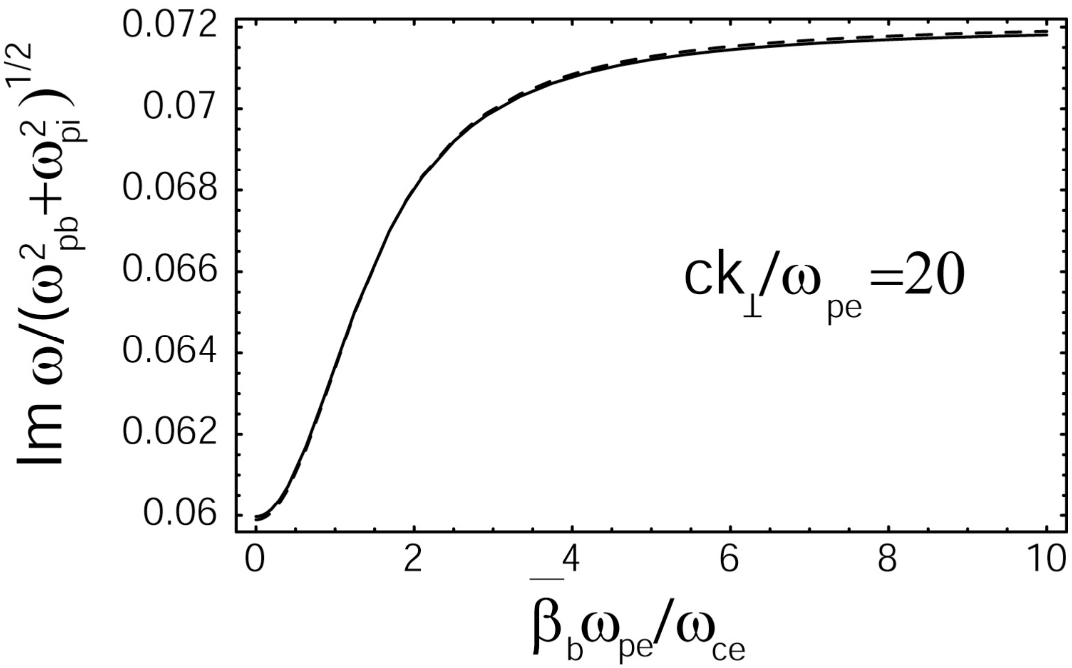
Two-stream instability for intense ion beam propagating through background plasma in a solenoidal magnetic field*



Plot of normalized growth rate of two-stream instabilty versus normalized axial wavenumber. Dashed line is the solution of the full electromagnetic dispersion relation, and the solid line is obtained from the approximate electrostatic dispersion relation.

* E. A. Startsev, R.C. Davidson and M. Dorf, Phys. Plasmas **15,** 062107 (2008).

Multispecies Weibel instability for intense ion beam propagating through background plasma in a solenoidal magnetic field*



Plots of normalized growth rate of multicpecies Weibel instabilty versus normalized beam velocity. Dashed line is the solution of the full electromagnetic dispersion relation, and the solid line is obtained from the approximate dispersion relation.

* E. A. Startsev, R.C. Davidson and M. Dorf, Phys. Plasmas **15,** 062107 (2008).

Conclusions

- A wide variety of collective instabilities in intense beams and beam-plasma systems have been investigated.

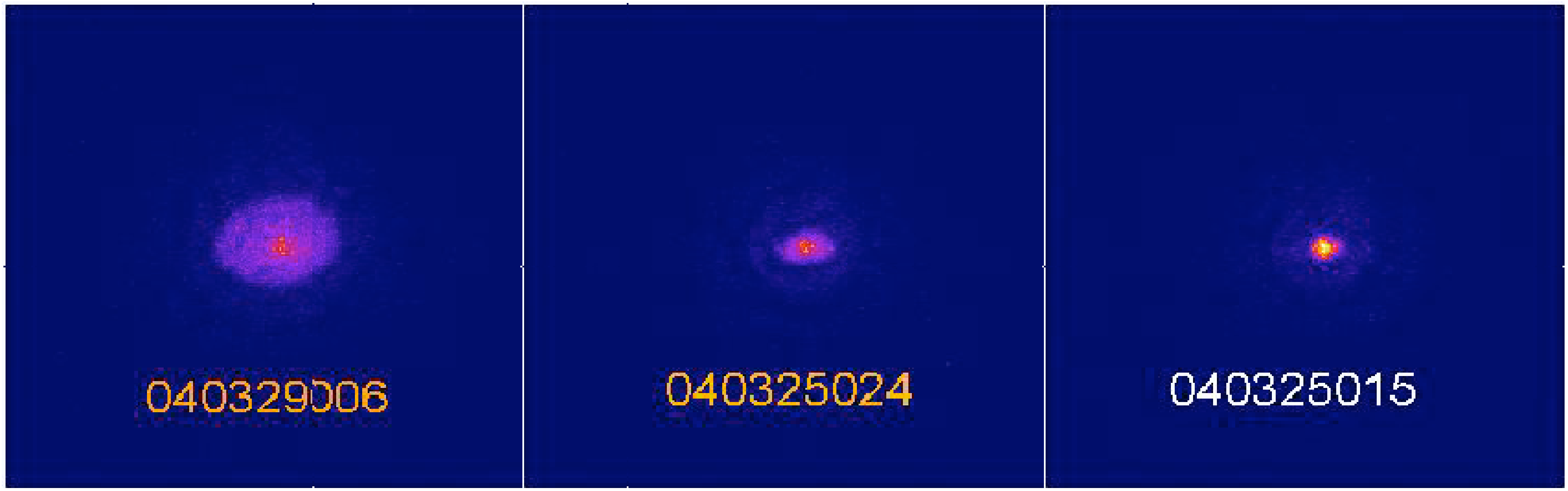
- Growth rate reduction or elimination mechanisms have been identified.

- Numerical simulations play a critical role in determining threshold conditions and nonlinear dynamics.

- Related publications can also be found at http://nonneutral.pppl.gov

Back-up Vugraphs

Measurements on the Neutralized Transport Experiment (NTX) demonstrate achievement of smaller spot size using volumetric plasma*



Neither plasma plug nor volumetric plasma. | Plasma plug. | Plasma plug and volumetric plasma.

* P. K. Roy, S. S. Yu et al. ,Phys. Rev. Lett. 95, 234801 (2005).

Class of Exact Dynamically-Compressing Neutralized Beam Equilibria with Linear Velocity Tilt in a Solenoidal Magnetic Field

In the definitions of $W_\perp$ and W_z, the quantities $z_b(t)$ and $r_b(t)$ solve

$$\frac{d^2 z_b(t)}{dt^2} = \frac{z_{b0}^2 \mathrm{v}_{T0}^2}{z_b^3(t)}$$

$$\frac{d^2 r_b(t)}{dt^2} + \mathring{\Omega}_L^2 r_b(t) = \frac{r_{b0}^2 \mathrm{v}_{\perp 0}^2}{r_b^3(t)}$$

and are related to the rms axial and transverse dimensions of the charge bunch.

The constants $z_{b0}^2 \mathrm{v}_{z0}^2$ and $r_{b0}^2 \mathrm{v}_{\perp 0}^2$ play the roles of scaled transverse and longitudinal emittances.

Dynamically-Compressing Neutralized Beam Equilibria

Many choices of beam equilibria are possible. As one example, consider

$$f_b(W_\perp, W_z) = const.\sqrt{(1 - W_z)}e^{-W_\perp}, \text{ where } 0 \le W_z < 1$$

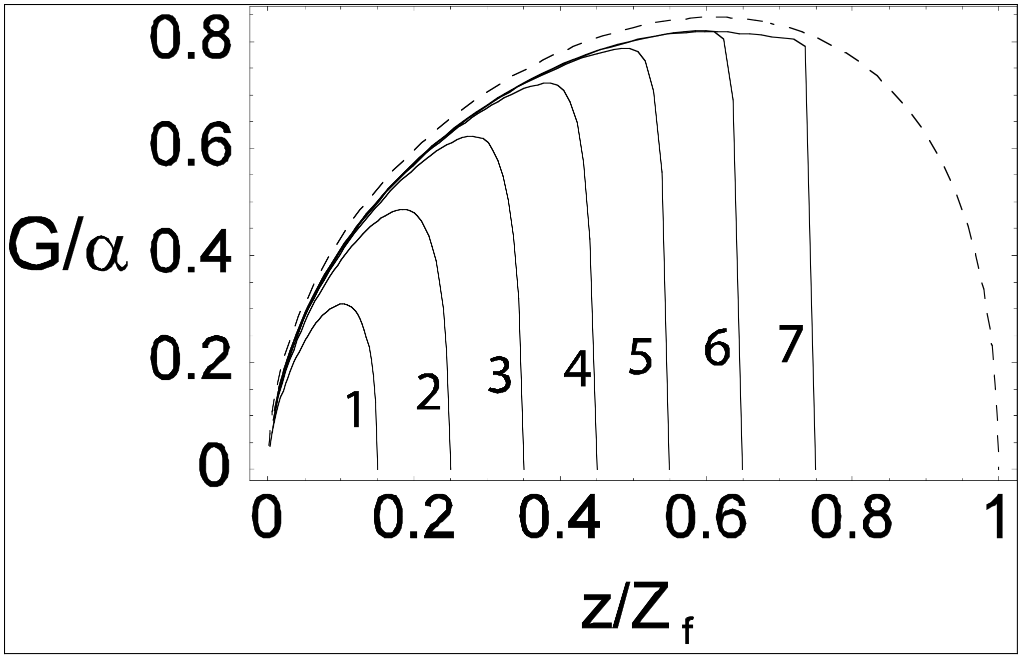
Then, the density profile $n_b(r, z, t)$ is given dynamically by

$$n_b(r, z, t) = \hat{n_{b0}} \left[\frac{r_{b0}^2}{r_b^2(t)} \frac{z_{b0}}{z_b(t)}\right] \left(1 - \frac{z^2}{z_b^2(t)}\right) e^{-r^2/r_b^2(t)}, \text{ where } 0 \le z^2 < z_b^2(t)$$

where $r_b(t)$ and $z_b(t)$ solve the envelope equations introduced earlier. For this choice of $f_b(W_\perp, W_z)$ note that the line density is parabolic with

$$\lambda_b(z, t) = \lambda_{b0} \left[\frac{z_{b0}}{z_b(t)}\right] \left(1 - \frac{z^2}{z_b^2(t)}\right), \text{ where } 0 \le z^2 < z_b^2(t)$$

A longitudinal velocity tilt can significantly reduce the growth rate of the electron-ion two-stream instability*



Normalized instability gain function is plotted as a function of axial distance at different times, obtained numerically (solid curves) and compared with the analytical estimate (dashed curve).

*E. A. Startsev and R. C. Davidson, Nucl. Instr. Meth. Phys. Res. **A577,** 79 (2007).