Some kinematic aspects of neutralized drift compression*

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A fully neutralized beam that drift-compresses as it propagates in the z direction is considered. Simple expressions are derived for the time at which the beam has minimal spatial extent, and for the z position at which it has minimal duration. A sample implementation (in the Python programming language), and output from a typical application, are presented.

Time of minimal pulse length

At any time t_0 once the beam is fully neutralized, it is possible to compute the time t_m of peak spatial compression (when the beam has its minimal RMS spatial length $z_{\mathsf{rms},m}$). One can then compute, for the beam at time t_m, a number of quantities of interest. These include $z_{\mathsf{rms},m}$; the "plane of shortest beam" $z = z_m$, which is the mean z of the particles; and the RMS pulse duration $\tau_{\mathsf{rms},m}$ through that plane (or some other measure of duration through that plane). We consider N particles, each particle i having its peculiar position z_{i0} and fixed velocity v_i at $t = t_0$. The output quantities that are likely to be of greatest interest are given by numbered equations. At any time,

$$z_{\mathsf{rms}} = \left[\frac{\sum_i (z_i - \bar{z})^2}{N}\right]^{1/2}$$

and

$$\frac{dz_{\mathsf{rms}}}{dt} = \frac{1}{z_{\mathsf{rms}} N} \sum_i (z_i - \bar{z})(v_i - \bar{v}) = \frac{1}{z_{\mathsf{rms}} N} \sum_i z_i (v_i - \bar{v}) \ .$$

The shortest length is achieved at the time when this derivative is zero:

$$\sum_i z_i (v_i - \bar{v}) = 0 \ .$$

The v_i are constant, so the z_i at any time can be computed from the starting values $\{z_{i0}\}$:

$$z_i = z_{i0} + v_i (t - t_0) \ ,$$

so that the condition for shortest length becomes:

$$\sum_i (z_{i0} + v_i (t_m - t_0))(v_i - \bar{v}) = 0 \ ,$$

or

$$\sum_i z_{i0} (v_i - \bar{v}) = -(t_m - t_0) \sum_i v_i (v_i - \bar{v}) \ .$$

Thus the required time shift $\tau_m = t_m - t_0$ is given by

$$\tau_m = -\frac{\sum_i z_{i0} (v_i - \bar{v})}{\sum_i v_i (v_i - \bar{v})} \ , \tag{1}$$

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and we have:

$$t_m = t_0 + \tau_m \ , \tag{2}$$

$$z_m = \bar{z}_0 + \bar{v}\tau_m \ , \tag{3}$$

and

$$z_{\mathsf{rms},m} = \left[\frac{\sum_i (z_{i0} + v_i \tau_m - z_m)^2}{N} \right]^{1/2} \ . \tag{4}$$

The travel time of particle i to z_m is $(z_m - z_{i0})/v_i$, so that the mean travel time is:

$$\bar{\tau}_m = \frac{\sum_i (z_m - z_{i0})/v_i}{N} \ ,$$

and the RMS travel time (pulse duration) is:

$$\tau_{\mathsf{rms},m} = \left[\frac{\sum_i [(z_m - z_{i0})/v_i - \bar{\tau}_m]^2}{N} \right]^{1/2} \ . \tag{5}$$

Axial location of minimal pulse duration

A similar calculation yields the plane z_d of true minimal pulse duration $\tau_{\mathsf{rms},d}$; typically z_d will vary only slightly from z_m as computed above, and $\tau_{\mathsf{rms},d}$ should be quite close to $\tau_{\mathsf{rms},m}$.

At any axial station z, the travel time τ_i of particle i is:

$$\tau_i = \frac{z - z_{i0}}{v_i} \ ,$$

with mean travel time

$$\bar{\tau} = \sum_i \tau_i / N \ ,$$

and RMS travel time (pulse duration)

$$\tau_{\mathsf{rms}} = \left[\frac{\sum_i (\tau_i - \bar{\tau})^2}{N} \right]^{1/2} \ .$$

The derivative is:

$$\frac{d\tau_{\mathsf{rms}}}{dz} = \frac{1}{\tau_{\mathsf{rms}} N} \sum_i (\tau_i - \bar{\tau})(\tau_i' - \bar{\tau}')$$

where

$$\tau_i' = \frac{d\tau_i}{dz} = \frac{1}{v_i} \ .$$

The derivative becomes:

$$\frac{d\tau_{\mathsf{rms}}}{dz} = \frac{1}{\tau_{\mathsf{rms}} N} \sum_i (\tau_i - \bar{\tau})(v_i^{-1} - \overline{v^{-1}}) \ ,$$

and the condition for minimal pulse duration is:

$$\sum_i \tau_i (v_i^{-1} - \overline{v^{-1}}) = 0 \ ,$$

or

$$\sum_i \frac{z - z_{i0}}{v_i}(v_i^{-1} - \overline{v^{-1}}) = 0 \ .$$

The value of z for which this condition obtains, z_d, is given by:

$$z_d \sum_i (v_i^{-2} - v_i^{-1}\overline{v^{-1}}) = \sum_i z_{i0}(v_i^{-2} - v_i^{-1}\overline{v^{-1}}) \ ,$$

or

$$z_d = \frac{\sum_i z_{i0}(v_i^{-2} - v_i^{-1}\overline{v^{-1}})}{\sum_i (v_i^{-2} - v_i^{-1}\overline{v^{-1}})} \ . \tag{6}$$

The mean time shift from t_0 for particle arrival at $z = z_d$ is:

$$\tau_d = \frac{1}{N} \sum_i \frac{z_d - z_{i0}}{v_i} \ , \tag{7}$$

and the mean arrival time at that plane is:

$$t_d = t_0 + \tau_d \ . \tag{8}$$

The RMS arrival time is:

$$\tau_{\mathsf{rms},d} = \left[\frac{\sum_i [(z_d - z_{i0})/v_i - \tau_d]^2}{N} \right]^{1/2} \ . \tag{9}$$

The mean particle position at time t_d is:

$$\bar{z}_d = \frac{\sum_i [z_{i0} + v_i \tau_d]}{N} \ ,$$

with RMS:

$$z_{\mathsf{rms},d} = \left[\frac{\sum_i [z_{i0} + v_i \tau_d - \bar{z}_d]^2}{N} \right]^{1/2} \ . \tag{10}$$

Python-language implementation, and output

The `np` particle positions are in the array `zp`, and their velocities in `vp`.

```
# --- instantaneous moments
zbar = sum(zp)/np
vbar = sum(vp)/np
zrms = sqrt(sum((zp-zbar)**2)/np)
taurmscrude = zrms/vbar
taubar = sum((zp-zbar)/vp)/np
taurms = sqrt(sum(((zp-zbar)/vp-taubar)**2)/np)
print ' ';print 'instantaneous moments'
print 't =',t
print 'zbar =',zbar
print 'vbar =',vbar
print 'zrms =',zrms
print 'taubar =',taubar
print 'taurmscrude =',taurmscrude
print 'taurms =',taurms
# --- at time of minimal pulse length
taum = - sum(zp*(vp-vbar))/sum(vp*(vp-vbar))
tm = t + taum
zm = zbar + vbar*(tm-t)
zrmsm = sqrt(sum((zp+vp*(tm-t)-zm)**2)/np)
taubarm = sum((zm-zp)/vp)/np
taurmsm = sqrt(sum(((zm-zp)/vp-taubarm)**2)/np)
print ' ';print 'at time of minimal pulse length'
print 'taum =',taum
print 'tm =',tm
print 'zm =',zm
print 'taurmsm =',taurmsm
print 'zrmsm =',zrmsm
print 'taubarm =',taubarm
# --- at z of minimal pulse duration
vpinvbar = sum(1./vp)/np
zd = sum(zp*(1./vp**2-vpinvbar/vp))/sum(1./vp**2-vpinvbar/vp)
taud = sum((zd-zp)/vp)/np
td = t + taud
taurmsd = sqrt(sum(((zd-zp)/vp-taud)**2)/np)
zbard = sum(zp+vp*taud)/np
zrmsd = sqrt(sum((zp+vp*(td-t)-zbard)**2)/np)
print ' ';print 'at z of minimal pulse duration'
print 'taud =',taud
print 'td =',td
print 'zd =',zd
print 'taurmsd =',taurmsd
print 'zrmsd =',zrmsd
print 'zbard =',zbard
```

Using particle positions and velocities from a 1-D test code, which had been advanced to a time slightly past peak compression, the following output was obtained. Note that $\tau_m, \tau_d < 0$ in this case. As can clearly be seen, the two formulations yield very similar results.

```
instantaneous moments
t = 3.798e-06
zbar = 16.1792849226
vbar = 9727761.89018
zrms = 0.0128757423499
taubar = -1.16579114943e-11
taurmscrude = 1.32360788589e-09
taurms = 1.30433323629e-09

at time of minimal pulse length
taum = -5.33558428714e-08
tm = 3.74464415713e-06
zm = 15.6602519877
taurmsm = 1.04641894803e-09
zrmsm = 0.0103180309813
taubarm = -5.33559510509e-08

at z of minimal pulse duration
taud = -5.23948308449e-08
td = 3.74560516916e-06
zd = 15.6695994749
taurmsd = 1.0463214719e-09
zrmsd = 0.0103189635271
zbard = 15.6696004839
```

Discussion

If the beam is not perfectly neutralized, these calculations might still yield good estimates if applied slightly upstream of best longitudinal focus; the instantaneous z_{rms} and/or τ_{rms} might be monitored, and as they start to level off the calculation carried out. In such a case this might provide useful guidance for when a simulation "snapshot" of the maximally compressed beam might be taken, and/or at what plane the target might optimally be placed.

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