LLNL-PROC-407596

# ESTIMATES OF ENERGY FLUENCE AT THE FOCAL PLANE IN BEAMS UNDERGOING NEUTRALIZED DRIFT COMPRESSION*

J. J. Barnard, LLNL, Livermore, CA 94550, USA, P. A. Seidl, J. E. Coleman, D. Ogata, LBNL, Berkeley, CA 94720, USA, D. R. Welch, Voss Scientific Corp., Albuquerque, NM, 87108, USA

*Abstract*

We estimate the energy fluence (energy per unit area) at the focal plane of a beam undergoing neutralized drift compression and neutralized solenoidal final focus, as is being carried out in the Neutralized Drift Compression Experiment (NDCX) at LBNL. In these experiments, in order to reach high beam intensity, the beam is compressed longitudinally by ramping the beam velocity (i.e. introducing a velocity tilt) over the course of the pulse, and the beam is transversely focused in a high field solenoid just before the target. To remove the effects of space charge, the beam drifts in a plasma. The tilt introduces chromatic aberrations, with different slices of the original beam having different radii at the focal plane. The fluence can be calculated by summing the contribution from the various slices. We develop analytic formulae for the energy fluence for beams that have current profiles that are initially constant in time. We compare with envelope and particle-in-cell calculations. The expressions derived are useful for predicting how the fluence scales with accelerator and beam parameters..

## INTRODUCTION

Recently, experiments have been carried out on the Neutralized Drift Compression Experiment (NDCX) at Lawrence Berkeley National Laboratory to investigate the use of injected plasma into a final drift compression line, final focus magnet, and target chamber to eliminate the effects of space charge [1], which, in turn, allows for maximum longitudinal compression and transverse final focusing. These beams have short final pulse duration $\Delta t_f$ and small focal spot radius $r_{spot}$ (defined at $2^{1/2}$ times the rms radius, when averaged over all beam particles). Because of the possibility of creating high beam intensities in a short pulse, the beams are being used to generate so called warm dense matter (WDM) conditions [2,3]. The main figures of merit for experiments are the beam fluence (beam energy per unit area integrated over the pulse) $E$ and $\Delta t_f$, since the attainable temperature is determined by $E$ as long as $\Delta t_f$ is much shorter than the hydrodynamic timescale for expansion. In this paper, we provide an analytic estimation of $E$ which can be useful for designing experiments that maximize $E$.

In the following sections, we first describe a simplified model for a final drift and focus section. We then outline the derivation of the estimate and.compare with more detailed numerical calculations (envelope and particle in cell), and finally we describe how we have used these three approaches to help design experiments for NDCX at LBNL.

## MODEL FOR ANALYTIC ESTIMATE

We assume that after the beam is accelerated to final velocity $v_0$, and energy $qV_0$ with charge state $q$, the beam exits the accelerator with 4 rms unnormalized transverse emittance $\varepsilon$. The beam passes through an induction bunching module gap that increases the velocity of the tail to $v_t$ and decreases the velocity of the head to $v_h$. The "tilt" is defined as $\Delta \equiv (v_t-v_h)/v_0$. The beam drifts a distance $L$ to the target, longitudinally compressing as it propagates, due to the tilt. A distance $f$ from the target, when the beam has radius $r_0$ the beam enters a solenoid of strength $B_{sol}$ and length $l_{mag}$, and exits the solenoid with the envelope converging angle $r_1'$ and radius $r_1$, setting the beam onto a final trajectory that focuses onto the target with radius $r_{spot}$. (Throughout this paper, envelope radii $r$, with or without subscripts, are defined as $2^{1/2}$ times the rms radius). A plasma is assumed to fill the drift section, the final solenoid and the target chamber (that includes the distance between the target and the solenoid). We further assume that the plasma density sufficiently exceeds the beam density so that the space charge forces within the beam are negligible. This implies that each slice of the beam retains the velocity $v \equiv v_0(1+\delta)$ it obtained in the bunching gap, and so each slice will have a slightly different focal length, and hence slightly larger focal spot at the target than the focal spot of the longitudinal center of the beam ($\delta$=0). Although formal analytic solutions to the kinetic equations describing drift compression have been obtained [4], exact closed form-scaling relations for the fluence have not, as of yet, been derived.

## ANALYTIC ESTIMATE OF FLUENCE $E$

The envelope equation for the beam radius $r$ for a beam without space charge may be written: $r'' = -\frac{k_c^2}{4} r + \frac{\varepsilon^2}{r^3}$. Here $k_c = qB_{sol}/(mv)$, and prime is derivative with respect to longitudinal position $z$. Within the solenoid, the emitttance term is small relarive to the focusing term, so we may solve the envelope equation: $r_1 = r_0 \cos\frac{k_c z}{2} + \frac{2r_0'}{k_c}\sin\frac{k_c z}{2}$. We assume that $r_0'$=0, as the change in $r'$ going through the solenoid is expected to be large. The condition that the beam comes to a focus at a distance after the magnet $f$-$l_{mag}$ is: $r_1' = \left(r_0/\left(f - l_{mag}\right)\right)\cos k_c l_{mag}/2 = (k_c r_0/2)\sin k_c l_{mag}/2$. This may be expressed as: $\eta_{mag} = \theta\tan\theta/(1+\theta\tan\theta)$. Here $\eta_{mag} = l_{mag}/f$, and $\theta = k_c l_{mag}/2$. The contribution to the spot

size from the emittance is thus: $r_{spot}^2\big|_{\text{emittance}} = \frac{\varepsilon^2}{r_1'^2} = \frac{\varepsilon^2 f^2}{r_0^2} F_1(\theta)$.

Here $F_1(\theta) = 1/(\cos\theta + \theta\sin\theta)^2$. For "off-momentum" slices the beam spot will be larger by the amount $dr_{spot} = r_1' df$ where the focal length is given by: $f = r_1 / r_1'$ so the change in focal length for off-momentum particles is: $df = dr_1 / r_1' - r_1 dr_1' / r_1'^2$. The contribution to the spot size from chromatic aberrations is thus $dr_{spot}\big|_{\text{chromatic}} = (\theta/\sin\theta) r_0 \delta$. So the spot radius from both emittance and chromatic aberrations can be written:

$$r_{spot}^2(\delta) = r_{spot}^2\big|_{\text{emittance}} + dr_{spot}^2\big|_{\text{chromatic}} = \varepsilon^2 f^2 F_1(\theta)/r_0^2 + r_0^2 \delta^2 F_2(\theta)$$

Here $F_2(\theta) = (\theta/\sin\theta)^2$. For the "thin lens" approximation $\theta \ll 1$, $F_1(\theta) \approx 1$ and $F_2(\theta) \approx 1$, and the "thick lens" approximation $\theta = \pi/2$, $F_1(\theta) = 4/\pi^2$ and $F_2(\theta) = \pi^2/4$. A more direct derivation of $r_{\text{spot}}(\delta)$ may be obtained by expanding the envelope equation in $\delta$ and integrating.

To calculate the central fluence $E(r=0)$, we must integrate the intensity from each slice. Since the phase space at the focus is rotated by $\sim\pi/2$ relative to the beginning of the soleonid, we expect (and assume) the spatial distribution for each slice to be close to a gaussian distribution in radius with an rms radius equal to $r_{\text{spot}}/2^{1/2}$ (for each $\delta$). This amounts to adding many different gaussians with different widths, yielding a non-gaussian distribution. The number of particles per unit area $n(r)$ at radius $r$ at the focus integrated over the pulse is thus:

$$n(r) = \int_{-l_b/2}^{l_b/2} \frac{dn(r)}{ds} ds = \int_{-\Delta/2}^{\Delta/2} \frac{dn(r)}{ds}\frac{ds}{d\delta} d\delta = \frac{l_b}{\Delta}\int_{-\Delta/2}^{\Delta/2} \frac{dn(r)}{ds} d\delta$$

Here $l_b$ is the length of the bunch before compression, and d$s$ is an element of beam along the beam length corresponding to an element of velocity tilt d$\delta$. Note that we are assuming that the velocity tilt imposed on the beam in the induction gap is linear, so that $\delta/\Delta = s/l_b$. By assumption:

$$\frac{dn(r)}{ds} = \frac{N_0/l_b}{\pi\sigma^2}\exp[-r^2/\sigma^2]$$

Here $\sigma = r_{\text{spot}}/2^{1/2}$, and $N_0$ is the total number of particles in the bunch, so that $N_0 = 2\pi\int_0^\infty dr\, r\, dn(r)/ds$. The bunch here is assumed to be constant current before bunch compression. The integral for $n(r)$ may be expressed:

$$n(r) = \frac{4N_0}{\pi f \varepsilon \Delta\sqrt{F_1(\theta)F_2(\theta)}} \int_1^{\sigma_{\max}/\sigma_{\min}} \frac{\exp[-r^2/(u^2\sigma_{\min}^2)]}{u\sqrt{u^2-1}} du \quad \text{where}$$

$$\sigma_{\min} = \frac{1}{\sqrt{2}}\frac{\varepsilon f\sqrt{F_1(\theta)}}{r_0} \quad \text{and} \quad \sigma_{\max} = \frac{1}{\sqrt{2}}\sqrt{\frac{\varepsilon^2 f^2 F_1(\theta)}{r_0^2} + \frac{r_0^2\Delta^2 F_2(\theta)}{4}}.$$

For $r=0$, the integral may be carried out:

$$n(r=0) = \frac{4N_0}{\pi f \varepsilon \Delta\sqrt{F_1(\theta)F_2(\theta)}} \tan^{-1}\left(\frac{r_0^2\Delta\sqrt{F_2(\theta)}}{2f\varepsilon\sqrt{F_1(\theta)}}\right) \quad . \qquad (1)$$

In the limit, that $\Delta$ approaches zero, the central integrated density is just that of an uncompressed emittance limited beam $n(r=0,\Delta=0) = 2N_0 r_0^2/[\pi f^2 \varepsilon^2 F_1(\theta)]$. The fluence $E$ is given by $E \approx qV_0\, n(r=0)$. In figure 1, we have plotted the fluence normalized to the fluence at $\Delta=0$, as a function of the argument of the inverse tangent to show potential gains if the chromatic aberrations were corrected (as in a time dependent correction concept now under study for NDCX). For the NDCX experiments listed in Table 1 (cases b and c), the argument has a range of 4 to 16, with corresponding values of $n(r=0)/n(r=0,\Delta=0)$ ranging 0.31 to 0.092, respectively, indicating potential fluence increases of 3 to 11, respectively if chromatic aberrations are corrected.

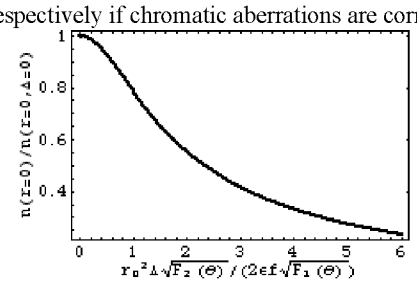


Figure 1. Fluence normalized to $\Delta=0$ fluence as function of quantity $r_0^2\Delta F_2(\theta)^{1/2}/[2\varepsilon f F_1(\theta)^{1/2}]$.

We may also estimate the spot radius of the integrated pulse. Integrating over all slices, the radius of the integrated pulse is given by:

$$r_{spot}^2 = \frac{\varepsilon^2 f^2}{r_0^2} F_1(\theta) + r_0^2 \eta^2 \Delta^2 F_2(\theta) \qquad (2)$$

Here $\eta \equiv \left(\int_{-l_b/2}^{l_b/2} I(s)(\delta(s)/\Delta)^2 ds\right)^{1/2} / \left(\int_{-l_b/2}^{l_b/2} I(s) ds\right)^{1/2}$, where $I(s)$ is the current as a function of longitudinal position $s$ along the beam before drift compression, with s=0 corresponding to the center of the beam. For $I(s)$=constant, then $\eta = 1/12^{1/2} \approx 0.29$. (For a parabolic pulse $\eta = 1/20^{1/2} \approx 0.22$). The quantity $r_{\text{spot}}$ is minimized when $r_0$ is such that the two terms in the equation for $r_{\text{spot}}^2$ are equal, namely: $r_{0_\text{opt}}^2 = \varepsilon f[F_1(\theta)F_2(\theta)]^{1/2}/(\eta\Delta)$ for which $r_{\text{spot_opt}}^2 = 2\varepsilon f \eta \Delta[F_1(\theta)F_2(\theta)]^{1/2}$. Note that, although there is an optimum $r_{0_\text{opt}}$ that minimizes the rms radius of the integrated spot $r_{\text{spot_opt}}$[5], increasing $r_0$ increases $E$ monotonically, only saturating as the inverse tangent in eq. 2 approaches $\pi/2$. However, the area over which $E$ is large decreases as $r_0$ increases beyond $r_{0_\text{opt}}$.

There are a number of assumptions that are built into equation (1). One assumption is that $r_0$ is constant for all $\delta$. When we rederive equation 1 replacing $r_0$ with $r_0 + \alpha L\delta$, where $\alpha$ is a constant (as can be expected from aberations from the induction bunching module), we find that the integral for $n(r=0)$ is unchanged, to lowest order in the parameter $\alpha L\delta/r_0$ as long as $\alpha L\delta/r_0 \ll 1$, with finite correction only in second order. Another assumption in the model, is that $r_0'$ is small, which usually is true relative to $r_1'$, but its impact on $r_{\text{spot}}$ has not yet been quantified analytically.

## COMPARISONS WITH NUMERICAL RESULTS

We have compared our analytic model with predictions of an envelope model and a particle in cell code LSP. The envelope model makes the same assumption as the

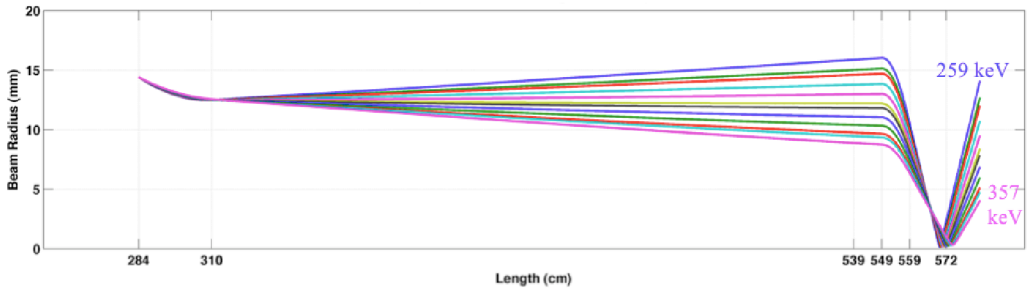


Figure 2. Calcuation of E using an envelope/slice model. Each color represents the trajectory of a different slice with different d. See text for further details.

Table 1. Comparison of $E$ calculated using envelope model, LSP, and Eq. (1)

| | $B_{sol}$ (T) | Initial pulse t(ns) | $qV_0$ (keV) | $r$ $z$=284 (mm) | $r'$ z=284 (mm) | $I$ at focus (A) | Δt_f (ns) | L (cm) | Δ | $f$ (cm) | r_0 (mm) | $E$ en-velope (J/cm$^2$) | $E$ LSP (J/cm$^2$) | $E$ Eq. (1) (J/cm$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a | 0 | 200 | 300 | 21.5 | -23.80 | 3.08 | 1.69 | 144 | 0.17 | | | 0.06 | | |
| b | 8 | 282 | 300 | 9.55 | -9.82 | 4.01 | 1.83 | 144 | 0.24 | 23 | 19.2 | 0.39 | 0.30 | 0.59 |
| c | 8 | 400 | 300 | 14.40 | -13.70 | 3.23 | 3.22 | 288 | 0.17 | 23 | 11.8 | 0.82 | 0.69 | 0.94 |

analytic model at the focal spot. Namely, take the spot radius for each slice as calculated by a numerical integration of the envelope equations, and assume a gaussian intensity profile for the slice at the target plane, summing the intensities over slices numerically. The advantage of the envelope model is that the finite $r'$ induced by the induction bunching module can be used as an initial condition at the beginning of the drift; the finite region where the beam is non-neutral can be accounted for; and so there are no assumptions about $r_0$ and $r_0'$.

The most detailed and accurate description of the drift compression and final focus is obtained using the LSP code [6,7]. LSP is a particle in cell code that includes fringe fields of the magnets and bunching module and models for calculating the plasma density and flow. The calculation includes first principle simulations of the beam through accelerator, drift, final focus, and chamber.

In figure 2, we plot an example of an envelope calculation using parameters from the NDCX experiment. The beam is 300 kV, 27 mA, singly charged potassium. The envelope calculation is initialized at $z$ = 284 cm, at the exit of the induction bunching module gap. The envelope slices are calculated assuming full space charge contribution, until the entrance to the neutralized section at $z$=310 cm. The beam propagates assuming full neutralization through the final focusing solenoid (549 < $z$ < 559 cm) to the focal plane ($z$= 572 cm).

Table 1 compares the final fluence $E$ for the numerical calculation of the envelope; LSP; and analytic model (eq. 1). The first row (a) corresponds to experiments without a final focusing solenoid. The second and third rows correspond to beamlines using a new NDCX induction bunching module, the final focusing solenoid (Bsol = 8 Tesla) and two beamline configurations: (b) with the present drift compression length ($L$=144 cm), and (c) with twice the drift compression length ($L$=288 cm) as the present setup. The two cases (b) and (c) show the effect of using a bunching module with a shorter bunch length (and relatively large tilt) with short drift length (since $L \approx l_b/\Delta$) versus using a larger bunch length and smaller tilt (and large drift length). This approximately keeps the "Volt-seconds" (roughly proportional to $\Delta\, l_b$) of the bunching module constant, which is constrained by the finite size of the induction core. As can be seen from the table the simulation yields somewhat smaller values of $E$, than the envelope results or analytic results, possibly due to imperfect neutralization, or effects of fringe fields, but the analytic calculation demonstrates the trends and scalings that can be useful for quick design estimates.

## CONCLUSION

We have estimated the energy fluence in a beam undergoing neutralized drift compression and neutralized solenoidal final focus, (eq. 1) and have compared it to envelope and particle in cell simulations. We find that the estimate is useful for understanding the general scaling of fluence on beam and accelerator parameters and for estimating system performance when alternative experiments are being considered.

## ACKNOWLEDGMENTS

This work was performed under the auspices of the U.S. DOE by LLNL under contract DE-AC52-07NA27344, and the Univ. of California, LBNL under Contract Number DE-AC02-05CH11231.

## REFERENCES

[1] P.A. Seidl et al, Proceedings of LINAC08 (these proceedings).
[2] F.M. Bieniosek et al, Nucl. Inst. Methods in Phys. Res. A, (submitted) 2008.
[3] J.J. Barnard et al, Nucl. Inst. Methods in Phys. Res. A.**577**, 275-283, (2007)
[4] R. C. Davidson and H.Qin, PRSTAB **8**, 064201 (2005).
[5] J.J. Barnard et al, Proceedings of the 2005 Part. Accel. Conf, 2568 - 2570 (2005).
[6] D.R. Welch et al, PRSTAB, **11**, 064701 (2008).
[7] A.B. Sefkow, et al, Nucl. Inst. Meth. in Phys. Res. A, 577, 289-297 (2007)