PROBLEM SET 1 DUE:
NE 290H BARNARD & LUND 1/28/2009

(20 POINTS) ① CONSIDER A ROUND UNIFORM ION BEAM WITH A CURRENT OF 1 AMPERE OF K^+ ($A = 39$) IONS, A KINETIC ENERGY OF 2 MeV, A BEAM RADIUS OF 2 cm AND NORMALIZED EMITTANCE OF 1 mm-mrad.

CALCULATE, FOR THESE BEAM PARAMETERS: (to 2 significant figures)

a) $\beta = v_0/c$

b) n = number density of ions in beam

c) $kT_\perp$ = transverse temperature (express in eV)

d) λ_D = transverse Debye LENGTH

e) Q = generalized perveance

f) Λ = plasma parameter

g) $\Delta\Phi$ = potential difference between center and edge of beam.

FOR REFERENCE:

$e = 1.6 \times 10^{-19}$ C [electron charge]

$k_B = 1.38 \times 10^{-23}$ J K^{-1} [Boltzmann's const]

$\epsilon_0 = 8.854 \times 10^{-12}$ F m^{-1} [permittivity of free space]

$c = 3 \times 10^8$ m s^{-1} [speed of light]

$m_u = 1.66 \times 10^{-27}$ kg

$m_u c^2 = 931.1 \times 10^6$ eV

[atomic mass unit]

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(2). Show that

$$\langle r \frac{\partial \varphi}{\partial r} \rangle = \frac{-\lambda}{4\pi\epsilon_0}$$

for a charge distribution in which $\rho(r,\theta) = \rho(r)$ only.

[λ = line charge density = $\int_0^\infty 2\pi r \rho(r)\, dr$

$\langle g \rangle = \frac{1}{\lambda} \int_0^\infty g(r)\, 2\pi r \rho(r)\, dr$]

where g is any beam quantity

PROBLEM (3)
[20 POINTS]

Let the single particle equation of motion of a particle be:

$$x'' = -\alpha(s)\, x^n$$

Here x is the usual transverse coordinate and s is the longitudinal coordinate.

Calculate the derivative with respect to s of the square of the emittance $\epsilon^2 = 16[\langle x^2\rangle\langle x'^2\rangle - \langle x x'\rangle^2]$.

Express $\frac{d\epsilon^2}{ds}$ in terms of $\langle x^2\rangle$, $\langle x x'\rangle$, $\langle x' x''\rangle$ and $\langle x^{n+1}\rangle$.

For what value of n is $\frac{d\epsilon^2}{ds}$ identically zero?