

20
POINTS

1) CONSIDER A ROUND UNIFORM ION BEAM WITH A CURRENT OF 1 AMPERE OF K^+ ($A=39$) IONS, A KINETIC ENERGY OF 2 MeV, A BEAM RADIUS OF 2 CM AND NORMALIZED EMITTANCE OF 1 mm-mrad.

CALCULATE, FOR THESE BEAM PARAMETERS: (to 10 significant figures)

- a) $\beta = v_0/c$
- b) n = number density of ions in beam
- c) kT_{\perp} = transverse temperature (express in eV)
- d) λ_D = transverse Debye LENGTH
- e) Q = generalized perveance
- f) Λ = plasma parameter
- g) $\Delta\Phi_{\perp}$ = potential difference between center and edge of beam.

FOR REFERENCE:

$e = 1.6 \times 10^{-19} \text{ C}$ [electron charge]

$k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$ [Boltzmann's const]

$\epsilon_0 = 8.854 \times 10^{-12} \text{ F m}^{-1}$ [permittivity of free space]

$c = 3 \times 10^8 \text{ m s}^{-1}$ [speed of light]

$m_0 = 1.66 \times 10^{-27} \text{ kg}$
 $m_0 c^2 = 931.1 \times 10^6 \text{ eV}$
 [atomic mass unit]

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2. Show that

$$\left\langle r \frac{\partial \phi}{\partial r} \right\rangle = \frac{-\lambda}{4\pi\epsilon_0}$$

for a charge distribution in which $\rho(r, \theta) = \rho(r)$ only.

$$\left[\lambda = \text{line charge density} = \int_0^{\infty} 2\pi r \rho(r) dr \right.$$

$$\left. \left\langle g \right\rangle = \frac{1}{\lambda} \int_0^{\infty} g(r) 2\pi r \rho(r) dr \right]$$

where g is any beam quantity

PROBLEM 3
[20 POINTS]

Let the single particle equation of motion of a particle be:

$$x'' = -\alpha(s) x^n$$

Here x is the usual transverse coordinate and s is the longitudinal coordinate.

Calculate the derivative with respect to s of the square of the emittance $\epsilon^2 = 16 [\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2]$.

Express $\frac{d\epsilon^2}{ds}$ in terms of $\langle x^2 \rangle$, $\langle x x' \rangle$, $\langle x' x'' \rangle$ and $\langle x^{n+1} \rangle$.

For what value of n is $\frac{d\epsilon^2}{ds}$ identically zero?