John Barnard
Steven Lund
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U.C. Berkeley

Intrabeam collisions, gas and electron effects in intense beams

1. Beam/beam coulomb collisions
2. Beam/gas scattering
3. Charge changing processes
4. Gas pressure instability
5. Electron cloud processes
6. Electron-ion instability

(2)

Gas and electron effects

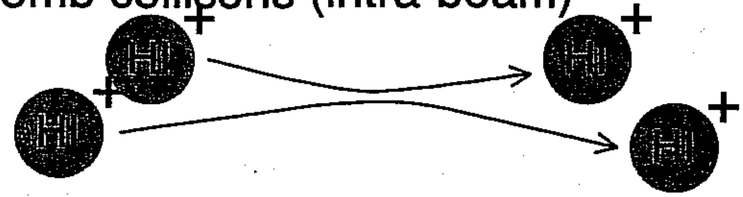
-Effects are quite different depending on q, m of species being accelerated

-Circular accelerators vs. Linacs
($t_{residence}$ ~ ms to days vs. 10's of μs)

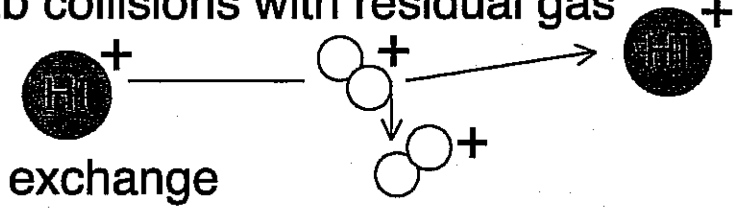
-Long pulse vs. short pulse
(t_{pulse} ~ 10's of μs vs. 10's of ns)

Processes:

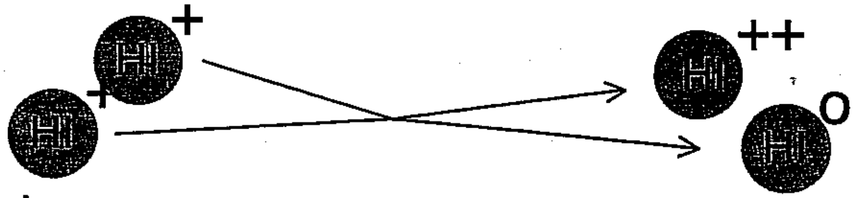
1. Coulomb collisons (intra-beam)



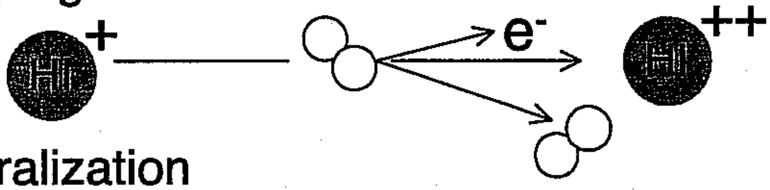
2. Coulomb collisions with residual gas



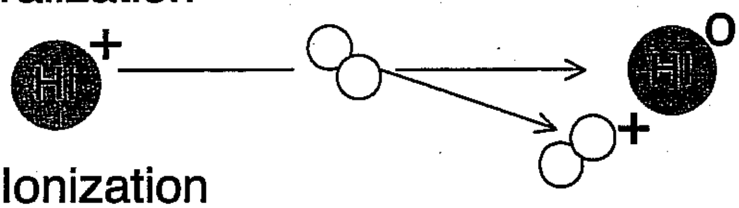
3. Charge exchange



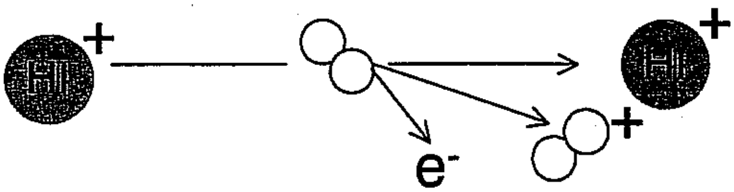
4. Stripping



5. Neutralization



6. Gas Ionization



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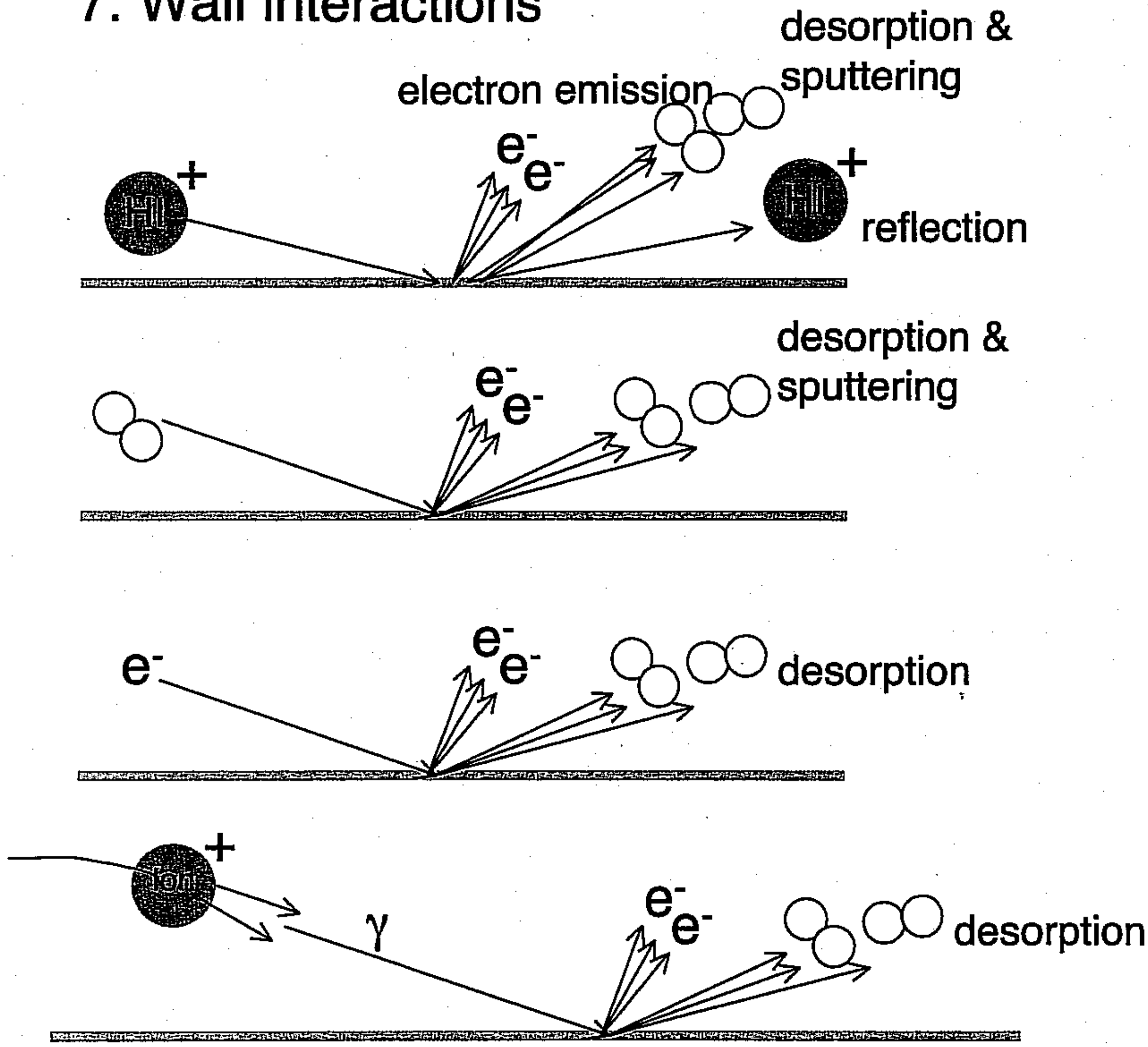
7. Wall interactions



γ	synchrotron photon

1. COLLISIONS WITHIN BEAM   REISER 6.4

CONSIDER EFFECTS OF COULOMB COLLISIONS IN A CONTINUOUS BEAM PROPAGATING THROUGH A SMOOTH FOCUSING CHANNEL WITH $T_{\perp 0} \neq T_{\parallel 0}$

(IF $T_{\perp 0} = T_{\parallel 0} \Rightarrow$ BEAM ALREADY RELAXED).

FROM ICHIMARU & ROSENBLUTH, PHYS FLUIDS 13, 2778, (1970):

$$\frac{dT_\perp}{dt} = -\frac{1}{2}\frac{dT_\parallel}{dt} = \frac{-(T_\perp - T_\parallel)}{\tau}$$

(SINCE $T_x = T_y = T_\perp$, $T_\parallel$ CHANGES AT TWICE THE RATE OF $T_\perp$)
(SINCE $2k_B T_\perp + k_B T_\parallel = \text{const}$)

τ = RELAXATION TIME

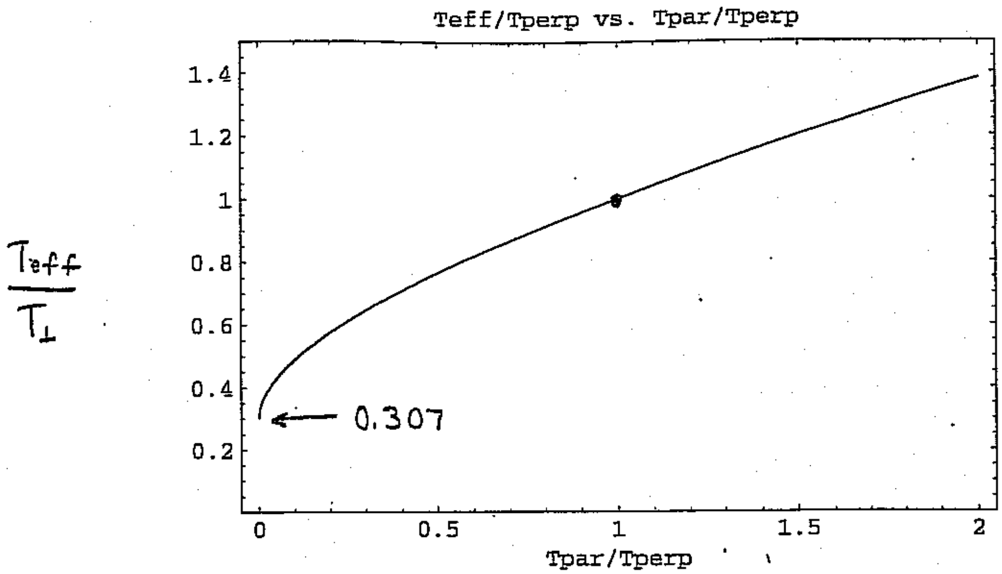
$$= \frac{15\,(k_B T_{eff}/mc^2)^{3/2}(4\pi\epsilon_0)^2 m^2 c^3}{8\pi^{1/2} q^4 \ln\Lambda \; n} = \left(\frac{15\,\pi^{1/2}}{8\ln\Lambda}\right)\nu_c^{-1}$$

COULOMB COLLISION APPROXIMATE RATE FOR LARGE ANGLES (PAGE 9 OF INTRODUCTION NOTES)

$$\ln\Lambda = \begin{cases} \ln \dfrac{(\epsilon_0 k T_{eff})^{3/2} 12\pi}{q^3 n^{1/2}} & \text{for } \lambda_D < r_b \\[2ex] \ln \dfrac{12\pi\epsilon_0 k T_{eff} r_b}{q^2} & \text{for } \lambda_D > r_b \end{cases}$$

$$T_{eff} = T_\perp \left[\frac{15}{4}\int_{-1}^{1} \frac{\mu^2(1-\mu^2)\,d\mu}{[(1-\mu^2) + \mu^2 (T_\parallel/T_\perp)]^{3/2}}\right]^{-2/3}$$

T_{eff} is an appropriate average of $T_\perp$ & $T_\parallel$



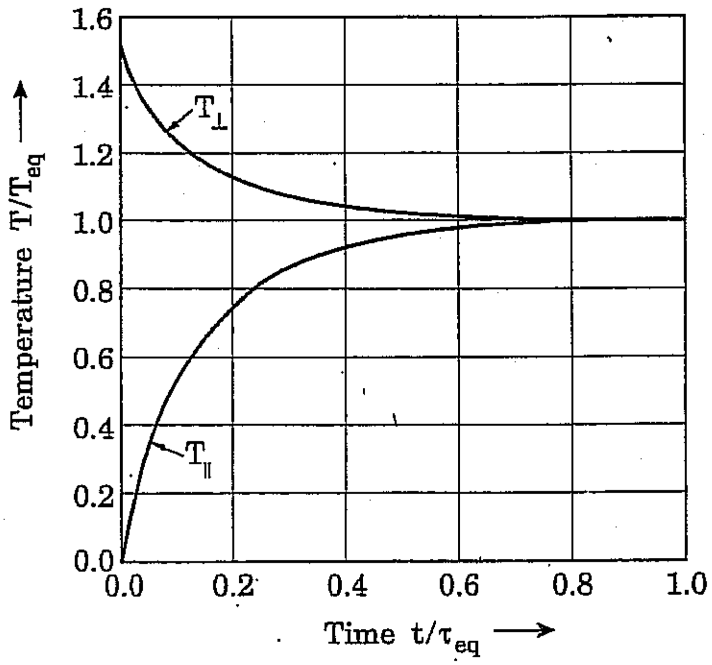
FOR $T_\parallel = 0$

$$T_\perp = \frac{2}{3}\,T_{\perp 0}\left(1 + \frac{1}{2}e^{-3t/\tau_{\text{eff}}}\right), \qquad (6.156a)$$

(APPROXIMATE SOLUTIONS)

$$T_\parallel = \frac{2}{3}\,T_{\perp 0}\left(1 - e^{-3t/\tau_{\text{eff}}}\right), \qquad (6.156b)$$

$\tau_{eff} = 0.42\,\tau_{eq}$



FROM REISER p.527

$\tau_{eq} = \tau(T_{eq})$

BOERSCH EFFECT

AREN'T COLLISIONS NEGLIGIBLE? (NOT ALWAYS)

PUTTING IN NUMBERS:

FOR IONS:

$$\tau_{eff} = 4.3 \cdot 10^{-4}\,s \; \frac{(A^{1/2})}{Z^4} \left(\frac{kT_{eff}}{1\,eV}\right)^{3/2} \left(\frac{15}{\ln\Lambda}\right) \left(\frac{10^{10}\,cm^{-3}}{n}\right)$$

$$\ln\Lambda = \ln\left[\frac{1.5\cdot 10^5\,(kT/1eV)^{3/2}}{Z^3\,(n/10^{10}\,cm^{-3})}\right]$$

EXAMPLE: 2 MeV INJECTOR

$\tau_{eff} \simeq 8.8\cdot 10^{-4}\,s$ for $A = 39$, $kT_{eff} = 0.3\,eV$, $Z = 1$, $\ln\Lambda = 8.5$, $n = 10^{10}\,cm^{-3}$

$$t_{transit} \simeq \frac{2d}{v} \approx \frac{2\,(2m)}{(.01)\,3\cdot 10^8} = 1.3\,\mu s$$

So $\tau_{eff} \gg t_{transit} \Rightarrow$ collisions are rare BUT

$$T_{\substack{accel\\cool}} = \frac{1}{2}\left(\frac{kT_0}{qV}\right) kT_0 = 2.5\cdot 10^{-9}\,eV \quad \text{for } kT_0 = 0.1\,eV,\ qV = 2\,MeV$$

$$T_{\parallel\,collisions} \simeq \frac{2}{3} T_{\perp 0}\left(1 - \exp\left(-3t/\tau_{eff}\right)\right) \simeq 2T_{\perp 0}\left(\frac{t_{transit}}{\tau_{eff}}\right) = .006\,eV \quad \text{for } T_{\perp 0} = 1\,eV$$

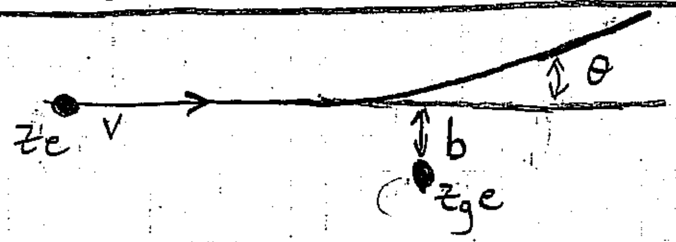
SO $T_{\parallel}$ FROM "BOERSCH EFFECT"

$\gg$ $T_{\parallel}$ FROM LONGITUDINAL COOLING

Coulomb Collisions in Residual Gas (Reiser 6.4.3)

Jackson Chapter 13

(Rutherford Scattering)



$$\frac{dp_x}{dt} = \frac{ZZ_g e^2}{4\pi\epsilon_0 r^2}\frac{b}{r} \implies \Delta p = \int_{-\infty}^{\infty} \frac{dp_x}{dt}\frac{dt}{dz}\,dz$$

$$= \frac{ZZ_g e^2 b}{4\pi\epsilon_0 v}\underbrace{\int_{-\infty}^{\infty}\frac{dz}{(z^2+b^2)^{3/2}}}_{2/b^2}$$

$$= \frac{2ZZ_g e^2}{4\pi\epsilon_0 v b}$$

$$\theta \cong \frac{\Delta p}{p} = \frac{2ZZ_g e^2}{4\pi\epsilon_0 p v b} \implies \frac{db}{d\theta} \sim \frac{1}{\theta^2}$$

Differential cross section for scattering with impact parameter b into solid angle $d\Omega$ at angle θ satisfies

$$\underbrace{2\pi b\,db}_{\text{AREA}} = \frac{d\sigma}{d\Omega}\underbrace{2\pi \sin\theta\, d\theta}_{\text{SOLID ANGLE}}$$

$$\implies \frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \left(\frac{2ZZ_g e^2}{4\pi\epsilon_0 p v}\right)^2 \frac{1}{\theta^4}$$

Electron screening puts cutoff at small θ (large b) so better to use

$$\frac{d\sigma}{d\Omega} = \left(\frac{2ZZ_g e^2}{4\pi\epsilon_0 p v}\right)^2 \frac{1}{(\theta^2+\theta_{min}^2)^2}$$

AVERAGE ANGLE SQUARED FOR A SINGLE SCATTERING IS:

$$\bar{\theta}^2 = \frac{\int \theta^2 \frac{d\sigma}{d\Omega} 2\pi \sin\theta \, d\theta}{\int \frac{d\sigma}{d\Omega} 2\pi \sin\theta \, d\theta} \simeq \frac{\int_0^{\theta_{max}} \frac{\theta^3}{(\theta^2+\theta_{min}^2)^2} d\theta}{\int_0^{\theta_{max}} \frac{\theta}{(\theta^2+\theta_{min}^2)^2} d\theta}$$

$$\simeq 2\theta_{min}^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right)$$

ASSUMES $\theta_{max}^2 >> \theta_{min}^2$

& $\ln\left(\frac{\theta_{max}}{\theta_{min}}\right) >> 1$

MULTIPLE COLLISIONS

AFTER TRAVERSING DISTANCE s,
AND UNDERGOING N_s COLLISIONS, THE
MEAN SQUARE ANGLE $\overline{\Theta}^2$ $\left[\sigma_s = \pi\left(\frac{zZ_g e^2}{4\pi\epsilon_0 pv}\right)^2 \frac{1}{\theta_{min}^2}\right]$

$$\overline{\Theta}^2 = N_s \bar{\theta}^2 = n_g \sigma_s s \bar{\theta}^2$$

$$= 16\pi n_g \left(\frac{zZ_g e^2}{4\pi\epsilon_0 mc^2 \gamma \beta^2}\right)^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right) s$$

JACKSON ARGUES θ_{max} ARISES FROM DISTRIBUTED
NATURE OF NUCLEUS (NOT POINT CHARGE)
AND θ_{min} ARISES FROM SCREENING OF ELECTRONS
OR UNCERTAINTY PRINCIPLE

$$\ln\frac{\theta_{max}}{\theta_{min}} \simeq \ln\left(204\, Z_g^{-1/3}\right)$$

$$\Delta\langle x'^2\rangle = \frac{1}{2}\,\overline{\Theta^2} \qquad \left[\text{since } \Delta\langle x'^2\rangle + \Delta\langle y'^2\rangle = \Delta\overline{\Theta^2}\right]$$

$$\epsilon = 4\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

FOR A MATCHED BEAM

$$k_p^2\langle x^2\rangle = \langle x'^2\rangle \qquad \text{where } k_p = \text{depressed betatron wavenumber}$$

$$\Rightarrow \quad \epsilon = \frac{4\langle x'^2\rangle}{k_p}$$

$$\Delta\epsilon = \frac{4\,\Delta\langle x'^2\rangle}{k_p} = \frac{2\,\overline{\Theta^2}}{k_p}$$

$$\Rightarrow \quad \boxed{\frac{d\epsilon}{ds} = \frac{32\pi}{k_p}\, n_g \left(\frac{Z Z_g e^2}{4\pi\epsilon_0 mc^2\gamma\beta^2}\right)^2 \ln\left(204\, Z_g^{-1/3}\right)}$$

IN TERMS OF NORMALIZED EMITTANCE:

$$\boxed{\frac{d\epsilon_N}{ds} = \frac{32\pi}{k_p}\, n_g \left(\frac{Z Z_g e^2}{4\pi\epsilon_0 mc^2}\right)^2 \frac{1}{\gamma\beta^3} \ln\left(204\, Z_g^{-1/3}\right)}$$

Example: $n_g = 10^{-7}$ torr $= 3.5\cdot10^{9}$ cm^{-3} $= 3.5\cdot10^{15}$ m^{-3}

$k_{p0} = 2.5$ m^{-1} $\quad k_p = 0.25$ m^{-1}

$Z_g = 7$; $Z = 19$, $A = 39$, $\beta = 0.01$, $\epsilon_N = 1\cdot10^{-6}$ m-rad

$\Rightarrow \dfrac{d\epsilon_N}{ds} = 3.7\cdot10^{-11}$ m^{-1} $\Rightarrow$ Need 27 km to equal original emittance!

(But more important for rings & low mass!)

PRESSURE "BUMPS" AND
BEAM LOSS FROM CHARGE CHANGING COLLISIONS

REFERENCE: WORKSHOP ON BEAM INDUCED PRESSURE RISE IN RINGS, BNL, Dec. 2003,

σ_s = STRIPPING CROSS SECTION

σ_{ce} = CHARGE EXCHANGE CROSS SECTION

σ_i = IONIZATION CROSS SECTION

V_{cm} = mean ion velocity in ion beam frame

(1) BEAM LOSS

$$\frac{dn_b}{dt} = -\sigma_s V_i n_b \bar{n} - \sigma_{ce} V_{cm} n_b^2 - \left.\frac{\partial n_b}{\partial t}\right|_{HALO}$$

(2) GAS EVOLUTION

$\bar{n}$ = average gas density

$$\frac{d}{dt}\bar{n} = \underbrace{\eta_G \sigma_i V_i n_b \bar{n}\left(\frac{V_{beam}}{V_{pipe}}\right)}_{\text{IONIZATION}} + \underbrace{\eta_{HI} \sigma_s V_i n_b \bar{n}\left(\frac{V_{beam}}{V_{pipe}}\right)}_{\text{STRIPPING}}$$

VOLUME OF BEAM

VOLUME OF PIPE

$$+ \underbrace{\eta_{HI} \sigma_{ce} V_{cm} n_b^2\left(\frac{V_{beam}}{V_{pipe}}\right)}_{\text{CHARGE EXCHANGE}} + \underbrace{q}_{\text{OUTGASSING}} - \underbrace{(S/A_p)\,\bar{n}}_{\text{PUMPING}}$$

S = Effective linear pumping rate [$m^3 s^{-1}/m$] $\quad A_p = \pi r_p^2$ = AREA OF PIPE

q = OUTGASSING rate $= \frac{2\pi r_p Q}{\pi r_p^2} = \frac{2Q}{r_p}$; $\quad Q = \frac{\#}{m^2 s}$

η_G = GAS MOLECULES DESORBED PER INCIDENT RESIDUAL GAS ION

η_{HI} = GAS MOLECULES DESORBED PER INCIDENT HEAVY ION STRIKING WALL

($V_{beam}/V_{pipe} \rightarrow \left(\frac{r_b^2}{r_{pipe}^2}\right) \nu_{rep}\, \Delta t_{beam}$ for a rep rated linac)

If we take $n_b \simeq$ constant

then we may express gas evolution equation as:

$$\frac{d\bar{n}}{dt} = \frac{\bar{n}}{\tau} + q_{eff}$$

with solution:

$$\bar{n} = (\bar{n}_0 + \tau q_{eff}) \exp[t/\tau] - \tau q_{eff}$$

HERE $\tau = \dfrac{1}{(\eta_g \sigma_i + \eta_{HI} \sigma_s)\left(\frac{V_{beam}}{V_{pipe}}\right) n_b v_i - S/A_p}$

$$q_{eff} = q + \eta_{HI} \sigma_{CE} V_{cm} n_b^2 \left(\frac{V_{beam}}{V_{pipe}}\right)$$

EQUILIBRIUM REACHED IF $\tau < 0$ (i.e. pumping exceeds desorption).

$$\Rightarrow \bar{n} = -\tau q_{eff} = \frac{q + \eta_{HI} \sigma_{ce} V_{cm} n_b^2 (V_{beam}/V_{pipe})}{S/A_p - (\eta_g \sigma_i + \eta_{HI} \sigma_s)\left(\frac{V_{beam}}{V_{pipe}}\right) n_b v_i}$$

INSTABILITY IF $n_b v_i \geq \dfrac{\frac{S}{A_p}\left(\frac{V_{pipe}}{V_{beam}}\right)}{\eta_g \sigma_i + \eta_{HI} \sigma_s}$

Instability first observed on the ISR proton storage ring, limiting current in rings in 1970's.

If $l_{beam} = l_{pipe}$

INSTABILITY CRITERION MAY BE WRITTEN

$$I > \frac{ZeS}{\eta_g \sigma_i + \eta_{HI} \sigma_s}$$

EXAMPLE: If $S = 100 \, \ell \, s^{-1} m^{-1} = 0.1 \, m^3 s^{-1} m^{-1}$

ISR

$\eta_g = 4$

$\sigma_i = 10^{-22} \, m^2 = 10^{-16} \, cm^2$; $\sigma_s = 0$

$Z = 1$ (protons)

$\Rightarrow I \leq 40$ Amperes

(PRESSURE RUNAWAYS WERE OBSERVED ON the ISR AT 14-18A, (BENVENUTI et al, IEEE Trans. on Nuc. Sci. NS-24, 1773, 1977)

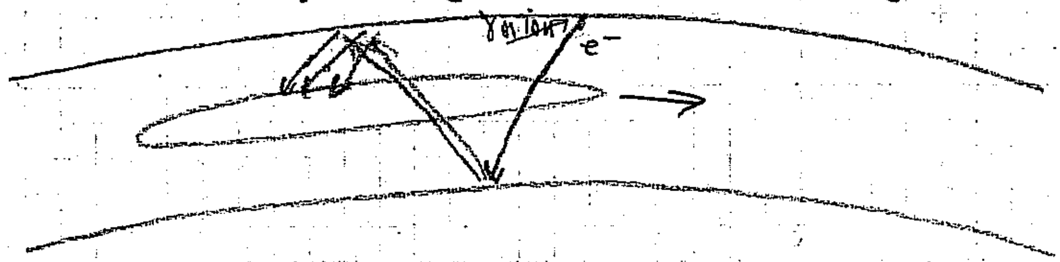
SEE "BEAM INDUCED PRESSURE RISE IN RINGS"
13th ICFA BEAM DYNAMICS MINI WORKSHOP, BNL, Dec. 9-12, 2003.
WEBSITE: http://www.c-ad.bnl.gov/icfa

"ELECTRON CLOUD EFFECTS"

REFERENCE: CERN e-CLOUD WORKSHOP
http://wwwslap.cern.ch/collective/ecloud02/proceedings.html

BASIC IDEA

IN ION storage rings or collider rings:



ELECTRONS ARE ATTRACTED TO POSITIVE POTENTIAL OF BEAM & ACCUMULATE

SOME SYMPTOMS:

1. BEAM LOSS & pressure rise
2. HIGH FREQUENCY CENTROID oscillations

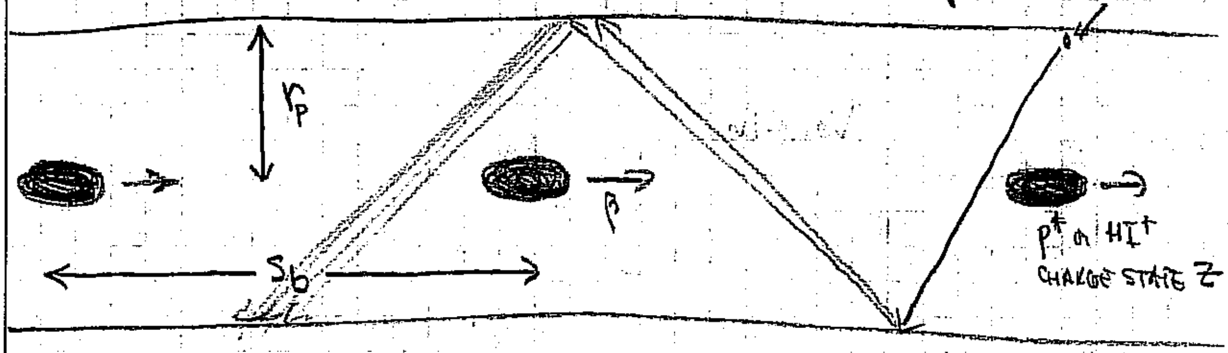
SOME ACCELERATORS WHICH SHOW EVIDENCE OF e^- EFFECTS
1. LANL PSR
2. CERN PS & SPS
3. BNL RHIC

COULD LIMIT PLANNED/UNDER CONSTRUCTION:
1. SNS ACCUMULATOR RING
2. LHC

cf. "Electron-cloud effects in High Intensity Proton Accelerators" J. Wei & R. Macek, CERN

BEAM INDUCED MULTIPACTING

a). MULTI-BUNCH



USING COULOMB COLLISION FORMULA FROM PAGE 9:

$$\Delta p_x \cong \frac{2 Z N_b e^2}{4\pi\epsilon_0 v r_p}$$

N_b = Number of ions of charge Z in bunch

$$\Delta E_e = m_e c^2\left[\sqrt{\frac{\Delta p_x^2}{m^2c^2}+1} - 1\right] = m_e c^2\left[\sqrt{\left(\frac{2 r_e Z N_b}{\beta r_p}\right)^2 + 1} - 1\right]$$

$$\cong 2 r_e^2 m_e c^2 \frac{Z^2 N_b^2}{\beta^2 r_p^2} \quad \text{for } \Delta E_e \ll m_e c^2 \quad \left(\text{or } \frac{2 r_e Z N_b}{\beta r_p} \ll 1\right)$$

(where $r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2} \simeq 2.8\times10^{-15}\,\text{m}$)

DEFINE A MULTIPACTING PARAMETER $\mathcal{J}_m$

$$\mathcal{J}_m = \frac{\text{TIME FOR ELECTRON TO CROSS PIPE}}{\text{TIME BETWEEN BUNCHES}} = \frac{2 r_p}{s_b}\frac{\beta}{\beta_e}$$

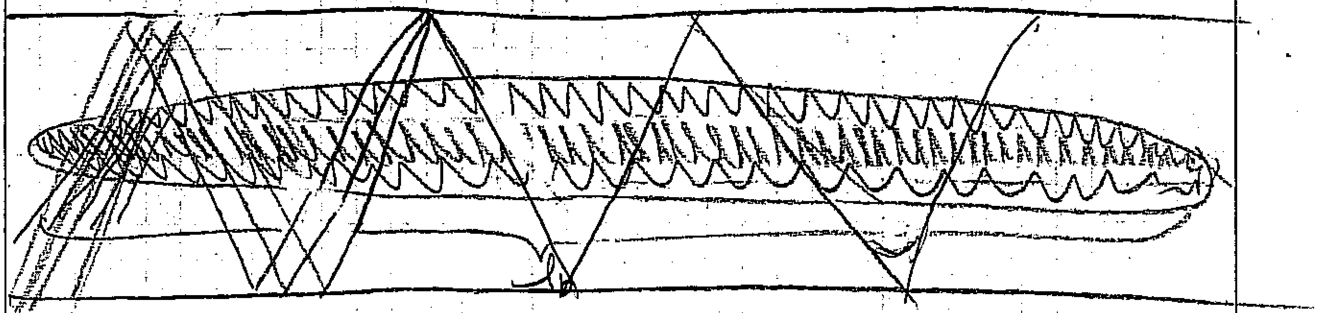
$$\cong \frac{\beta^2 r_p^2}{Z N_b r_e s_b}$$

RESONANCE CONDITION:

$$\mathcal{J}_m = 1$$

s_b = distance between bunches

b). SINGLE-BUNCH BEAM-INDUCED MULTIPACTING



$$\mathcal{J}_s = \frac{r_p}{l_b}\frac{\beta}{\beta_e} = \frac{\text{time for electrons to cross pipe}}{\text{passage time for half of the bunch}}$$

Recall:

$$\varphi = \begin{cases} \frac{\lambda}{2\pi\epsilon_0}\left[\frac{1}{2}\left(1-\frac{r^2}{r_b^2}\right) + \ln\frac{r_p}{r_b}\right] & 0 < r < r_b \\ \frac{\lambda}{2\pi\epsilon_0}\left[\ln\frac{r_p}{r}\right] & r_b < r < r_p \end{cases}$$

$$\frac{1}{2} m_e v_e^2 + q\varphi \simeq \text{const} \simeq 0$$

$$\beta_e \sim \frac{1}{2}\sqrt{\frac{2q\varphi}{m_e c^2}} \sim \sqrt{\frac{N_0 Z e^2}{l_b 4\pi\epsilon_0 m_e c^2}} \sim \sqrt{\frac{Z r_e N_0}{l_b}}$$

(AVERAGE e^- VELOCITY)

$$\Rightarrow \mathcal{J}_s = \frac{\beta r_p}{\sqrt{r_e l_b N_0 Z}}$$

THE ENERGY GAIN OF THE ELECTRON, RELIES ON THE DENSITY CHANGING OVER THE COURSE OF THE BUNCH.

$$\Delta E_e \sim \frac{m_e c^2}{2}\left[\frac{Z r_e N_0(s)}{l_b}\right] - \frac{m_e c^2}{2}\left[\frac{Z r_e N_0(s+\Delta s)}{l_b}\right]$$

$$\sim \frac{m_e c^2}{2}\left(\frac{\partial N_0}{\partial z}\Delta z\right)\left(\frac{Z r_e}{l_b}\right)$$

$$\Delta E_e \sim \frac{m_e c^2}{2}\left(\frac{\partial N_0}{\partial z}\Delta z\right)\left(\frac{Z r_e}{l_b}\right)$$

$$\Delta z = \frac{v_p}{v_e}\beta c = \beta r_p \sqrt{\frac{l_b}{Z r_e N_0}}\,; \qquad \frac{\partial N_0}{\partial z} \sim \frac{N_b}{l_b}$$

So $\Delta E_e \approx m_e c^2 \left(\frac{Z N_0 r_e}{l_b^3}\right)^{1/2} \beta r_p$

U[illegible] $\xi \leq 1 \Rightarrow$ Electron build up possible within bunch

WHAT IS STEADY STATE ELECTRON DENSITY?

Electrons can build up until E_r at pipe ~ 0.

$$\Rightarrow \lambda_e = \lambda_I$$

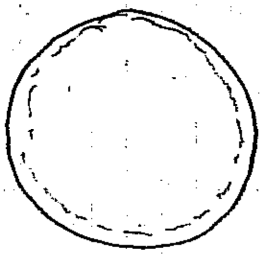
$$\pi r_p^2 n_e = \pi r_b^2 Z n_i$$

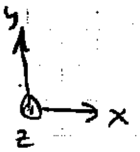
$$n_e = \left(\frac{r_b}{r_p}\right)^2 Z n_i$$

ELECTRON - ION INSTABILITY

(SEE ALSO R.C. DAVIDSON & H. Qin, Physics of Intense Charged Particle Beams in High Energy Accelerators, p 503 FOR KINETIC TREATMENT).

CONSIDER A UNIFORM DISTRIBUTION OF ELECTRONS (AT REST) WHICH HAS THE SAME RADIUS (OR SLIGHTLY SMALLER RADIUS) AS A UNIFORM DENSITY ION BEAM, THAT IS MOVING AT VELOCITY V_z (OUT OF THE PLANE OF THE PAPER).





$$E_x = \frac{\lambda_i(r)\,(x - x_i)}{2\pi\epsilon_0 r_0\, r} = \frac{\rho_i}{2\epsilon_0}(x - x_i)$$

THE EQUATION OF MOTION FOR THE CENTROID OF THE ELECTRONS IS OBTAINED FROM

$$m_e \ddot{x} = \frac{-e\rho_i}{2\epsilon_0}(x - x_i) + \frac{e\rho_e}{2\epsilon_0}(x - x_e)$$

or

$$\frac{d^2 x_e}{dt^2} = -\frac{\omega_{pi}^2}{2}\left(\frac{m_i}{q}\frac{e}{m_e}\right)(x_e - x_i)$$

here $\omega_{pi}^2 = \dfrac{q^2 n_i}{\epsilon_0 m_i} = \dfrac{q\rho_i}{\epsilon_0 m_i}$

(THE CENTER OF oscillation for the electrons is the center of the ion beam).

x_e = centroid of electron beam

x_i = centroid of ion beam

THE EQUATION OF MOTION FOR THE CENTROID OF THE IONS IS GIVEN BY

$$\frac{d^2 x_i}{dt^2} = -\omega_{\beta o}^2 x_i - \left[\frac{m_e N_e}{m_i N_i}\right]\left(\frac{\omega_{\beta i}^2}{2}\frac{m_i}{q}\frac{e}{m_e}\right)(x_i - x_e)$$

THE TOTAL MOMENTUM KICK TO EACH SPECIES MUST BE EQUAL & OPPOSITE

$$\Rightarrow \quad \frac{d^2 x_i}{dt^2} = -\omega_{\beta o}^2 x_i - f\,\frac{\omega_{\beta i}^2}{2}(x_i - x_e)$$

HERE $f \equiv \dfrac{e N_e}{q N_i}$ = fractional neutralization

Now $\dfrac{d}{dt}$ = total derivative $= \dfrac{\partial}{\partial t} + v_z \dfrac{\partial}{\partial z}$

$\Rightarrow$ THE ION & ELECTRON EQUATIONS MAY BE WRITTEN

$$\left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z}\right)^2 x_i = -\omega_{\beta o}^2 x_i - f\,\frac{\omega_{\beta i}^2}{2}(x_i - x_e)$$

$$\frac{\partial^2}{\partial t^2} x_e = -\frac{\omega_{\beta i}^2}{2}\left(\frac{m_i}{q}\frac{e}{m_e}\right)(x_e - x_i)$$

Now let $x_e = x_e \exp[i(\omega t - kz)]$; $x_i = x_i \exp[i(\omega t - kz)]$

$$\Rightarrow (-\omega^2 + 2\omega k v_z - k^2 v_z^2)\, x_i = -\omega_{p0}^2 x_i - f\frac{\omega_{pi}^2}{2}(x_i - x_e)$$

$$-\omega^2 x_e = -\frac{\omega_{pi}^2}{2}\left(\frac{m_i}{m_e}\frac{e}{q}\right)(x_e - x_i)$$

$$\Rightarrow \left[(\omega - kv_z)^2 - \omega_{p0}^2 - f\frac{\omega_{pi}^2}{2}\right] x_i = -\frac{f\omega_{pi}^2}{2} x_e$$

$$\left[\omega^2 - \frac{\omega_{pi}^2}{2}\left(\frac{m_i}{m_e}\frac{e}{q}\right)\right] x_e = -\frac{\omega_{pi}^2}{2}\left(\frac{m_i}{m_e}\frac{e}{q}\right) x_i$$

Multiplying the above equations and dividing by $x_e x_i$, yields the dispersion relation:

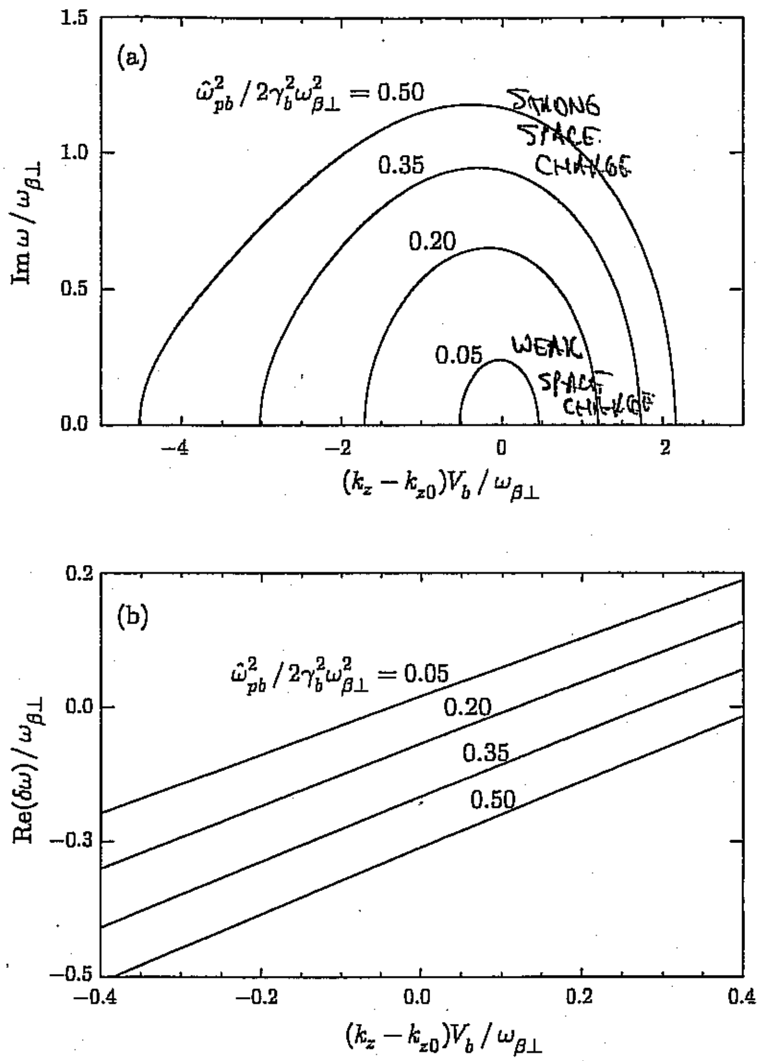
$$\underbrace{\left[(\omega - kv_z)^2 - \omega_{p0}^2 - f\frac{\omega_{pi}^2}{2}\right]}_{\text{ION BETATRON FREQUENCY (INCREASED BY SPACE CHARGE OF ELECTRONS)}}\underbrace{\left[\omega^2 - \frac{\omega_{pi}^2}{2}\left(\frac{m_i}{m_e}\frac{e}{q}\right)\right]}_{\text{ELECTRON OSCILLATING IN POTENTIAL WELL OF ION}} = +\underbrace{\frac{f\omega_{pi}^4}{4}\left(\frac{m_i}{m_e}\frac{e}{q}\right)}_{\text{COUPLING}}$$

So a beam with high spatial frequency undergoing betatron oscillations in the comoving frame, $\therefore\ kv_z - \omega \cong \sqrt{\omega_{p0}^2 + \frac{f\omega_{pi}^2}{2}}$ will resonate with electrons oscillating in the ion well if

$$\omega \cong \frac{\omega_{pi}}{\sqrt{2}}\sqrt{\frac{m_i e}{m_e q}}$$

Giving rise to instability!

From Davidson & Qin
Physics of Intense Charged Particle Beams, 2001. (22)



$\omega_{\beta\perp}^2 \equiv \omega_{\beta 0}^2$

$\frac{\hat{\omega}_{pb}^2}{2\gamma_b^2 \omega_{\beta\perp}^2} = \left(1 - \frac{\theta^2}{\theta_0^2}\right)$

Figure 10.11. Plots of (a) normalized growth rate ($Im\omega/\omega_{\beta\perp}$), and (b) normalized real frequency $(Re\omega - \omega_e)/\omega_{\beta b}$ versus shifted axial wavenumber $(k_z - k_{z0})V_b/\omega_{\beta\perp}$ obtained from the dispersion relation (10.103) for the unstable branch with positive real frequency. System parameters correspond to $v_{T\|b} = 0 = v_{T\|e}$, $m_b/m_e = 1836$ (protons), $(\gamma_b - 1)m_b c^2 = 800$ MeV, $r_b/r_w = 0.5$, and $f = 0.1$. Curves are shown for several values of normalized beam intensity $\hat{\omega}_{pb}^2/2\gamma_b^2\omega_{\beta\perp}^2$ ranging from 0.05 to 0.5.

$k_{z0} V_z = \omega \mp \sqrt{\omega_{\beta 0}^2 + f\omega_i^2/2}$; $\omega = \frac{\omega_{pi}}{2}\sqrt{\frac{m_i e}{m_e q}}$

From Davidson & Qin, 2001 (23)

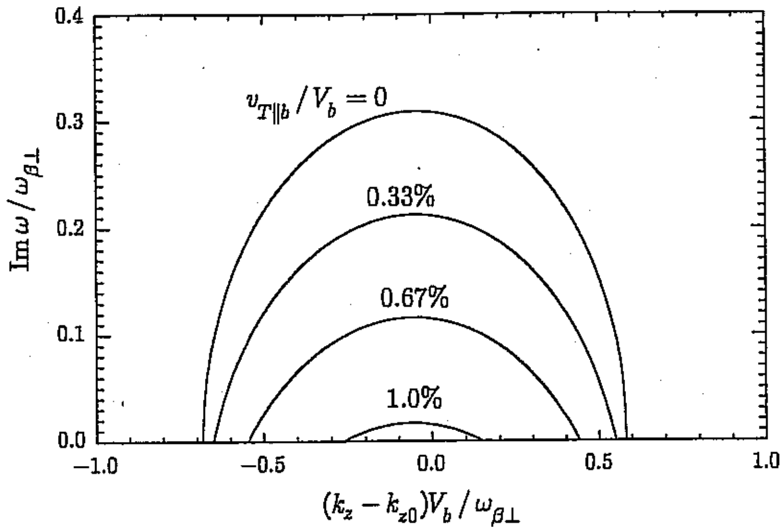


Figure 10.12. Plot of normalized growth rate ($Im\omega/\omega_{\beta\perp}$), and normalized real frequency $(Re\omega - \omega_e)/\omega_{\beta\perp}$ versus positive real frequency. System parameters correspond to $\hat{\omega}_{pb}^2/2\gamma_b^2\omega_{\beta\perp}^2 = 0.07$, $v_{T\|e} = v_{T\|b}$, $m_b/m_e = 1836$ (protons), $(\gamma_b - 1)m_bc^2 = 800$ MeV, $r_b/r_w = 0.5$, and $f = 0.1$. Curves are shown for several values of normalized ion thermal spread $v_{T\|b}/V_b$ ranging from 0 to 0.01.

velocity V_b [Eq. (10.105)], it is expected that Landau damping by parallel ion kinetic effects can have a strong stabilizing influence at modest values of $v_{T\|b}/V_b$. That this is indeed the case is evident from Fig. 10.12, which shows a substantial reduction in maximum growth rate and eradication of the instability over the instability bandwidth as $v_{T\|b}/V_b$ is increased from 0 to 0.01.

We now make use of the nonlinear particle perturbative simulation method [60, 61] described in Sec. 8.5 to illustrate several important properties of the two-stream instability in intense beam systems (Sec. 10.4.2).

PREVENTIVE MEASURES (from J. Wei & L. Macek, CERN electron cloud workshop 2003).

- SUPPRESS ELECTRON GENERATION
  - SURFACE TREATMENT OF THE VACUUM PIPE
  - KICKER MAGNETS IN GAPS
  - VACUUM PORTS SCREENED TO REDUCE E-field
  - CLEARING ELECTRODES
  - HIGH VACUUM
  - SOLENOIDS - TO REDUCE MULTIPACTING

Summary of Electron, Gas, Pressure, & Scattering Effects

1. Coulomb collisions within beam can transfer energy from $\perp$ to $\parallel$ and provide lower limit on $T_{\parallel}$, higher than from accelerative cooling.

2. Coulomb interactions with residual gas nuclei provide a source of emittance growth (but not important for higher mass and linac residence times).

3. Pressure instability from desorption of residual gas by stripped beam ions hitting wall or beam-ionized residual gas atoms, forced to wall by E-field of beam. Limits current in rings or high rep rate linac.

4. Electrons can cascade and reach a "quasi" equilibrium population of similar line charge to the ion beam. Electron-ion two stream instability is unstable, and can lead to transverse instability, similar to what is observed in some proton rings.