Neutralized drift compression offers the possibility of very short pulses

For a parabolic pulse the longitudinal envelope equation (including longitudinal thermal spread) for bunch length l is:

$$\frac{d^2 l}{dt^2} = \frac{16\varepsilon_z^2}{l^3} + \frac{4v_0^2 g Q_a l_a}{l^2} \qquad \Longrightarrow \qquad \left(\frac{\Delta v}{v}\right)_{tilt}^2 = 20\left(\frac{\delta v}{v}\right)_a^2 \left[C^2 - 1\right] + 8gQ_a\left[C - 1\right]$$

Thermal Spread + Space Charge

where $\varepsilon_z^2 \equiv 25\left(\langle \delta v_z^2 \rangle \langle \delta z^2 \rangle - \langle \delta v_z \delta z \rangle^2\right)$

So if initial velocity spread δv/v ~ 5 x 10^{-4}, initial tilt Δv/v <~ 1, and initial perveance in drift section Q_a = ~0, final compression ratio $C=l_0/l_f$ ~ 10^2 - 10^3 are possible.

For pulse durations at end of accelerator of ~ 250 ns, final pulse durations of order ~ 0.2 - 1 ns are possible.

How big can C be?

① $$\left(\frac{\Delta V}{V}\right)^2_{tilt} = 20\left(\frac{\delta v}{v}\right)^2_a \left[C^2 - 1\right] + 8 g Q_a \left[C - 1\right]$$

$\Rightarrow$ C_{max} occurs if $\frac{\Delta V}{V} = \frac{\Delta V}{V}\Big|_{max} \cong 1$

$\rightarrow$ If $Q_a = 0$

$$\Rightarrow C^2_{max} = \frac{\left(\frac{\Delta V}{V}\right)^2_{max}}{20\left(\frac{\delta v}{v}\right)_a} + 1$$

$$C_{max} = \sqrt{\frac{\left(\frac{\Delta V}{V}\right)^2_{max}}{20\left(\frac{\delta v}{v}\right)^2_a} + 1} \simeq \frac{\frac{\Delta V_{tilt}}{V_a}}{4.5\left(\frac{\delta v}{v}\right)_a}$$

$$\frac{\Delta V}{V}\Big|_{max} \leq 1$$

$$\left(\frac{\delta v}{v}\right)_a \cong \frac{\delta v_z}{v_z}\Big|_{inj} \underbrace{\left(\frac{\varepsilon_{acc}}{\varepsilon_{inj}}\right)}_{\gtrsim 1} \underbrace{\left(\frac{V_{inj}}{V_{acc}}\right)}_{\sim \frac{1}{3}} \underbrace{\left(\frac{\Delta t_{inj}}{\Delta t_{acc}}\right)}_{\sim 1}$$

So if $\frac{\delta v_z}{v_z}\Big|_{inj} = 5 \times 10^{-4}$

$$\Rightarrow C_{max} \sim \sqrt{\frac{1}{20 \cdot (1.7 \cdot 10^{-4})^2} + 1} \sim 1315$$

(2) Limit on C from Kinematics:

Craig Olson noted:

$$\frac{\Delta V}{V} = \frac{2\Delta z}{z_{drift}}$$

$$\Delta z \leq l_{b\,final}$$

$$z_{drift} = \frac{l_{b0}}{\Delta V_{tilt} / V_0}$$

$$\rightarrow \boxed{\frac{\Delta V}{V} \leq 2 \frac{\Delta V_{tilt}}{V_0} \frac{1}{C}}$$

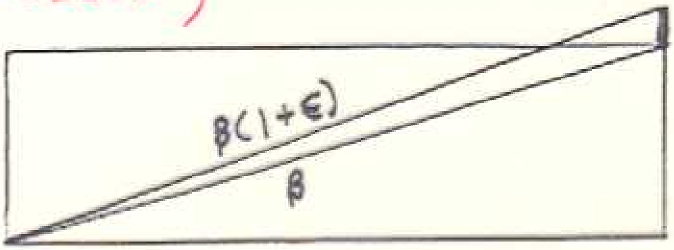
$$\Rightarrow C \leq \frac{2\, \Delta V_{tilt}/V_a}{(\Delta V / V_a)} = \frac{\Delta V_{tilt}/V_a}{(\delta V / V_a)}$$

If $\frac{\Delta V}{V} \sim 10^{-3}$

$$\Rightarrow C \lesssim 2000$$

[Limit on C from kinematics is to within a factor of order unity same limit as thermal pressure limit].

(FROM C. OLSON)



Δz = ERROR IN FINAL POSITION. MUST BE $\ll l$

$z_d \equiv$ DRIFT DISTANCE

$\tau_d \equiv$ DRIFT TIME

$$z_d = \beta c t_d$$

$$z_d^* = \beta(1+\epsilon) c \tau_d$$

$$\Delta z = \epsilon \beta c \tau_d = \epsilon z_d$$

$$\boxed{\epsilon_\beta = \frac{\Delta z}{z_d} \qquad \epsilon_\mathcal{E} = 2\epsilon_\beta}$$

$\Rightarrow$ FRACTIONAL ERROR IN ENERGY

$$\boxed{\frac{\Delta V}{V} \leq \frac{\sqrt{8 g Q_f}}{C}}$$

EXAMPLE:

IBX	DRIVER
$\beta = 0.03$	$\beta = 0.2$
$\Delta t = 50$ ns	$\Delta t = 10$ ns
$\Delta z = 45$ cm	$\Delta z = 60$ cm
$z_d = 30$ m	$z_d = 400$ m
$\epsilon_\beta = \frac{45}{3000} = 1.5\%$	$\epsilon_\beta = \frac{60}{40000} = 0.15\%$
$\epsilon_\mathcal{E} = 3\%$	$\epsilon_\mathcal{E} = \underline{\underline{0.3\%}}$
$Q_f \sim 10^{-3}$	$Q_f \sim 10^{-4}$
$C \sim 10$	$C \sim 20$