Simulations and Modeling of SSPX Plasma Evolution*

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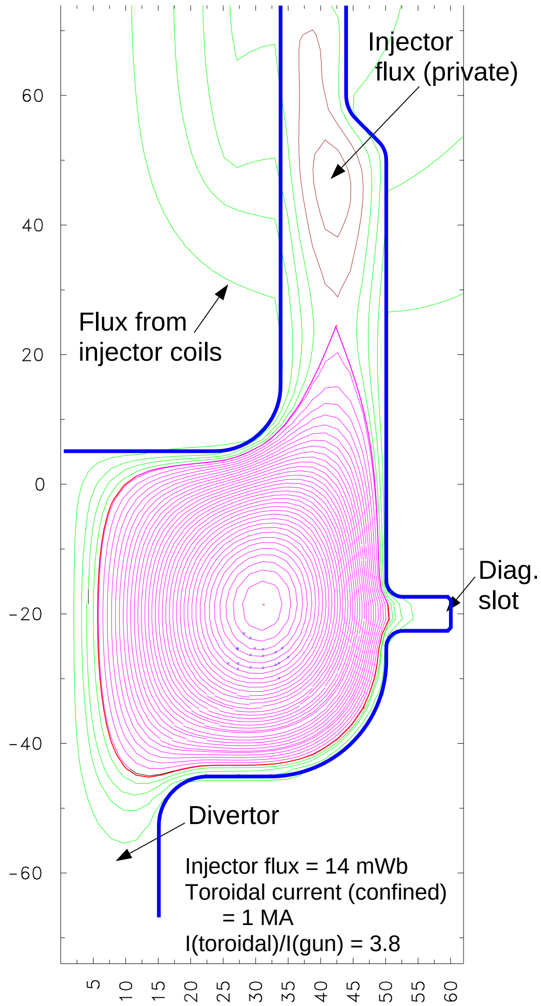
Spheromak Modeling - Outline

- CORSICA code modeling -- MHD axisymmetric equilibria, Taylor relaxed states, effects due to geometry and current profile.
- NIMROD 3D time-dependent, nonlinear resistive MHD simulations of formation, stability and reconnection.
- Analysis of fluctuation data from 8 Rogowski coils surrounding SSPX using the phase-velocity transform to identify macroscopic modes.

Application of CORSICA to Spheromaks



SSPX magnetic equilibrium

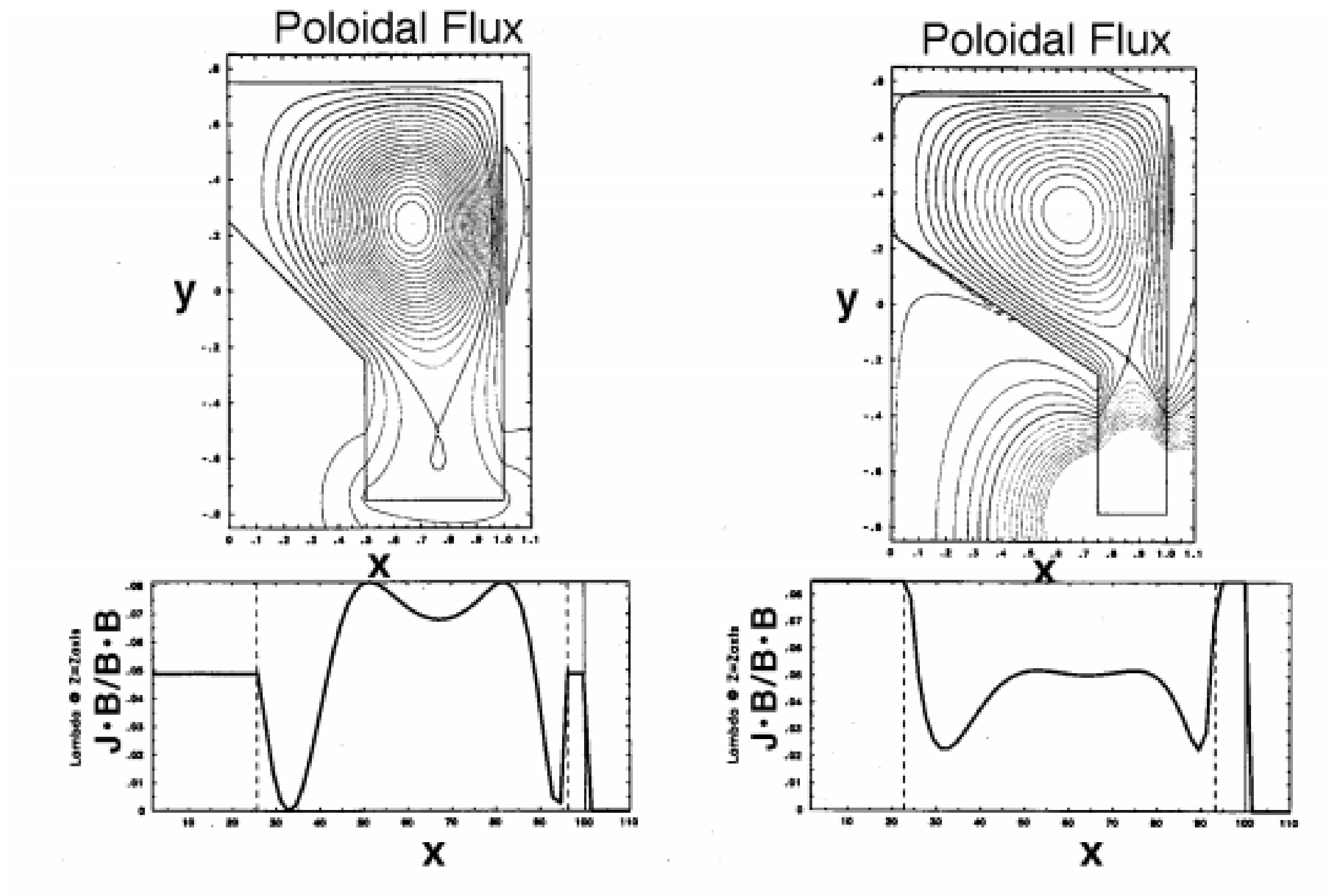
- **CORSICA modeling:**
  - **CORSICA extended, applied to spheromaks, RFP's**
  - **CORSICA used to model SSPX equilibria and guide experimental design/operation**
  - **CORSICA equilibrium and transport studies of SSPX with hyper-resistivity model**
  - **CORSICA/UEDGE core-edge coupling for edge plasma study**

  [CORSICA also supports users at Wisconsin & Washington on alternates applications]

CORSICA Studies of Spheromak Equilibrium -

Influence of Gun Geometry

- Here decreasing the gun region width widened the closed flux region while using current profiles similar to NIMROD simulations.



NIMROD Simulation of Spheromak Formation

- NIMROD is a 3D time-dependent, resistive MHD simulation code. NIMROD solves for thermal and momentum transport with resistive-MHD fluid equations with various physics options in Ohm's law. It is typically run on MPP computers using MPI and F90. NIMROD was developed by the NIMROD team- at SAIC, LANL, and LLNL (ref. -- http://nimrod.saic.com)

- Sovinec *et al.* (ref. -- C.R. Sovinec, *et al.,* Bull. Amer. Phys. Soc. **44**, 112 (1999)) have undertaken many simulations of spheromak-like plasmas beginning with (1) screw-pinch-like initial state in a simple cylinder with conducting boundary conditions, (2) a plasma driven by an electrostatic potential difference in a simple cylinder with some initial magnetic flux specified, and (3) a plasma driven by an electrostatic potential in a spheromak gun injector geometry with some initial magnetic flux distribution.

  Spheromak formation, magnetic reconnection, and the growth and saturation (leading sometimes to limit cycles) of non-axisymmetric modes have been studied.

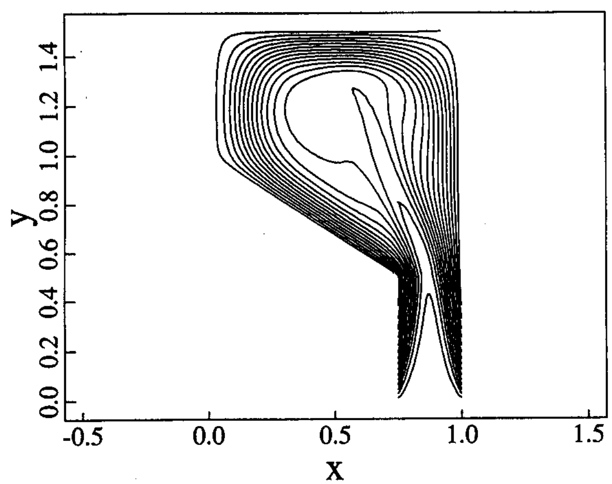
- We are extending Sovinec's work to study the influence of boundary conditions on the magnetic flux, the strength of the driving electrostatic potential, and geometry (size and shape of the conducting boundary) on spheromak formation and stability. A critical issue is whether closed field line structures can be formed and sustained.

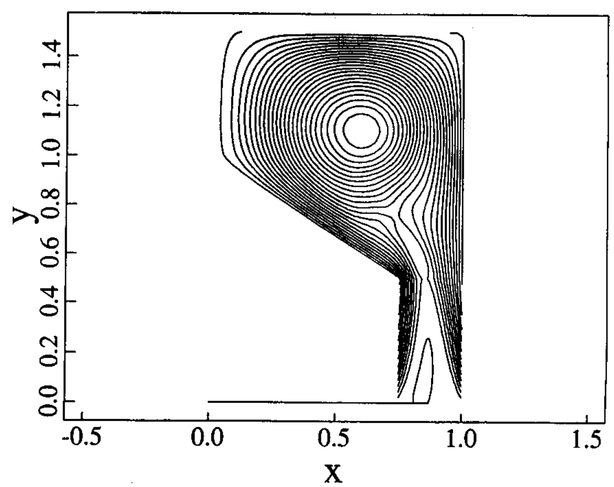
NIMROD Simulations of Spheromak Formation

- Three-dimensional, time-dependent, nonlinear resistive MHD simulations with the NIMROD code (Sovinec, *et al.*, LANL, and Schnack, *et al..,* SAIC).

- A simple model of the gun and flux-conserver regions is used. Radial currents are driven with an imposed electric potential across the gun electrodes and an initial radial magnetic field that varies in z, which in turn generate a toriodal magnetic field. The **JxB** forces drive additional currents and magnetic flux into the adjacent flux-conserver region. Resistivity and non-axisymmetric modes enable magnetic reconnection.

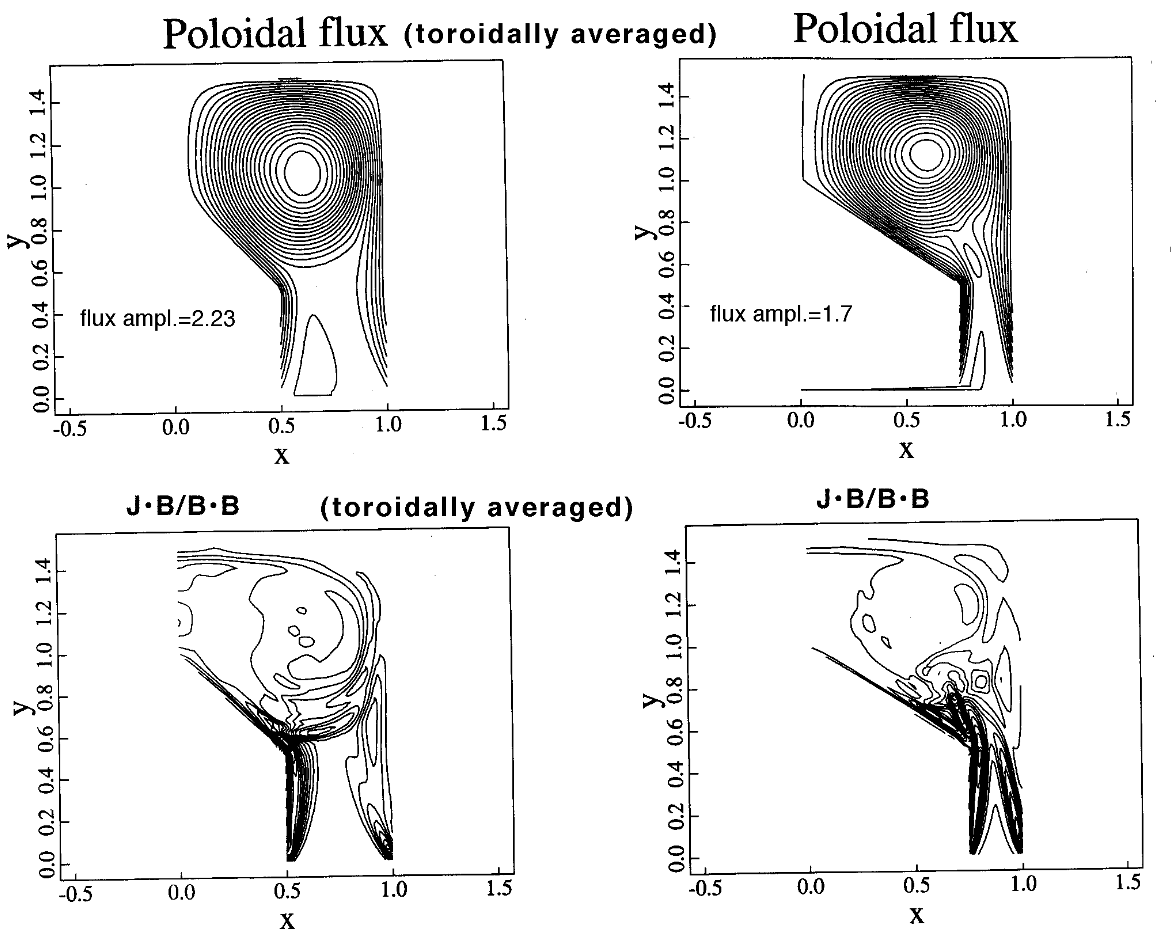
- Toroidally averaged poloidal flux at early and late times in a NIMROD spheromak simulation. Flux is steadily fed from the gun region to the flux conserver, where open field lines are reconnected to form closed field lines.





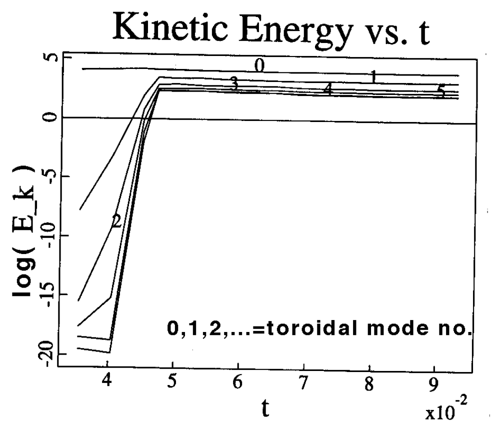
NIMROD Simulations of Gun-driven Spheromak Formation

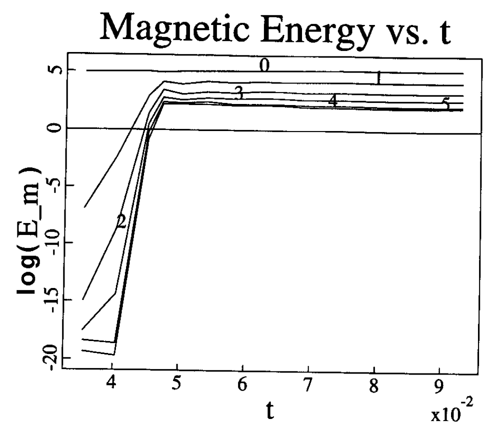
- Gun-driven spheromak simulations with $\mathbf{V}_0=\mathbf{E}_0\cdot\mathbf{w}_{gun}$=fixed & $\mathbf{w}_{gun}\rightarrow\mathbf{1/2}\ \mathbf{w}_{gun}$. n=0 closed flux region expands. Plasma is not in a Taylor relaxed state. Toroidally averaged poloidal flux and $\lambda=\mu_0\mathbf{J}\bullet\mathbf{B}/B^2$ contours at steady state.



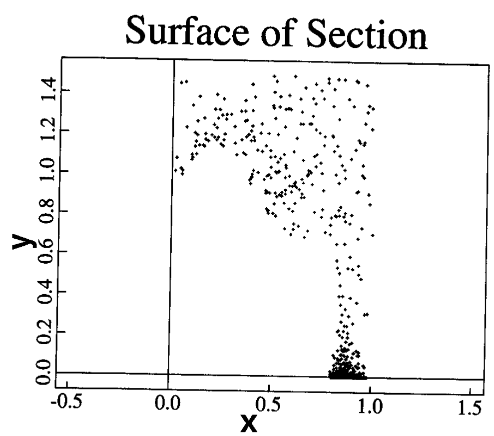
NIMROD Simulation of Gun-driven Spheromak Formation

- Gun-driven spheromak simulations achieve a non-axisymmetric steady-state as evidenced by the toroidal mode energy spectra vs. time:



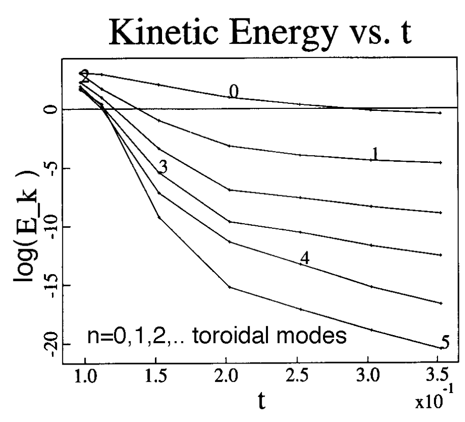


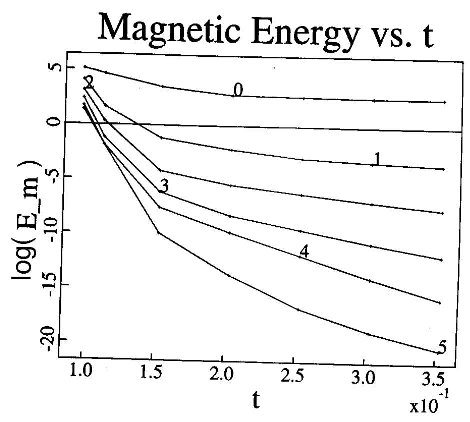
- Poincare surface-of-section plots for the 3D magnetic field lines (250 starting points) at steady state are dominantly chaotic and unconfined.



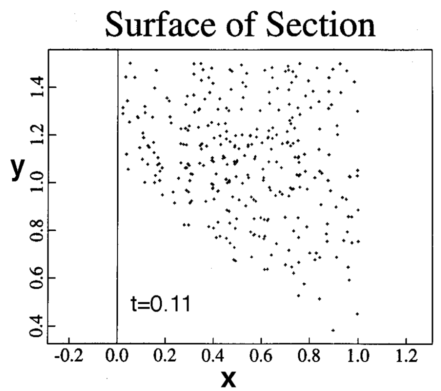
NIMROD Simulation of Decaying Spheromak

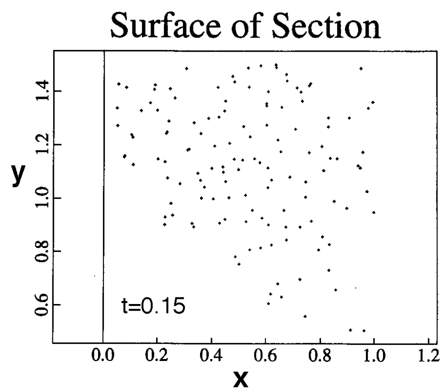
- After a gun-driven spheromak simulation reaches steady-state, we crowbar off the driving potential; and the spheromak decays. The toroidal mode energy spectra shows that finite *n* toroidal modes decay faster than does the *n* =0 component.





- At t=0.11 Poincare surface-of-section plots for magnetic field lines show better confinement, but the field lines are contorted; many are chaotic and poorly confined. Later in the decay, the magnetic well and the confinement weaken.

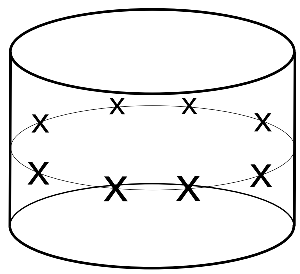




Fluctuation analysis

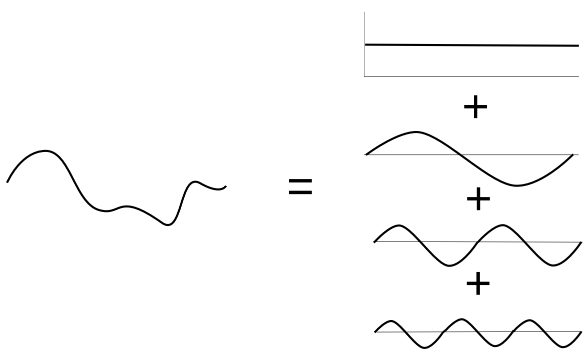
via Phase Velocity Transform

- SSPX Rogowski coils: Measure $\widetilde{B}(t)$ in 8 positions around equator



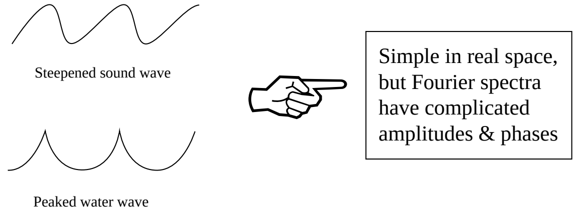
Want to recover toroidal structure of $\widetilde{\mathbf{B}}$ from this

- Standard to decompose data into *sine waves* via Fourier transform:



$$B(\varphi_l, t) \quad = \quad \sum_{n=-N/2}^{N/2} e^{in\varphi_l} \hat{B}_n(t)$$

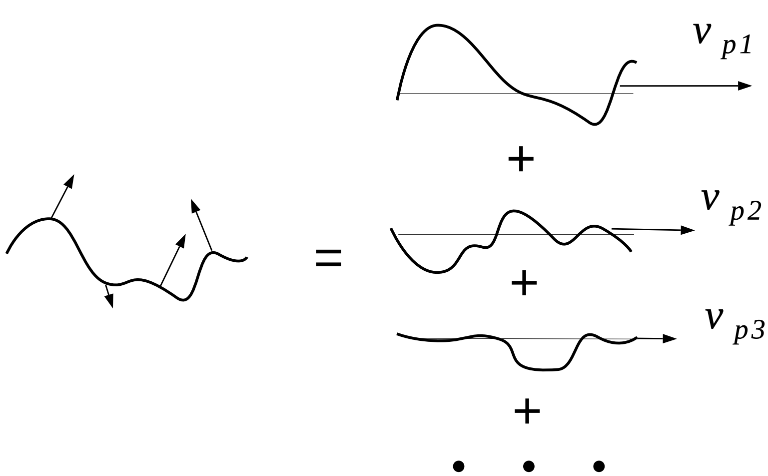
- But Fourier Transform can *complicate* many waves, not simplify!



Phase Velocity Transform ("PVT")

- PVT divides $f(x,t)$ into components that propagate without distorting

Test function   Phase velocity modes



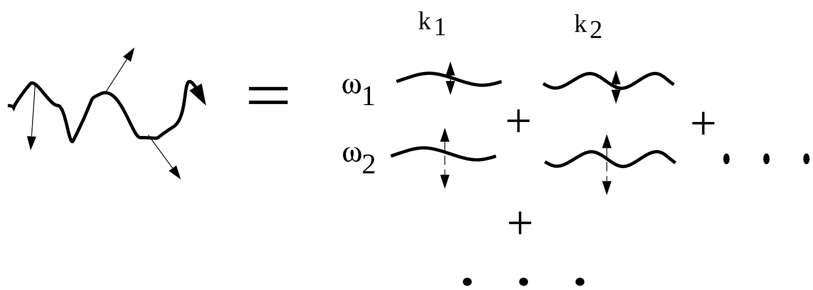
$$f(\mathbf{x},t) \quad = \quad \int dv_p\, f^{\dagger}(v_p, x - v_p t)$$

- Comments:
  - Each component has uniform phase velocity $\Longrightarrow$ "Phase Velocity Transform"
  - Can think of components as hard objects, moving at given speed

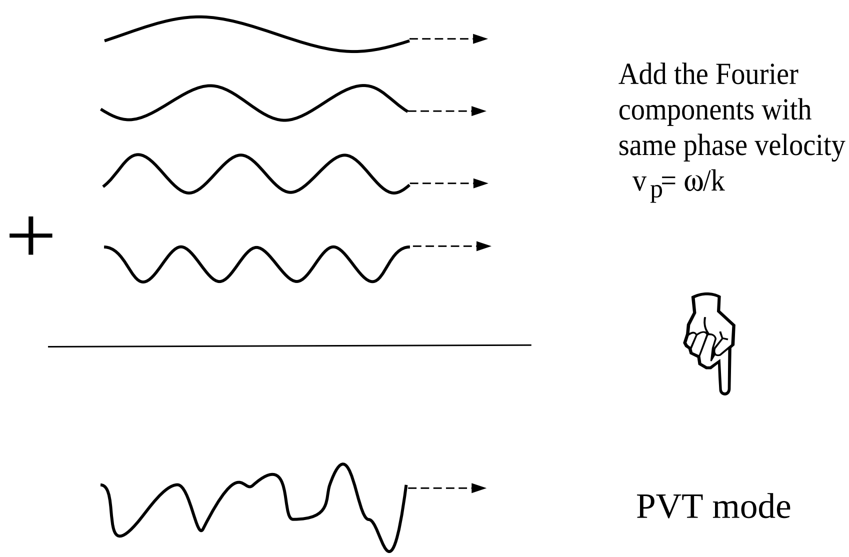
Deriving PVT

- Can obtain PVT from Fourier space-time transform:

  1) Divide $f(x,t)$ into propagating sine waves



  2) Combine all waves propagating with same velocity



- For discrete probes on periodic domain, PVT is:

$$\tilde{f}^{\dagger}(\Omega_p, \varphi, t) = \int dt' \sum_{l'=1}^{N} K_N[\varphi'_l - \varphi_l - \Omega_p(t'-t)] W(t-t') \partial_{\varphi'} f(\varphi', t')$$

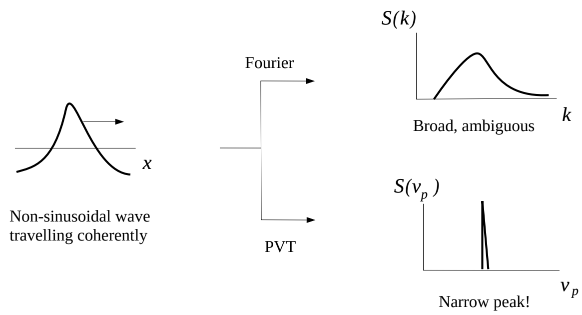
$K_N(\alpha) \equiv \sin[(N-2)\alpha/4] \sin[N\alpha/4] / \sin[\alpha/2]$

$W(t-t')$ = time window, to isolate global evolution stages

Advantages of PVT

1. PVT doesn't presume any preset wave form

   $\Longrightarrow$ good for finding weird nonlinear mode structures



2. PVT better for representing *propagating* waves

   - Many transforms assume x, t separation

$$
\begin{aligned}
f(x,t) &= \sum_k A_k(t)\hat{u}_k(x) \\
&= \text{stationary basis functions with varying amplitude}
\end{aligned}
$$

   - **But:** *Stationary* basis functions are awkward to represent *propagating* waves
   - In contrast, a PVT mode *is* a propagating wave

$$
f(\mathbf{x},t) = \int dv_p\, f^\dagger(v_p, x - v_p t)
$$

3. PVT takes better advantage of probe data quirks

   - Probe resolves time well, but space badly

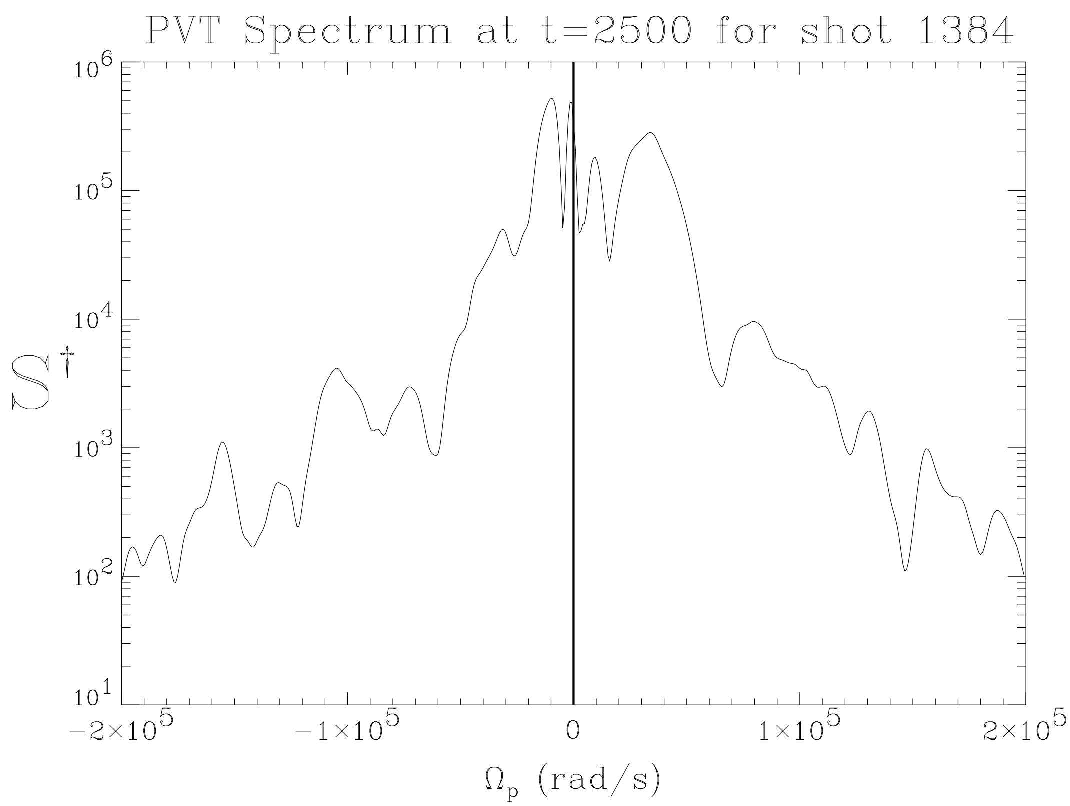
   - PVT uses good time resolution & propagation to resolve spatial structure

SSPX: PVT Spectrum

- The transformed signal, $I^\dagger(\Omega_p, \varphi, t)$, has 3 arguments

  $\Longrightarrow$ examine dependences individually

- Ω_p dependence: Gives phase velocity spectrum (Analogue of Fourier spectrum)

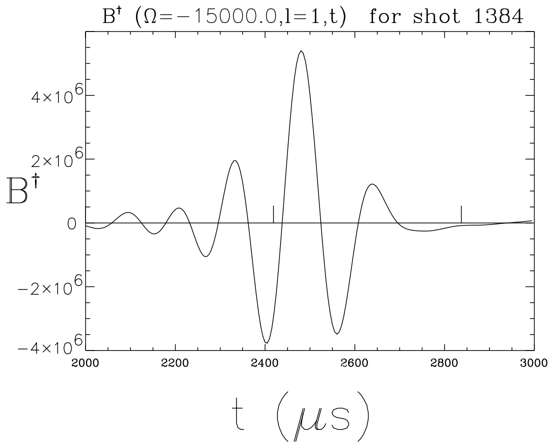


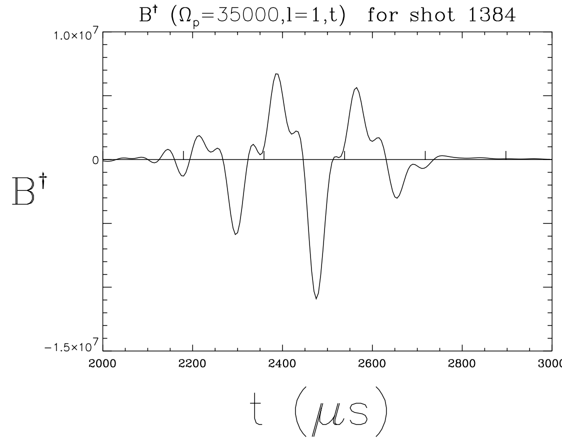
  - Has two main peaks, at $\Omega_p \simeq -1.5 \times 10^5$ and $\Omega_p \simeq 3.5 \times 10^5 rad/s$

    Peaks appear at about $\pm .1 v_A$ or $\pm c_s$

  - Want to see *structure* of underlying modes

• Time dependence: Watch Ω_p peaks pass by a probe:



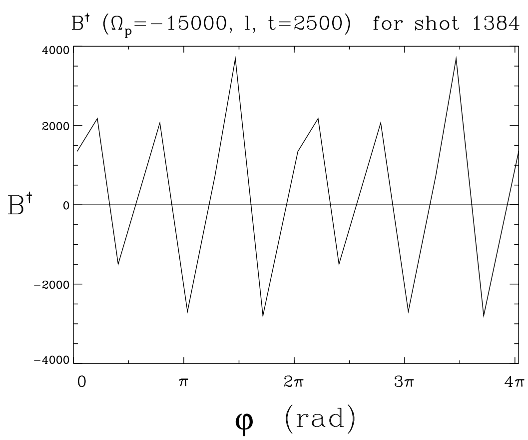


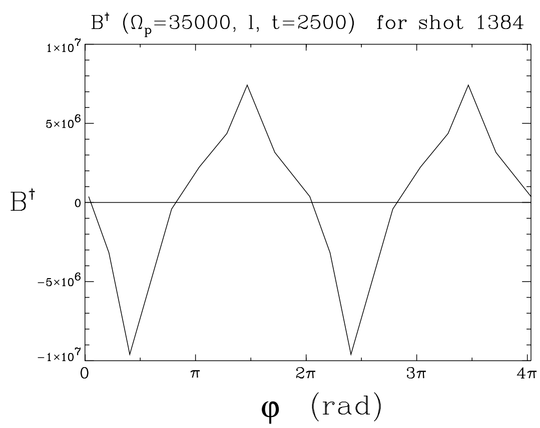
$n = 1$ mode: "shoulders" and deep narrow gullies (but worried about aliasing)

$n = 3$ mode: fairly sinusoidal structure

- # of peaks per rotation period (tick mark) shows mode structure
- Both modes build up and decay with main plasma

• Space dependence: gives instantaneous "snapshot" of mode



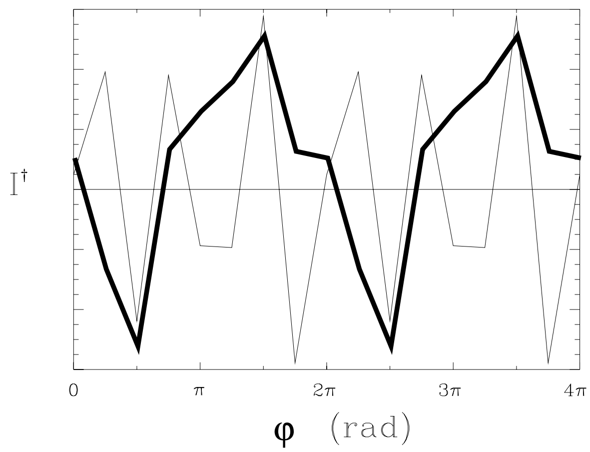


$n = 3$ mode: slightly modulated in both amplitude and wavelength

$n = 1$ mode: has flat top and deep narrow gullies (but worried about aliasing)

Further observations

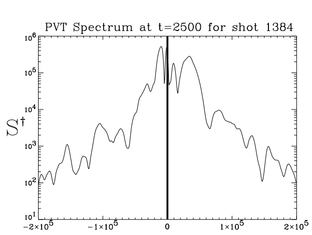
- Nonlinear mode interaction:
  Superimposed modes show $n = 3$ mode has higher amplitude and wavelength at $n = 1$ peaks



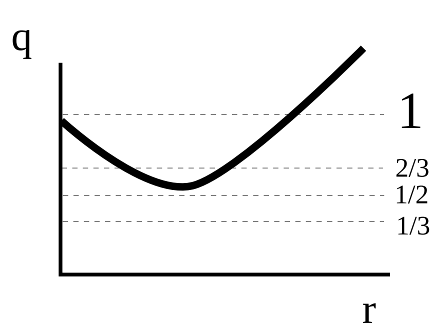
  - Suggests nonlinear mode coupling

    (as opposed to nonlinear mode *structure*)

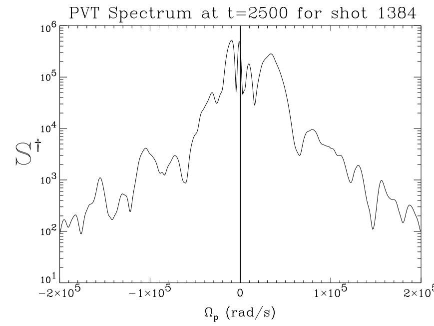
- Can infer q profile: Weak $n = 2$ mode means no $q = 1/2$ surface?!



? $\Rightarrow$



- Other peaks in PVT spectrum also significant



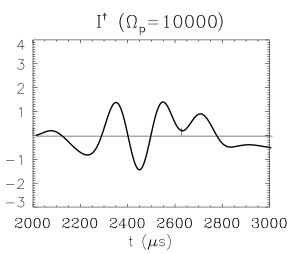
Stationary wave: Spectral peak at $\Omega_p = 0$

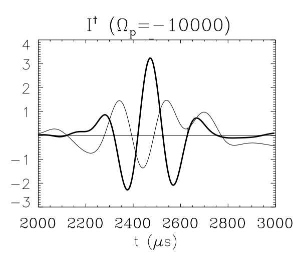
(could be imperfect probe calibration)

Mirror peaks:

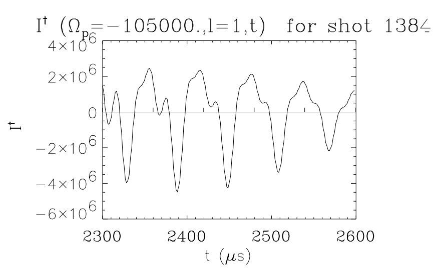
- Appear at $-\Omega_p$ of the two main peaks, with same n
- Represent a "standing wave" component

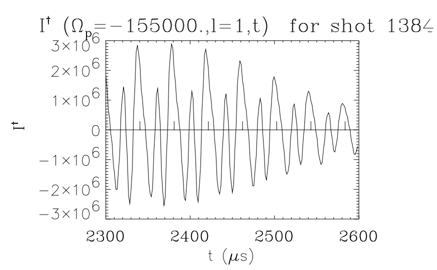
$\rightarrow$ suspect comes from vessel asymmetry or probe mis-calibration





High Ω_p peaks: Probably real, but aliasing makes it difficult to tell n





Conclusions

- Analyzed SSPX data with Phase Velocity Transform

  (basically group together Fourier harmonics with same $v_p = \omega/k$

- Phase Velocity Transform squeezes a lot of info from probe data
  - Nonlinear structure of dynamo modes
  - Nonlinear interaction between modes
  - Can see modes beyond usual spatial Nyquist limit
  - Degree of standing nature in modes (probe miscalibration?)
  - Clues about the q profile (weak $n = 2 \Longrightarrow q > 1/2$)
  - PVT "Inertial range" MST shows

- Future ideas:
  - Can make movies of modes propagating
  - Examine cause of modulational structure

    $\Longrightarrow$ Could have impact in number of fields.
  - Apply PVT analysis elsewhere

    (tokamak data, simulations ...)

Modeling of SSPX Plasma Evolution - Conclusions

- CORSICA code modeling -- MHD axisymmetric equilibria, Taylor relaxed states, effects due to geometry and current profile. CORSICA suggests how geometry influences how much closed flux can be formed. CORSICA+DCON can determine MHD stability (not illustrated here).
- NIMROD simulations of spheromak formation, stability and reconnection -- We are just beginning our studies. So far the plasmas are quite turbulent in the sense of having significant non-axisymmetric perturbations, and Poincare surface-section plots of the magnetic field lines indicate that it is difficult to form closed surfaces in the gun-driven plasma simulations except as a transient. Much more modeling work is needed.
- Analysis of fluctuation data from 8 Rogowski coils surrounding SSPX using the phase-velocity transform to identify macroscopic modes suggests strong $n=1$ modes and weaker $n=3$ and $n=2$ modes.