Modeling of RMF Current Drive in an FRC

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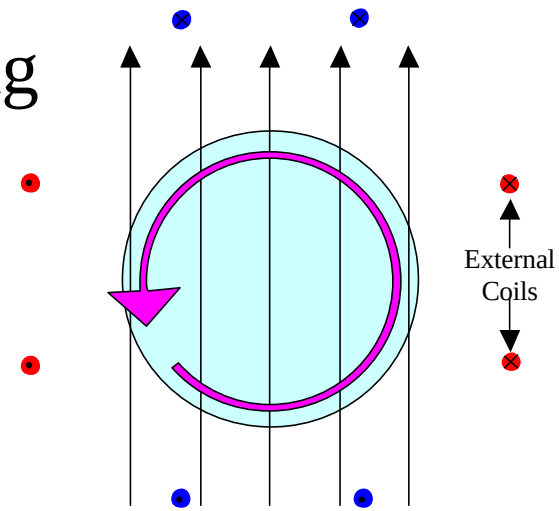
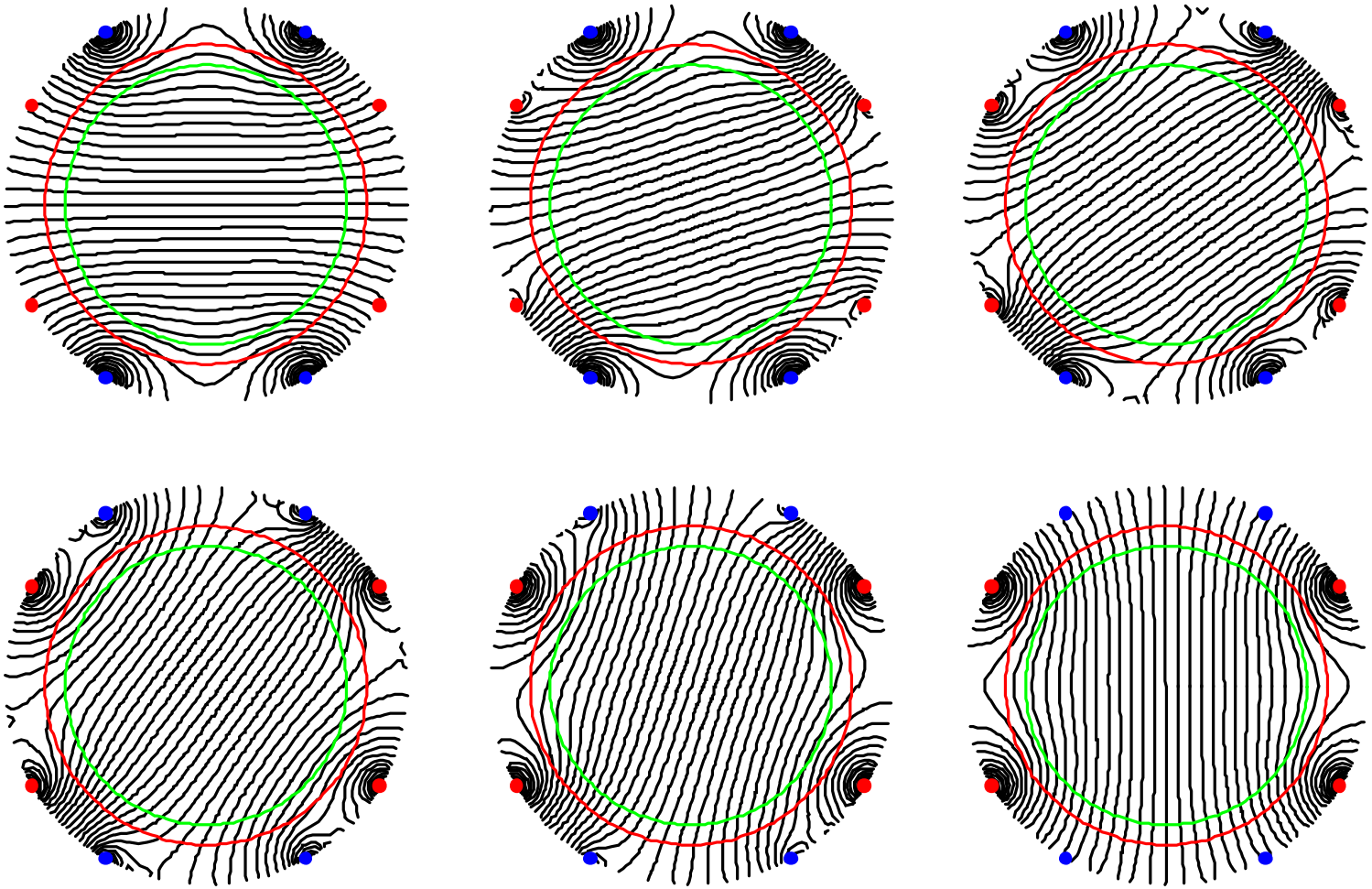
What Is RMF Current Drive?

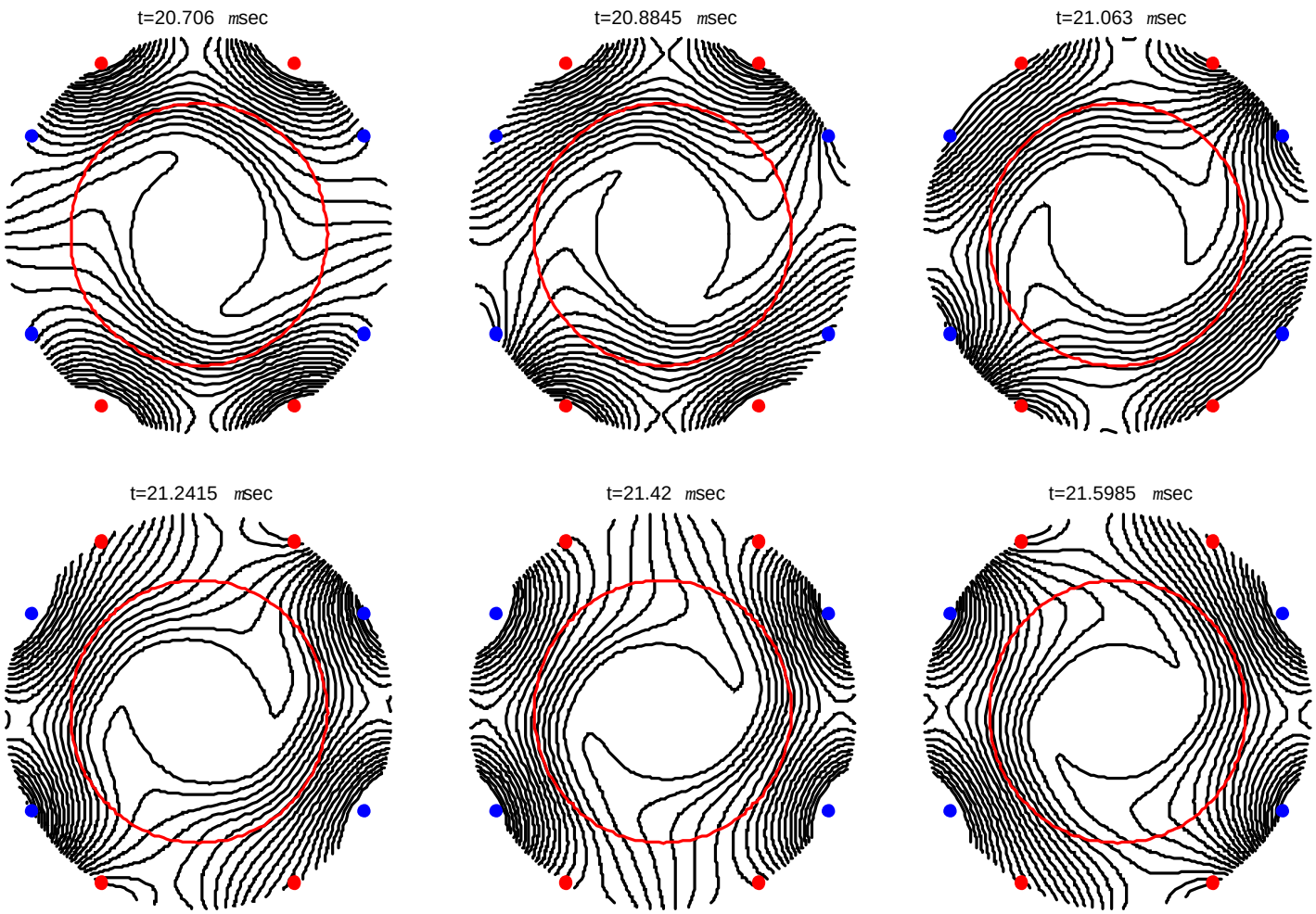
- Apply a rotating magnetic field to a long plasma column to drive current in the *q*-direction.
- $w_{ci} << w_{rmf} << w_{ce}$
- Electrons are tied to the rotating magnetic field lines, but the ions are unaffected.
- This drives an azimuthal electron current.



Vacuum RMF (*Real Coils*)



RMF Penetration (*Real Coils*)



Objective

- Develop MHD model (including Hall term)
  - Improve physics understanding
    - » RMF field penetration
    - » Poloidal flux build-up
    - » Sustainment
- Develop numerical support for RPPL experiments
  - Compare model predictions with experiment
  - *Tune* model by comparison with experiment (What are the required *knob* settings to get agreement?)

Equations

$$\frac{\partial n}{\partial t} + \nabla \bullet n\mathbf{u} = 0$$

$$Mn\left[\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \bullet \nabla \mathbf{u}\right] = \frac{\mathbf{J} \times \mathbf{B}}{c} - \nabla P - \nabla \bullet \Pi$$

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{u} \times \mathbf{B} - hc^2\left(\frac{\mathbf{J}}{c}\right) - \frac{c}{en}\left[\left(\frac{\mathbf{J}}{c} \times \mathbf{B}\right) - \nabla P_e\right]$$

$$\frac{\partial S}{\partial t} + \nabla \bullet S\mathbf{u} = \frac{g-1}{n^{g-1}}\left[h\mathbf{J}^2 + \nabla \cdot (k_\perp \nabla T) - \Pi : \nabla \mathbf{u} - R\right]$$

$$S = n^{2-g}T$$

$$P = nk_B T$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{J} = \frac{c}{4p}\nabla \times \mathbf{B}$$

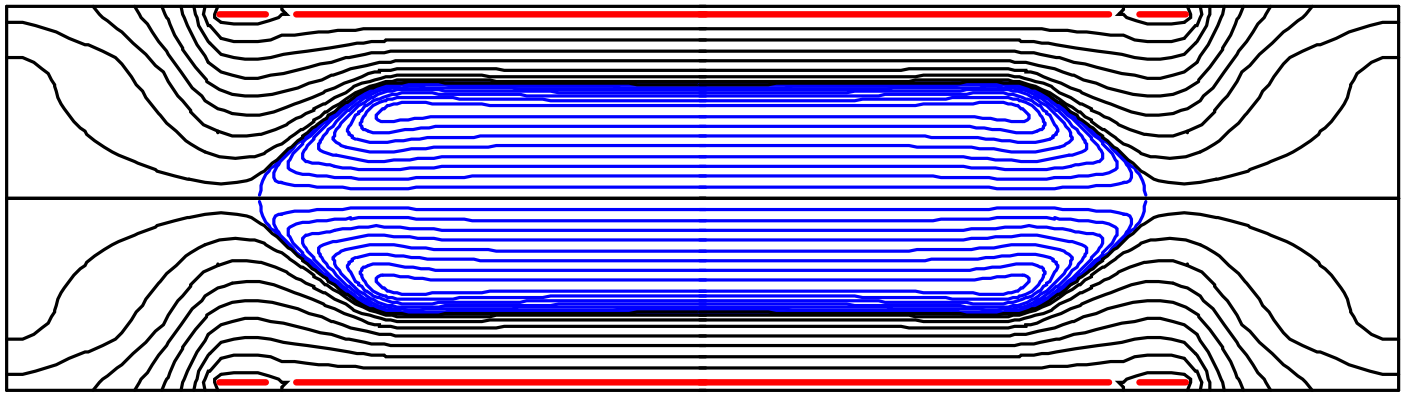
$$\Pi = -\frac{m}{Mn}\nabla \mathbf{u}$$

Transport

- Resistivity
  - Either classical, or a specified constant (usually isotropic)
- Viscosity (artificial)
  - $\Pi = -\frac{m}{Mn}\nabla\mathbf{u}, \quad m = f_{visco}\frac{\Delta r^2}{2\Delta t}$
- Thermal Conduction (*phenomenological*)
  - $k_\perp = \frac{f_{TC}}{(g-1)}\frac{n\Delta r^2}{\Delta t}$
- Radiation
  - $R = 10^{-18} f_{imp} n^2$

Phenomenological Inclusion of 3D-Effects

- To study FRC sustainment, two axial effects must be included:
  1. Average b condition: $\langle b \rangle = 1 - \frac{1}{2} x_s^2$
  2. Equalization of temperature and density (pressure) between inner and outer field lines.



- Assume long *square-ended* model.

Phenomenological Inclusion of 3D-Effects

$$\frac{\partial n}{\partial t} + \nabla \bullet n\mathbf{u} = \dot{n}_b + \dot{n}_{\parallel}$$

$$\dot{T} = \ldots + (g - 1)\frac{T}{n}\left(\dot{n}_b + \dot{n}_{\parallel}\right)$$

$$\dot{n}_b = \frac{n}{g t_b}\left(\frac{< b_D > - < b >}{< b >}\right)$$

$$\dot{n}_{i\parallel} = \frac{1}{2 g t_{\parallel}}\left(\frac{n_o T_o - n_i T_i}{\frac{1}{2}\left(T_i + T_o\right)}\right)$$

$$\dot{n}_{o\parallel} = -\frac{|B_{zo}|}{|B_{zi}|}\dot{n}_{i\parallel}$$

Numerical Model

- Numerically, employ finite difference mesh in the *r*-direction, spectral in the *q*-direction.
- Semi-implicit advance of vector potential and velocity.
- Current version of code is limited to $n \leq 1$ azimuthal modes.
- Typically use 100 grid points and $\Delta t = .25 \times 10^{-9}$, for 2,000,000 time-steps (for .5 msec calculation).

Numerical Model

- All variables are spectrally decomposed in the θ-direction.

$$Q(q) = Q_o + \left(\sum_{n=1}^{N} Q_n e^{inq} + c.c. \right)$$

- Temporal differencing is performed with a semi-implicit predictor-corrector algorithm[1].

[1]D.C. Barnes, et al. "Flexible, Semi-Implicit Algorithms For Bounded, Multi-Dimensional MHD Calculations" Los Alamos Internal Report LA-UR-90-3573

Predictor - Corrector

Predictor:

$$\left(\mathbf{u}^{*}-\mathbf{u}^{n}\right)-c_{u}\nabla^{2}\left(\mathbf{u}^{*}-\mathbf{u}^{n}\right)=a\,\frac{\Delta t}{r^{n}}\left[\frac{1}{c}\mathbf{J}^{n}\times\mathbf{B}^{n}-\nabla P^{n}-\nabla\bullet\Pi-r^{n}\mathbf{u}^{n}.\nabla\mathbf{u}^{n}\right]$$

$$\left(\mathbf{A}^{+}-\mathbf{A}^{n}\right)-c_{A}\nabla^{2}\left(\mathbf{A}^{+}-\mathbf{A}^{n}\right)=a\,\Delta t\left[\mathbf{u}^{*}\times\mathbf{B}^{n}-hc\mathbf{J}^{n}\right]$$

$$\left(\mathbf{A}^{*}-\mathbf{A}^{+}\right)-c_{H}\nabla^{2}\left(\mathbf{A}^{*}-\mathbf{A}^{+}\right)=-a\,\Delta t\,\frac{c}{en}\left[\frac{\mathbf{J}^{n}\times\mathbf{B}^{n}}{c}-\nabla\mathbf{P}_{e}^{n}\right]$$

Predictor - Corrector

Corrector:

$$\left(\mathbf{u}^{n+1}-\mathbf{u}^{n}\right)-c_u\nabla^2\left(\mathbf{u}^{n+1}-\mathbf{u}^{n}\right)=\frac{\Delta t}{r^n}\left[\frac{1}{c}\mathbf{J}^*\times\mathbf{B}^*-\nabla P^*-\nabla\bullet\Pi^*-r^n\mathbf{u}^*.\nabla\mathbf{u}^*\right]$$

$$n^{n+1}-n^n=-\Delta t\nabla\bullet n^n u^{n+1}$$

$$S^{n+1}-S^n=\ldots$$

$$\left(\mathbf{A}^{++}-\mathbf{A}^{n}\right)-c_A\nabla^2\left(\mathbf{A}^{++}-\mathbf{A}^{n}\right)=\Delta t\left[\mathbf{u}^{n+1}\times\mathbf{B}^*-hc\mathbf{J}^*\right]$$

$$\left(\mathbf{A}^{n+1}-\mathbf{A}^{++}\right)-c_H\nabla^2\left(\mathbf{A}^{n+1}-\mathbf{A}^{++}\right)=-\Delta t\frac{c}{en^{n+1}}\left[\frac{\mathbf{J}^*\times\mathbf{B}^*}{c}-\nabla\mathbf{P}_e^{n+1}\right]$$

Semi-implicit Coefficients

$$C_u = \left(n_u \Delta t^2 \max_q \left(\frac{B^2}{4p} + \frac{P}{g-1} \right) + m \Delta t \right) \left(\min_q (r) \right)^{-1}$$

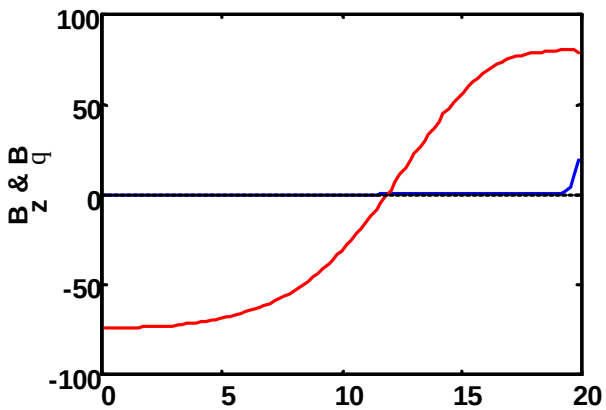
$$C_A = n_A \Delta t \left(\min_q \left(\frac{4p}{hc^2} \right) \right)^{-1}$$

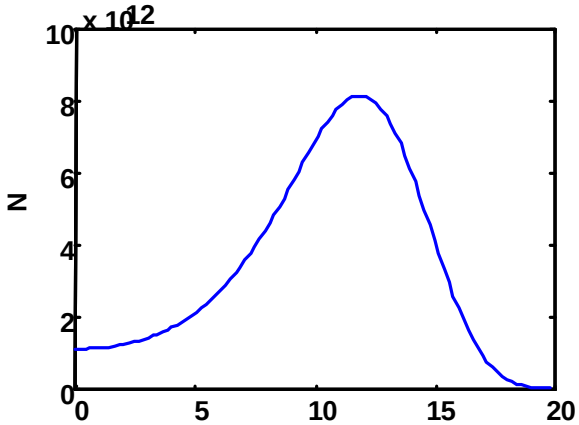
$$C_H = n_H \Delta t \frac{c}{4pe} \left(\min_q \left(\frac{n}{B} \right) \right)^{-1}$$

The numerical parameters n_u, n_A, and n_H, are chosen greater than 1, for numerical stability.

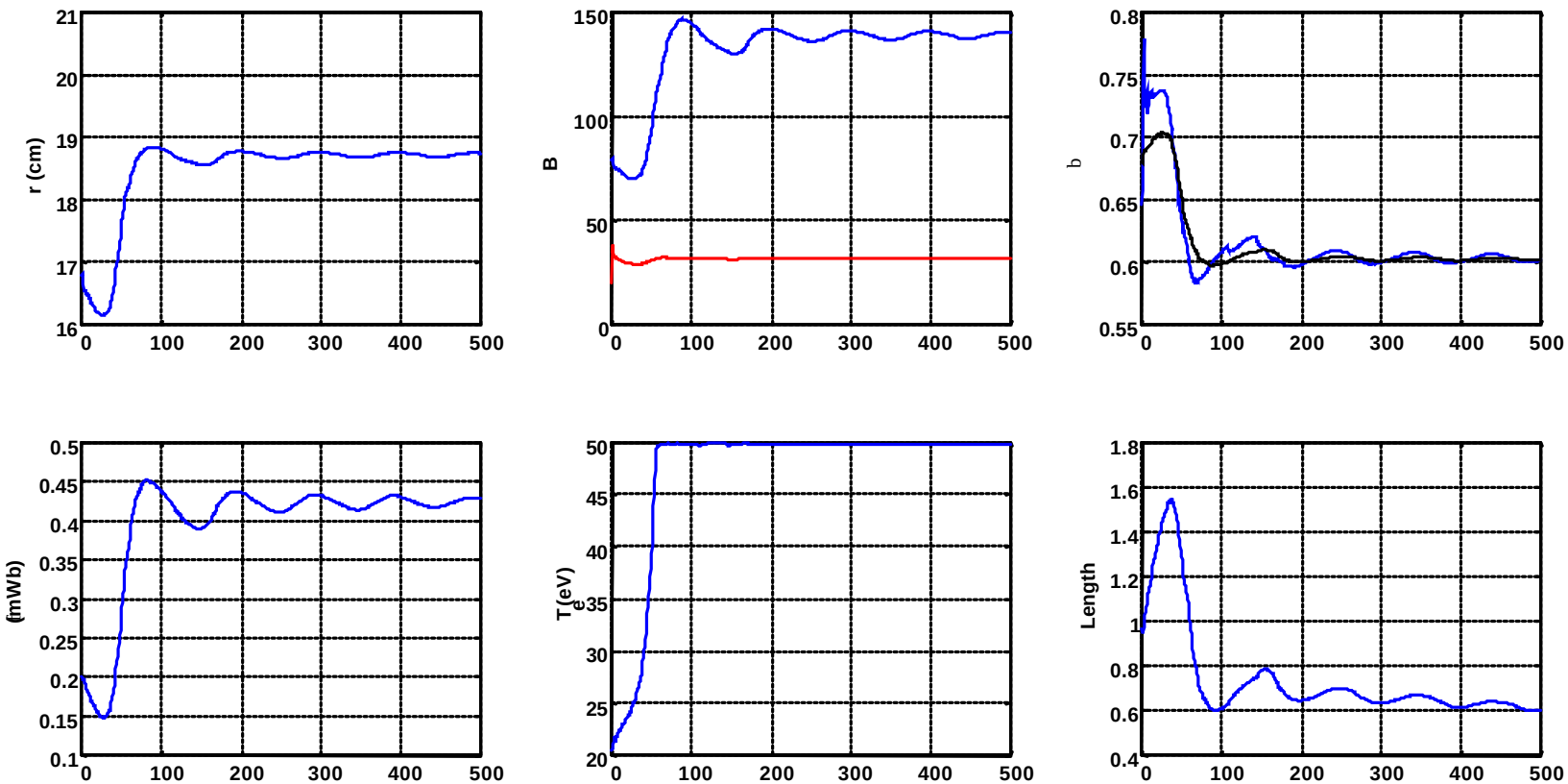
FRC Sustainment

- Initialize with an FRC profile, turn on RMF, and calculate evolution of profiles, trapped flux, etc.
- Parameters comparable to STX experiment were chosen. (T_e=20 eV, h = const (13 eV electrons).



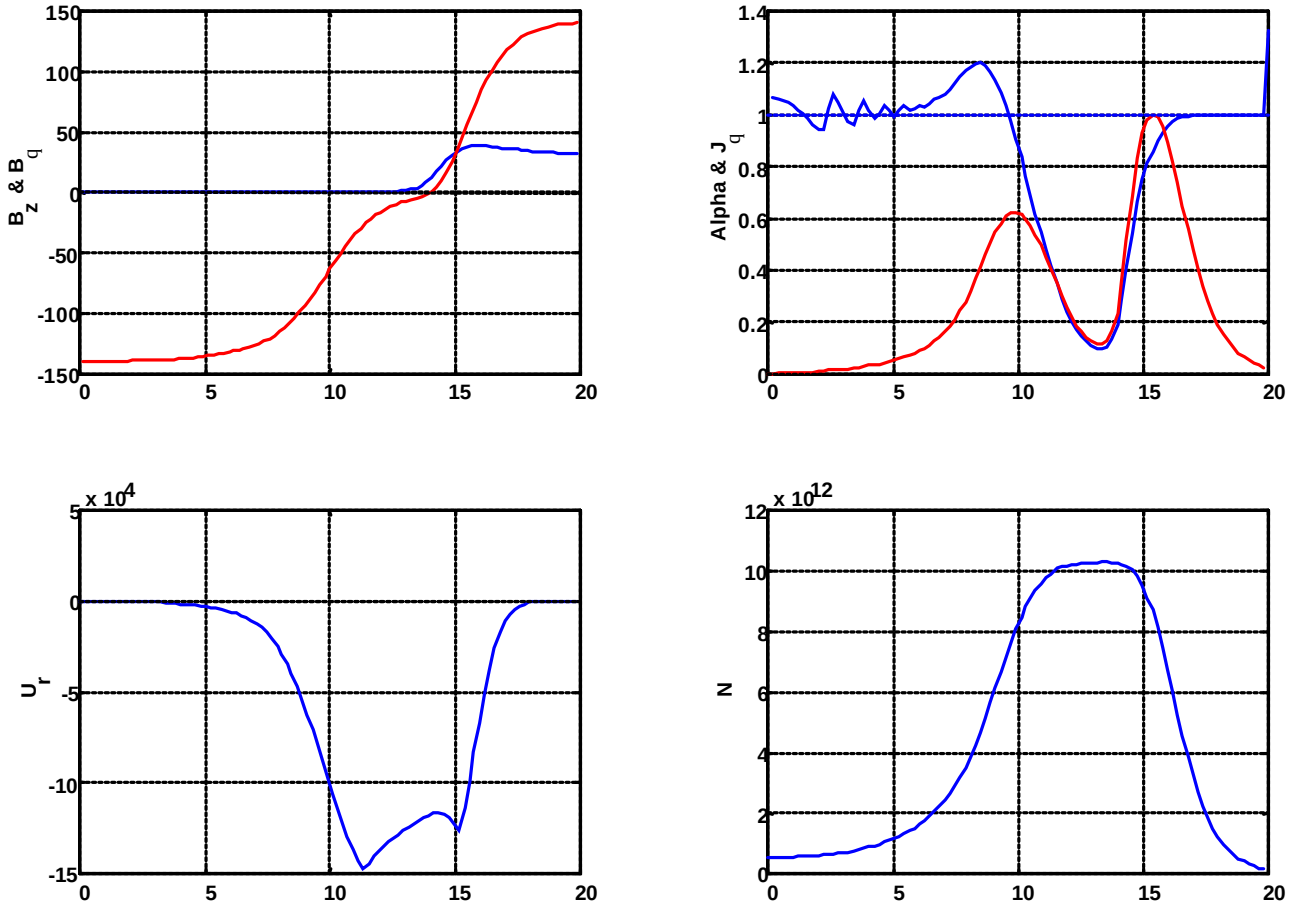


FRC Sustainment (Temporal Evolution)

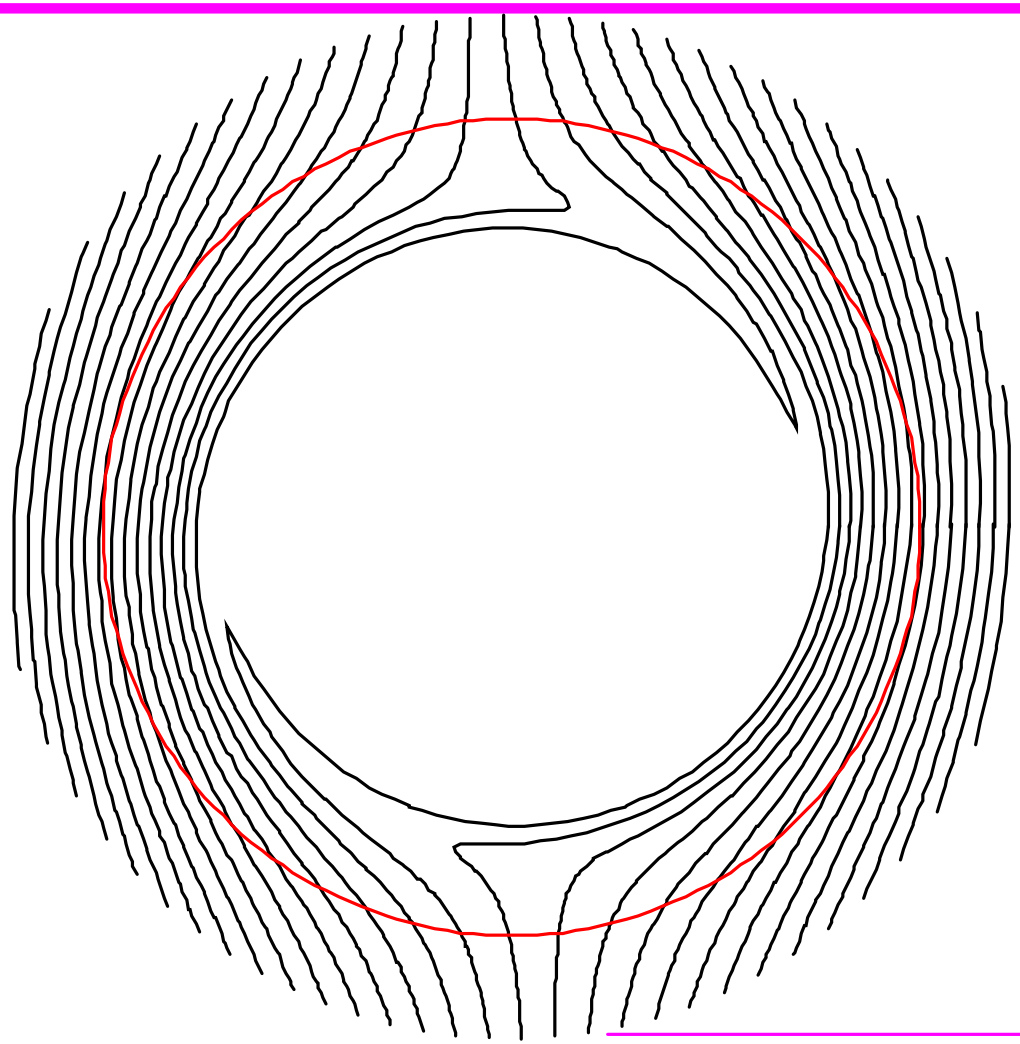


FRC Sustainment (Equilibrium Profile)

t=500 μsec, ncyc=2000000, dataSet=999

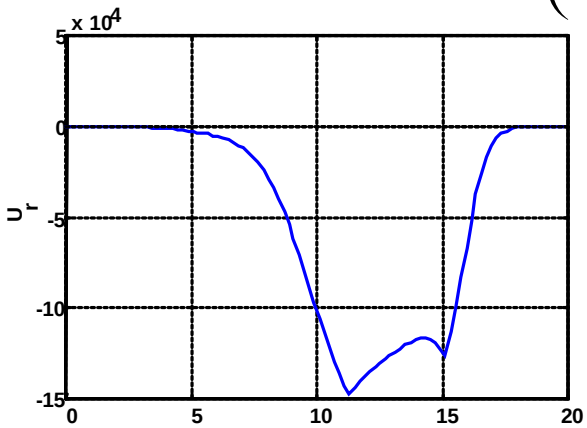


FRC Sustainment (Equilibrium RMF Field Lines)

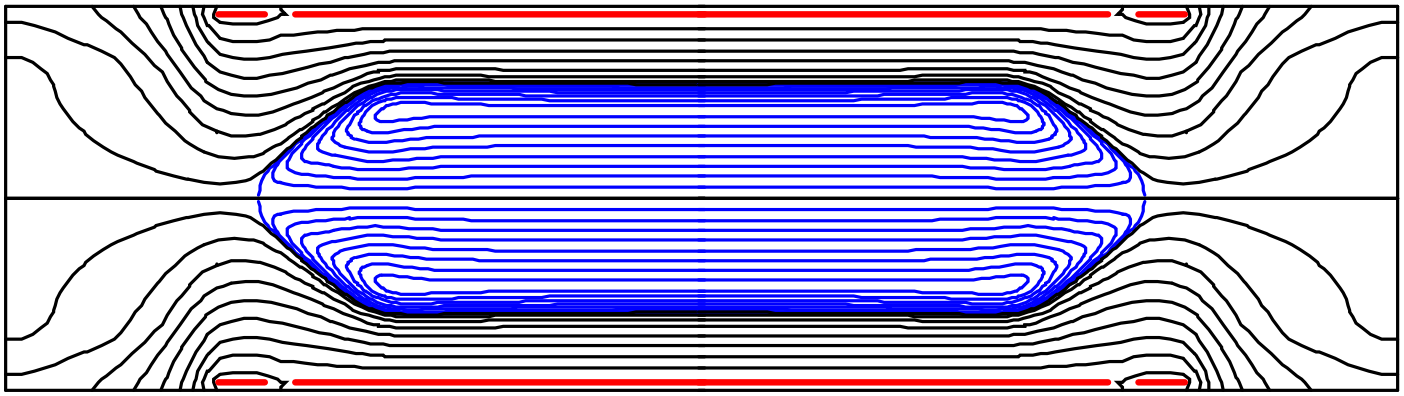


FRC Sustainment

$$\mathbf{E} = \mathbf{u}\times\mathbf{B} - hc^2\left(\frac{\mathbf{J}}{c}\right) - \frac{c}{en}\left[\left(\frac{\mathbf{J}}{c}\times\mathbf{B}\right) - \nabla P_e\right]$$

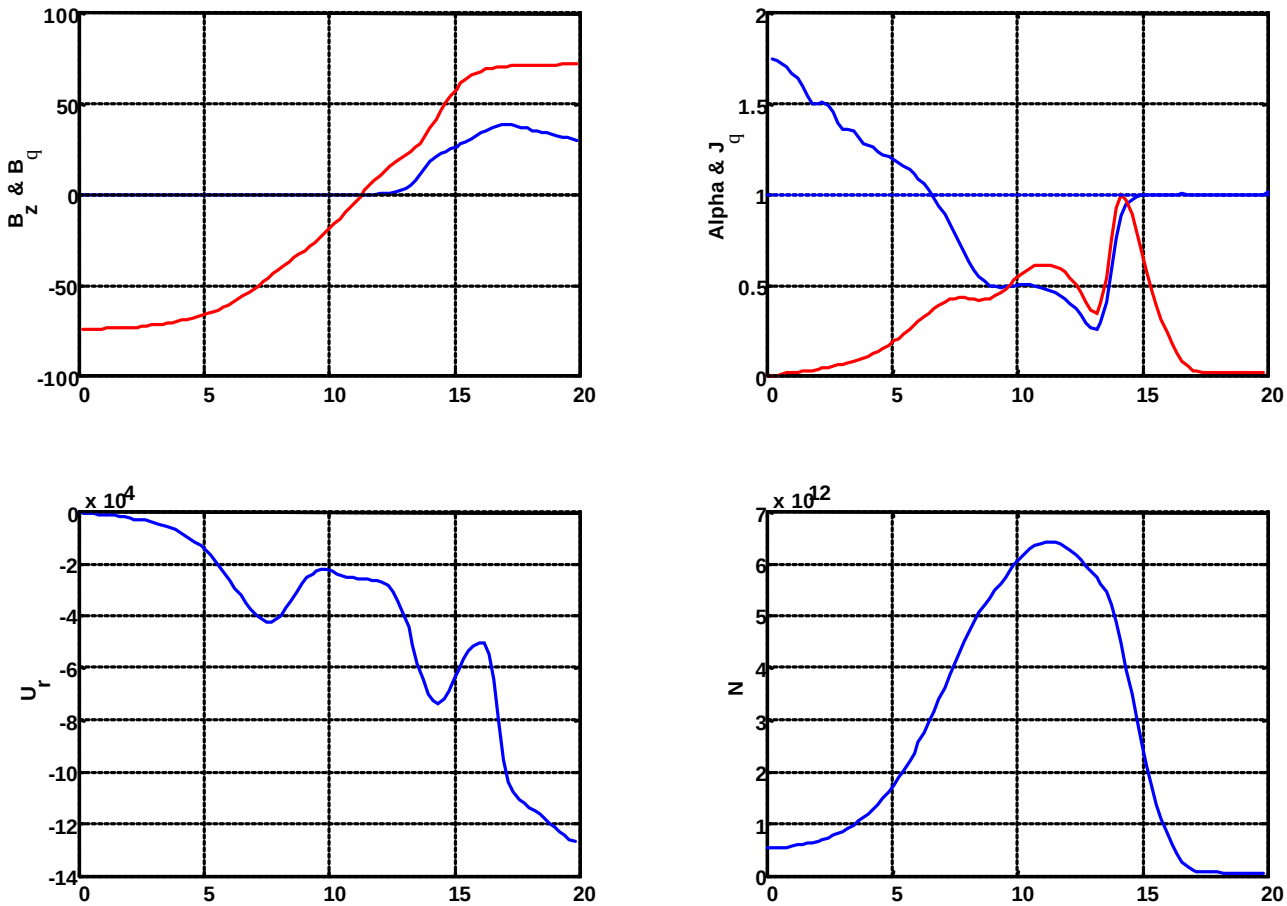


	Outer	Inner
u x B	-	+
*h*J	-	-
J x B	+	**0**



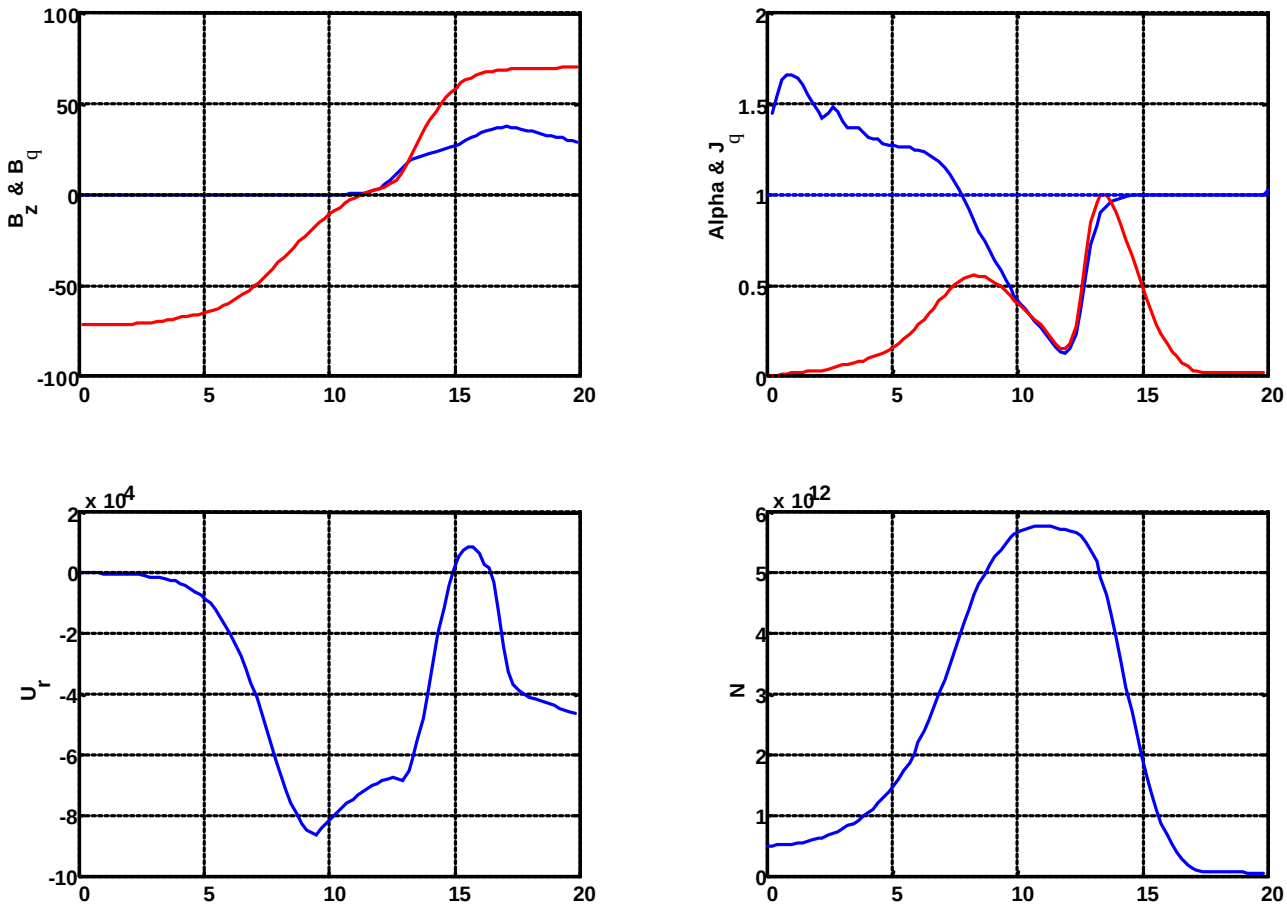
Flux Build-up Phase

t=15 *m*sec, ncyc=60,000



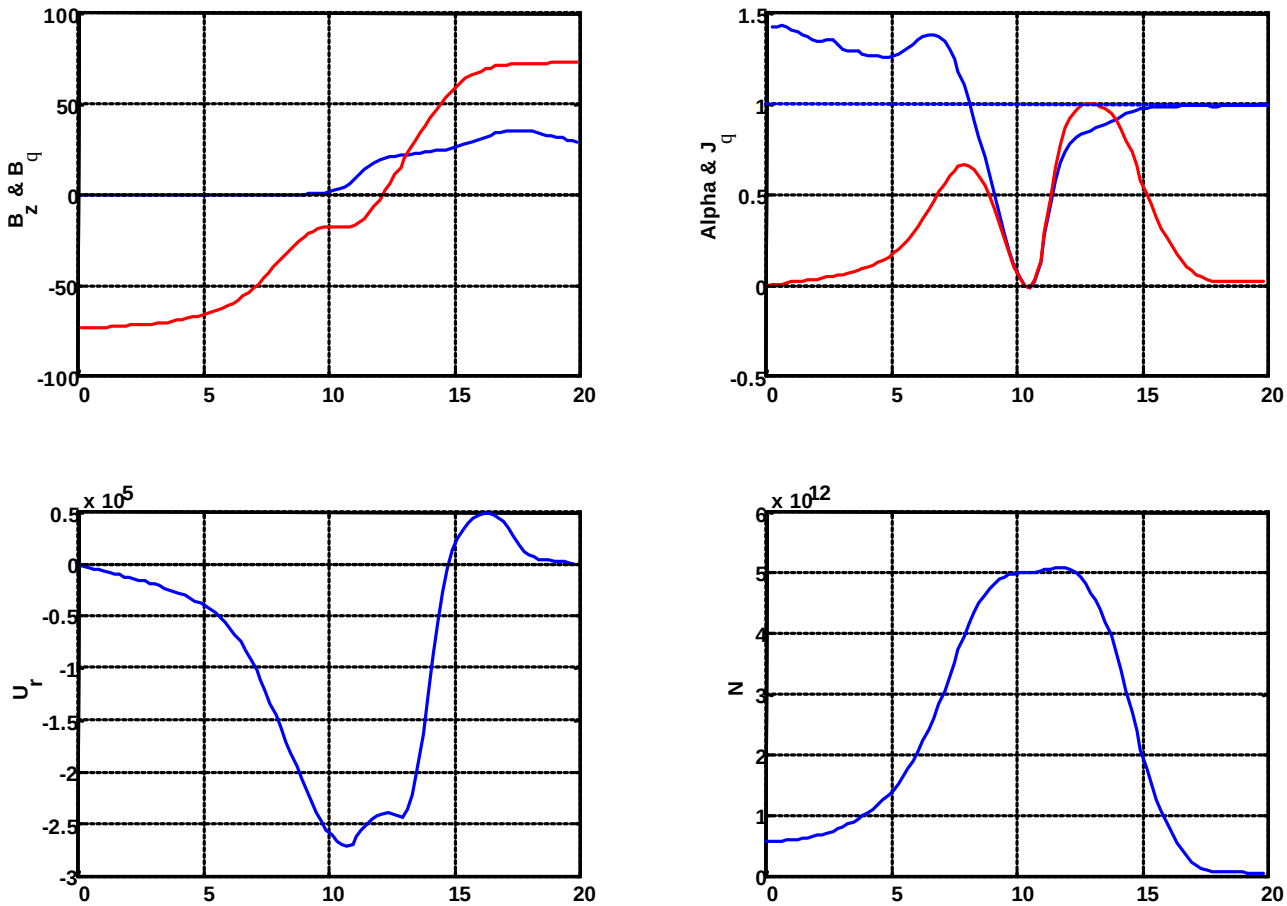
Flux Build-up Phase

t=25 *m*sec, ncyc=100,000



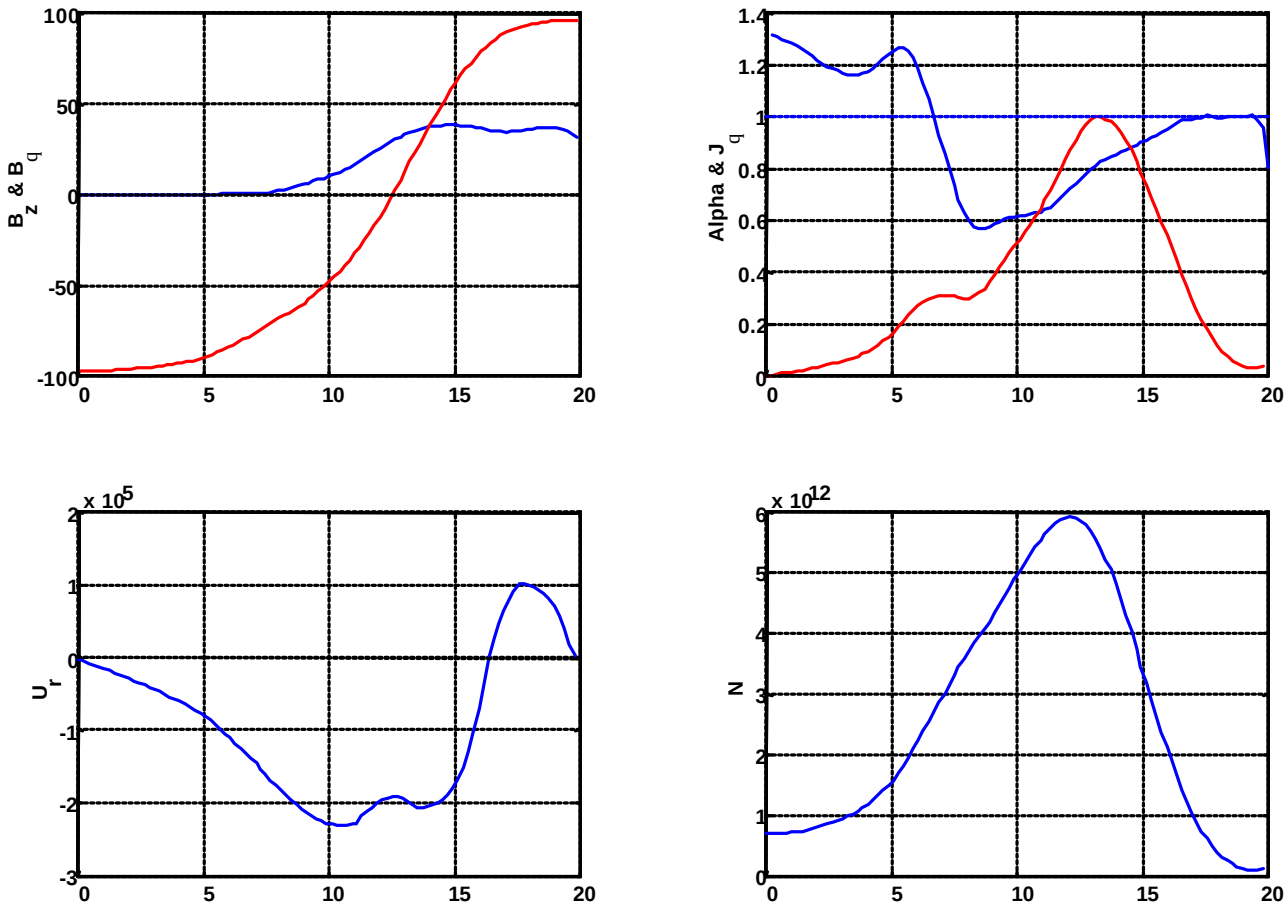
Flux Build-up Phase

t=37.5 *m*sec, ncyc=150,000



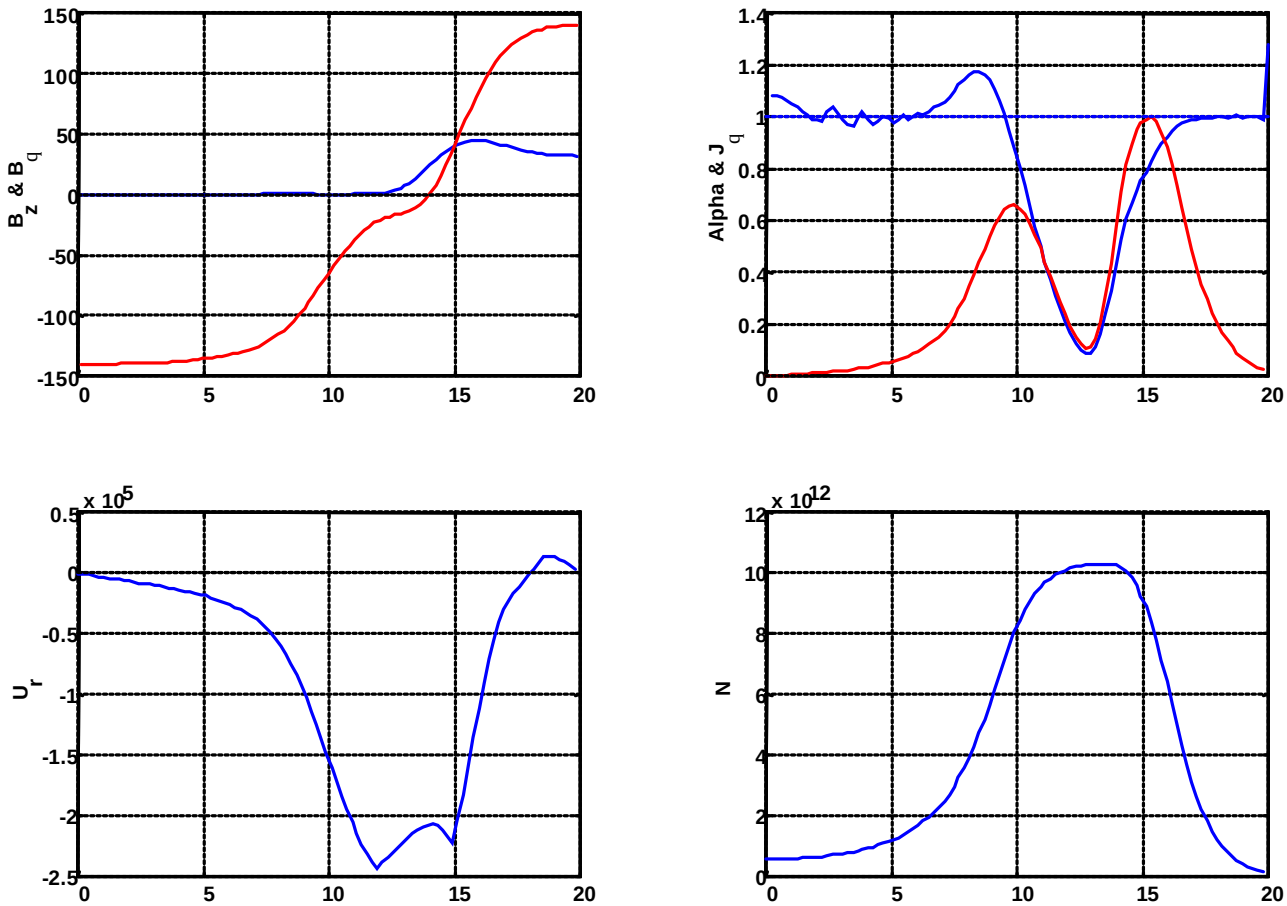
Flux Build-up Phase

t=50 *m*sec, ncyc=200,000



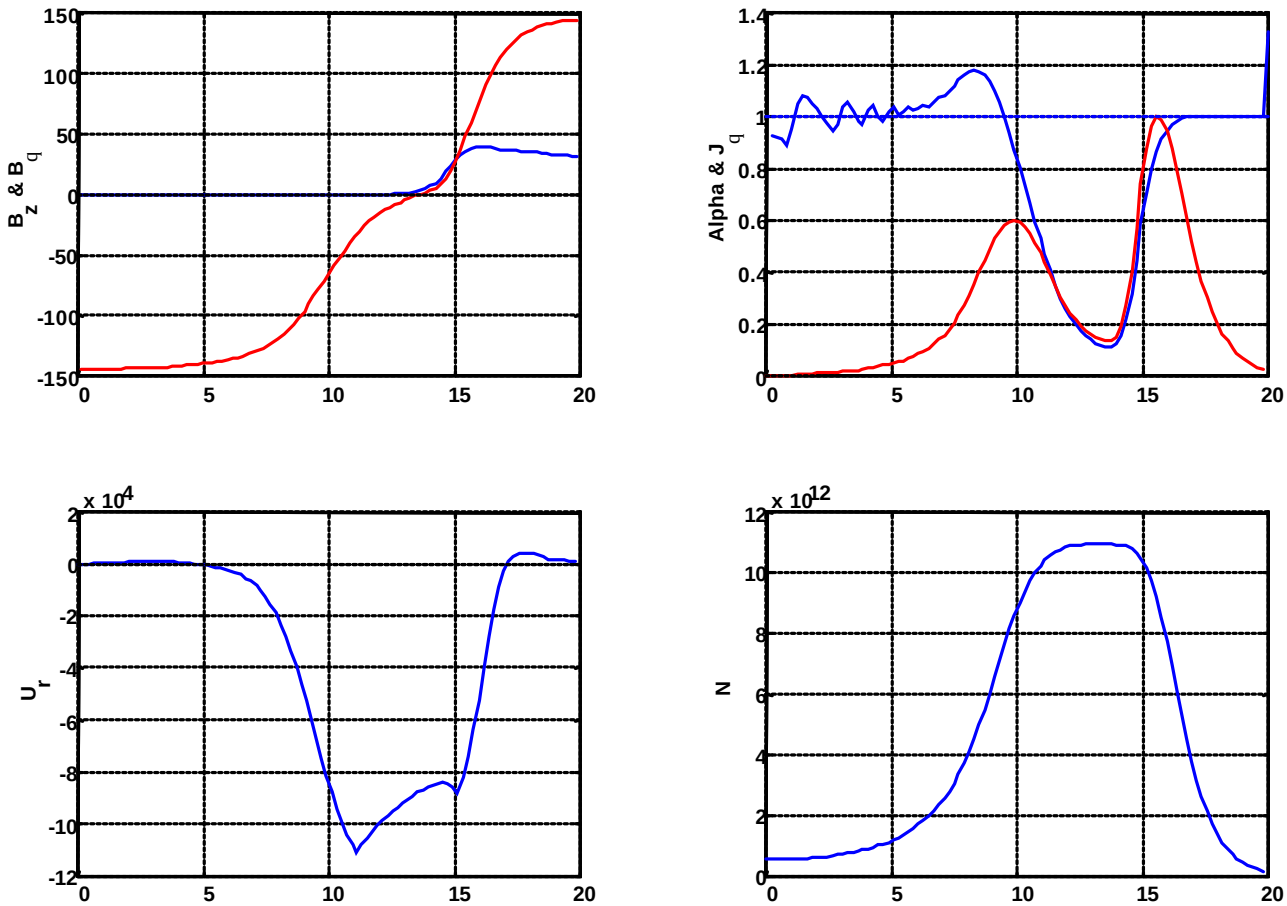
Flux Build-up Phase

t=75 *m*sec, ncyc=300,000

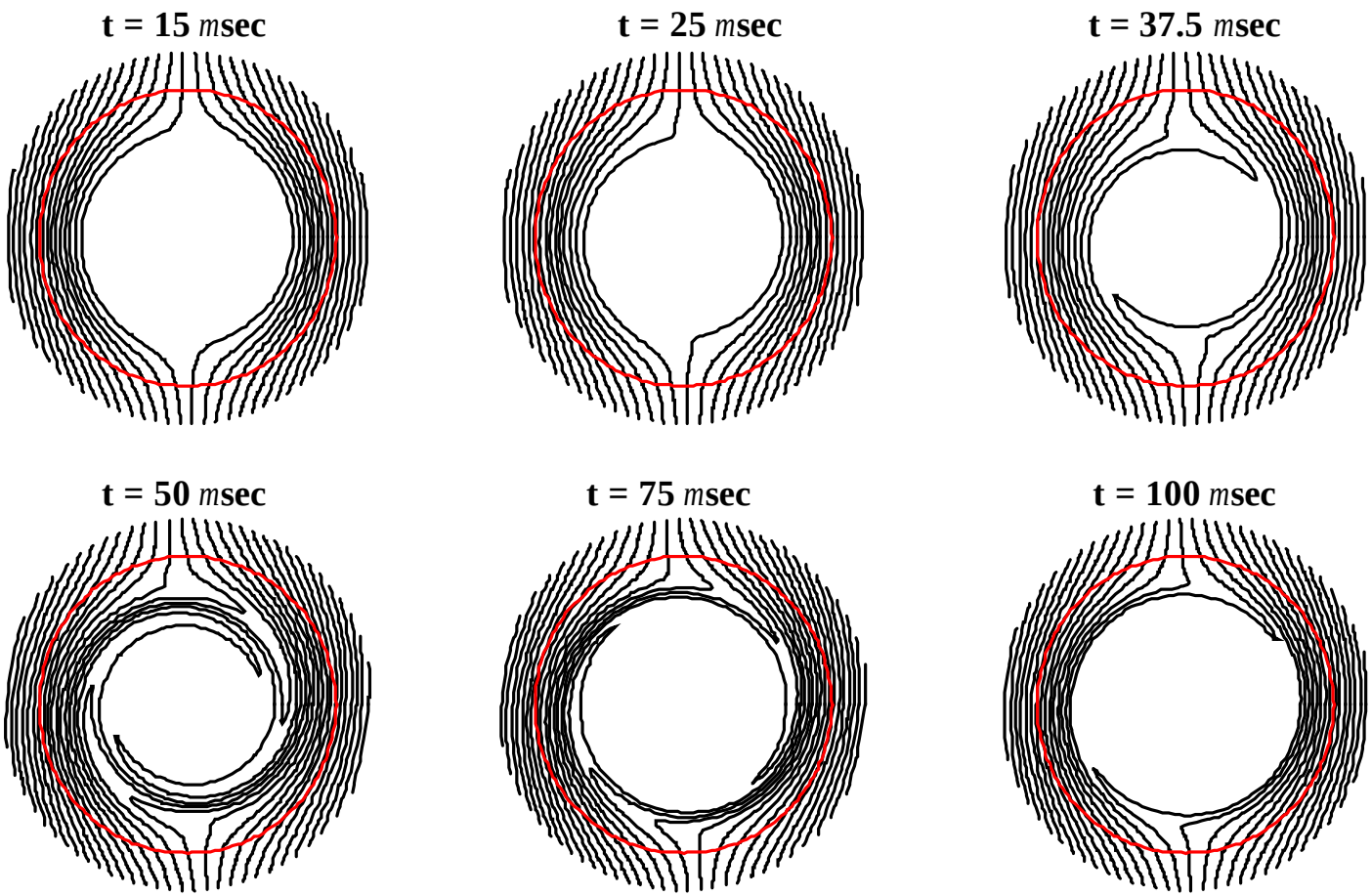


Flux Build-up Phase

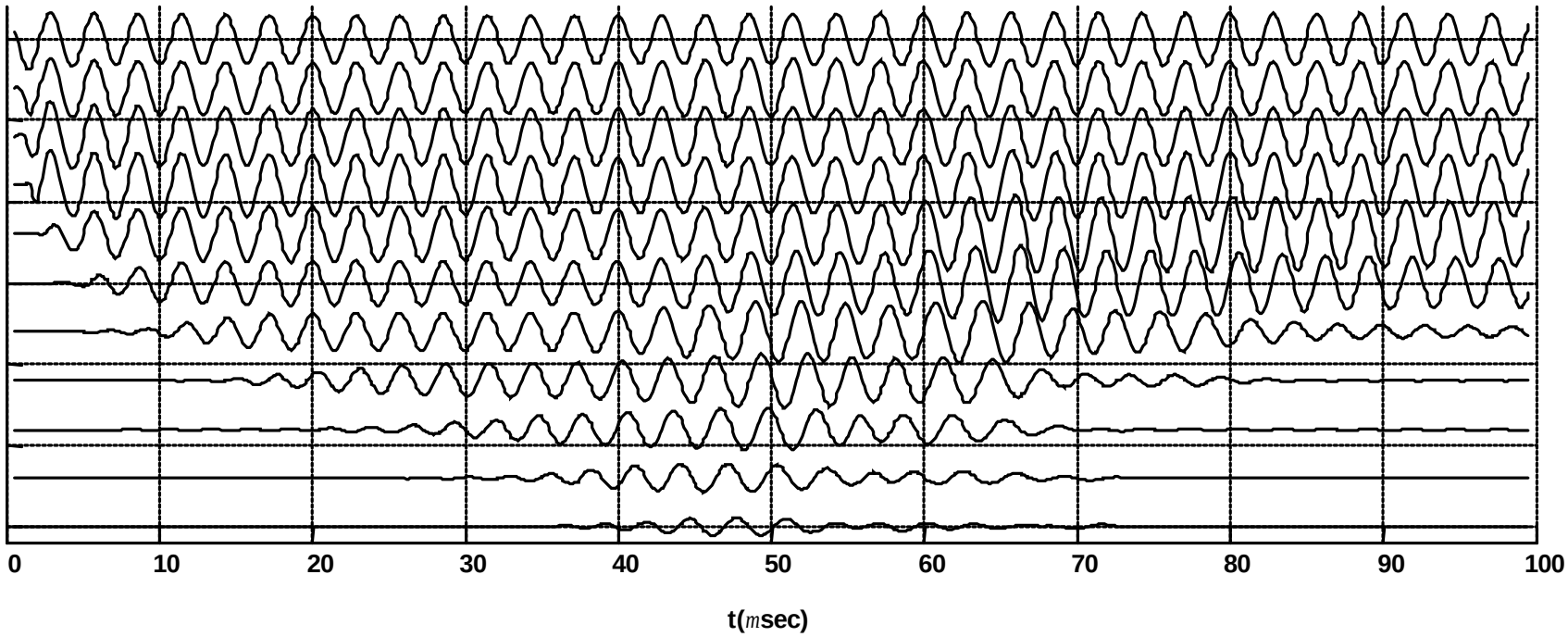
t=100 *m*sec, ncyc=400,000



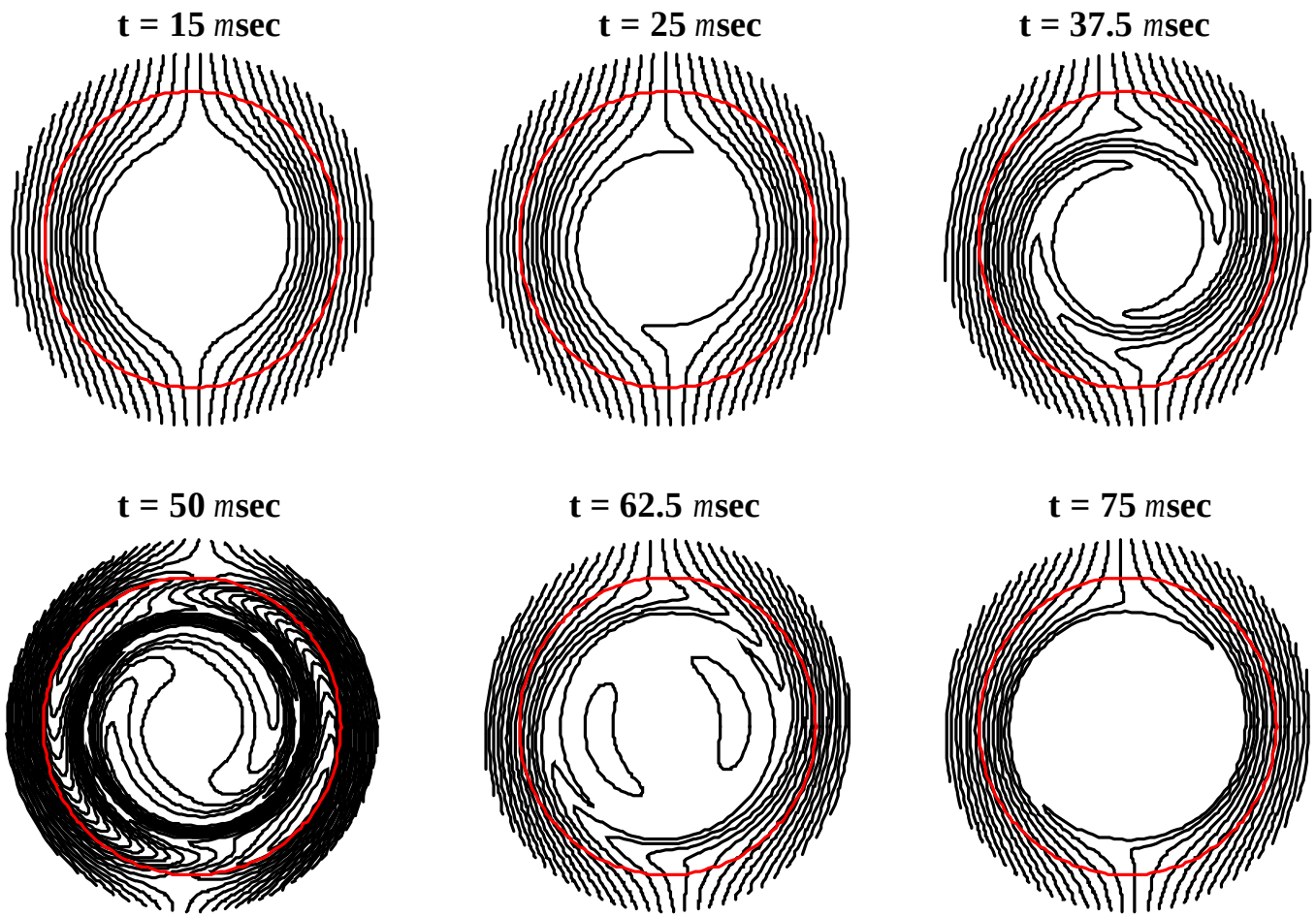
Flux Build-up Phase (Transverse Field Lines)



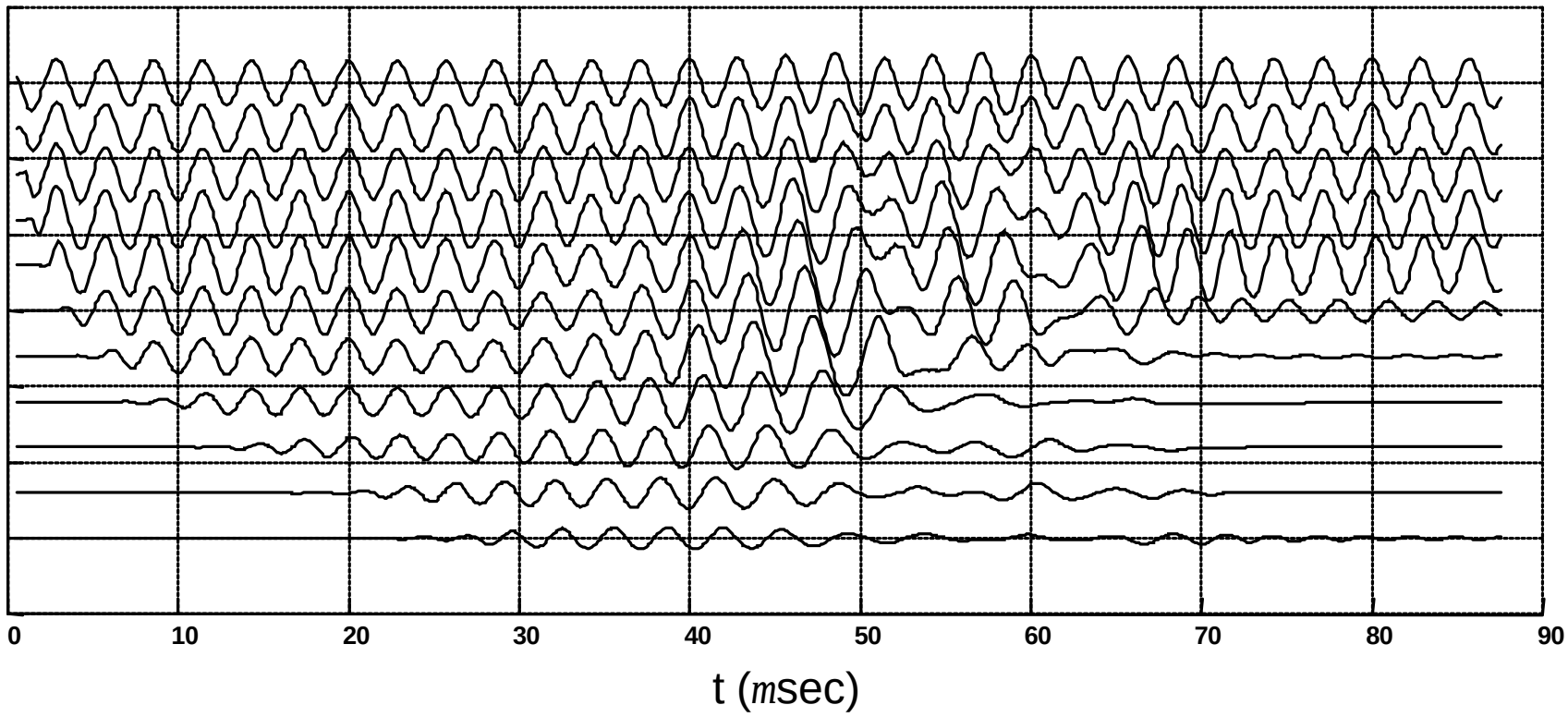
Internal B_q Probe Measurements



Flux Build-up (Transverse Field Lines)



Internal B_q Probe Measurements



Conclusions

- 2D (*r*-*q*) RMF code has provided new insights into the physics of RMF current drive.
  - Penetration
  - RMF Field expulsion
  - Mechanisms of sustaining an equilibrium
- Code has already proven itself to be a useful tool to assist in the interpretation of experimental data.