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NUMERICAL STUDY OF TILT MODE STABILITY IN FIELD-REVERSED CONFIGURATIONS

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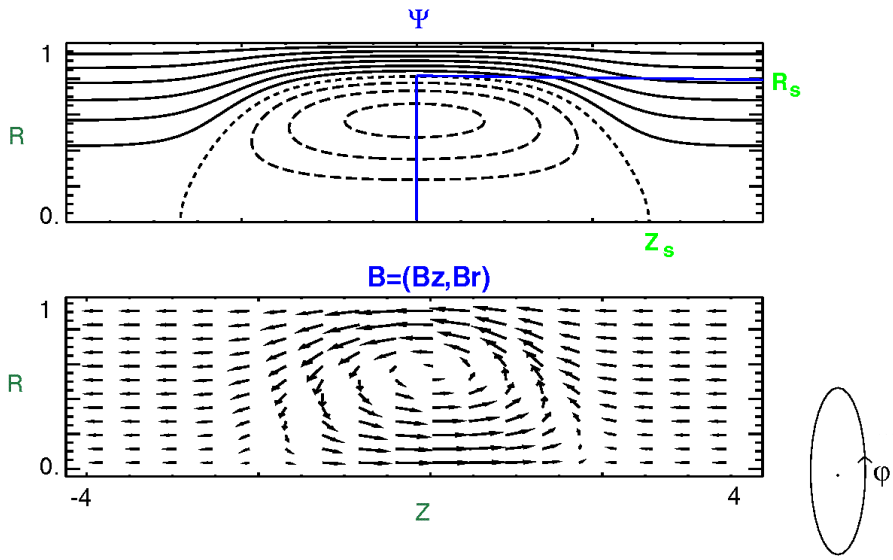


n - toroidal mode number: $\delta B \sim e^{in\varphi}$

$E = Z_s/R_s$ - elongation (E<1 - oblate; E>1 - prolate FRC)

$$\bar{s} \equiv \int_{R_o}^{R_s} \frac{r\,dr}{\rho_i R_s}$$

- key parameter for stability , $\bar{s} \sim 2\text{-}10$

large $\bar{s} \rightarrow$ MHD regime
small $\bar{s} \rightarrow$ kinetic

$n_i \sim 1\text{-}5\ 10^{15}\ cm^{-3}$	$T_e \sim 0.1\text{-}0.2$ keV	$L_c \sim 100\text{-}200$ cm
$B \sim 4\text{-}13$ kG	$T_i \sim 0.1\text{-}1$ keV	$R_c \sim 15\text{-}90$ cm

FRC stability code

2^d order accurate in time, explicit scheme

4th order spatial derivatives (3D finite difference)

Linearized or nonlinear

MHD - large $\bar{s}$

MHD / kinetic ions

Hybrid (fluid $\bar{e}$, particle ions) - small $\bar{s}$

For particles: δF scheme

Cylindrical grid for fields, 3D Cartesian grid for particles

Hybrid scheme (fluid electrons, kinetic ions)

$$\mathbf{E} = -\,\mathbf{V_e} \times \mathbf{B} - \frac{\nabla p_e}{e n_e} + \eta\,\mathbf{J}$$

$$\mathbf{V_e} = \frac{1}{e n_e}\,(\mathbf{J_i} - \mathbf{J})$$

$$\partial \mathbf{B}/\partial t = -\nabla\times\mathbf{E}, \qquad \mathbf{J} = \nabla\times\mathbf{B}$$

$$\partial p_e/\partial t + \mathbf{V_e}\cdot\nabla p_e + p_e\,\gamma(\nabla\cdot\mathbf{V_e}) = \eta\,(\gamma-1)\,J^2$$

$$n_e = n_i$$

Particles:

$$d\mathbf{x}/dt = \mathbf{v}$$

$$d\mathbf{v}/dt = \frac{e}{m}(\mathbf{E} - \eta\,\mathbf{J} + \mathbf{v}\times\mathbf{B})$$

Previous results

1. FRCs unstable in MHD regime (large $\bar{s}$)

   - n=1 tilt mode
   - n>1 - unstable
   - no nonlinear saturation

2. Sheared rotation stabilizing

   $V_s / V_A \gtrsim 1$

3. Shaping (n=1 mode)

   - elliptic separatrix $\gamma (V_A/R) \sim 1/E$
   - racetrack separatrix $\gamma \sim$ const (E>3)
   - oblate FRCs (E<1)

4. Kinetic effects (prolate FRCs) n=1 tilt mode

   - FLR effects $\omega^* \gtrsim \gamma \rightarrow \bar{s}/E \leq 0.2\text{-}0.5$
   - hybrid simulations

     γ is reduced but <u>no complete stabilization</u>

     $\bar{s} \sim 1$

Linearized hybrid simulation results

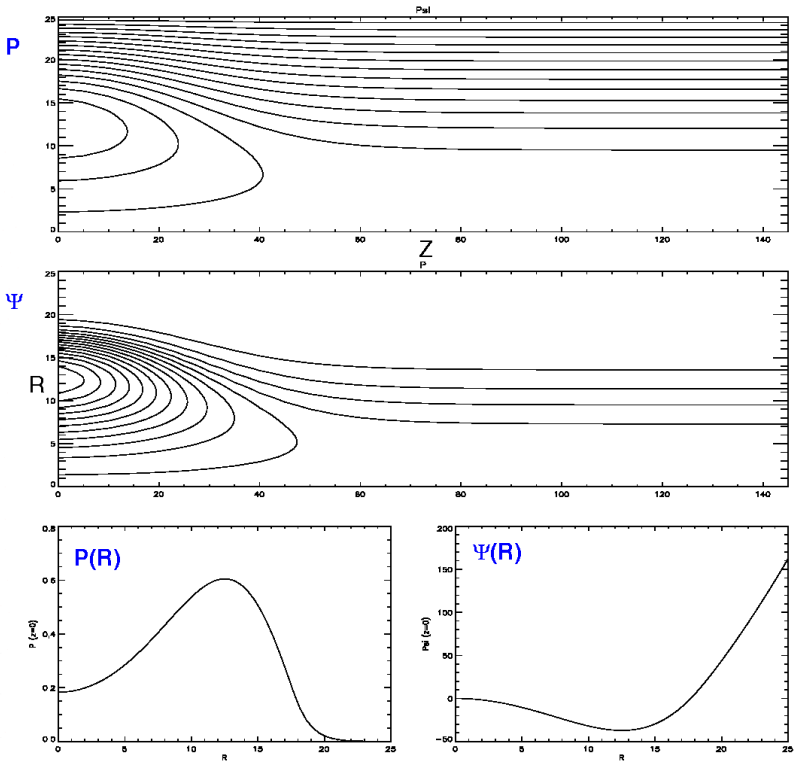
Table 1: Parameters for hybrid simulations

run	E	x_s	β_s	M_s	shape	$\bar{s}$	γ_{mhd}/γ_0	γ/γ_0
RH1	2.0	0.76	0.19	0.09	elliptic	9.2	1.5	1.4
RH2	1.7	0.78	0.17	0.55	elliptic	1.5	1.5	0.98
RH3	3.9	0.71	0.30	0.0	elliptic	7.4	1.8	1.6
RH4	3.9	0.71	0.30	0.0	elliptic	1.9	1.8	0.73
RH5	3.9	0.71	0.30	0.0	elliptic	0.9	1.8	0.34
RH6	4.1	0.76	0.15	0.0	racetrack	1.4	2.2	0.97
RH7	7.2	0.64	0.33	0.0	racetrack	6.2	2.54	2.01
RH8	7.2	0.64	0.33	0.0	racetrack	0.8	2.54	0.34

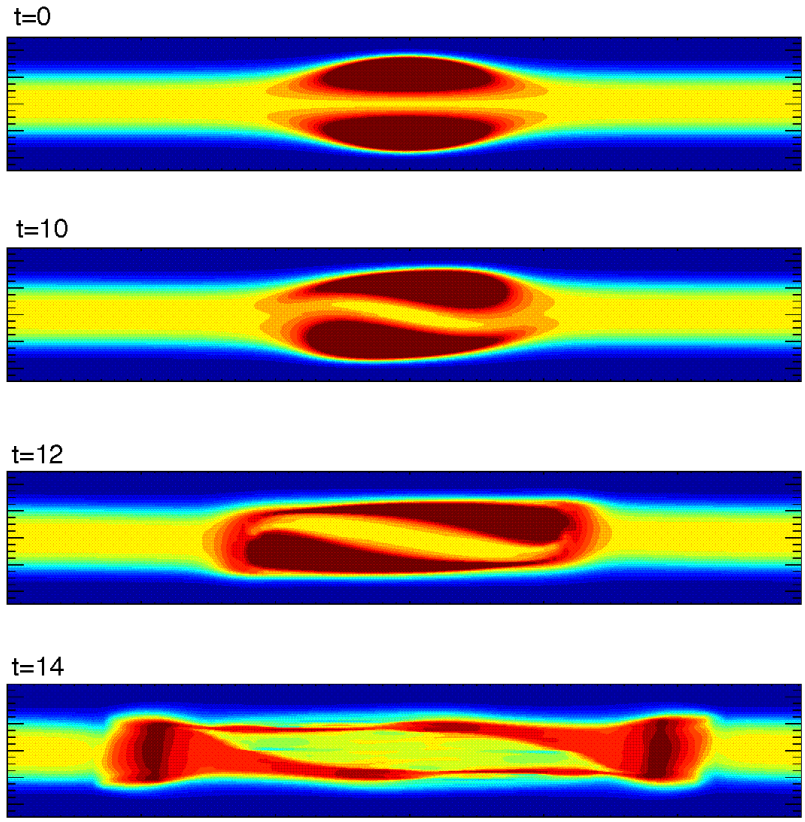
- Linear theory does not explain stability
- Non-linear effects in kinetic regime?

→ new result: non-linear stabilization in small $\bar{s}$ regime

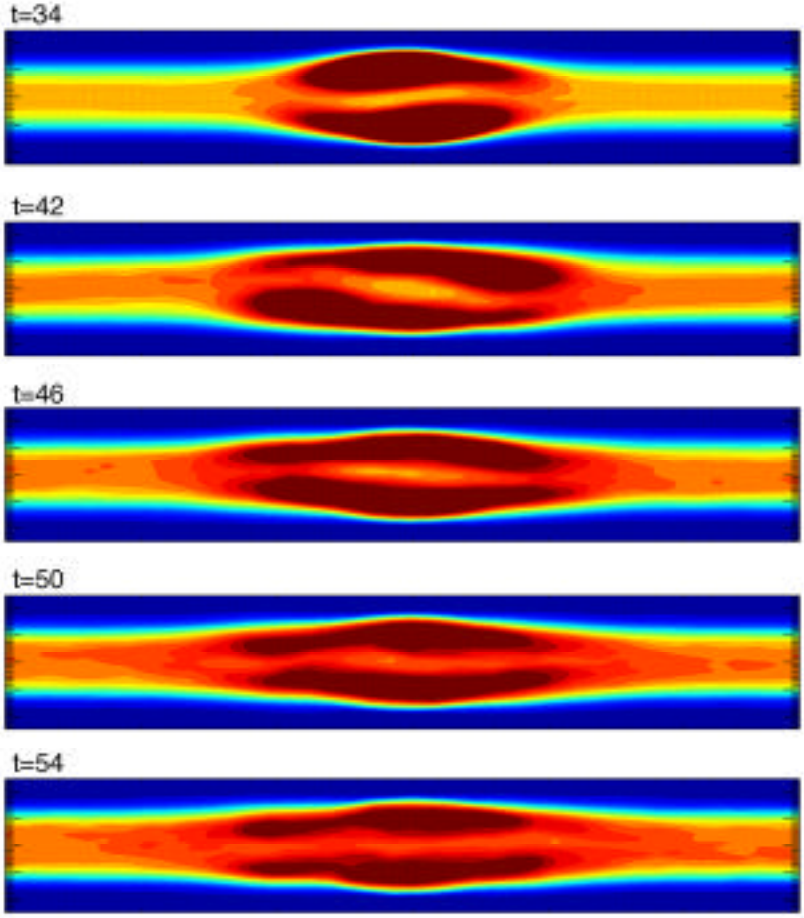
Equilibrium profiles for E=4 and s=2 hybrid simulations



MHD simulations with E=4, elliptical separatrix



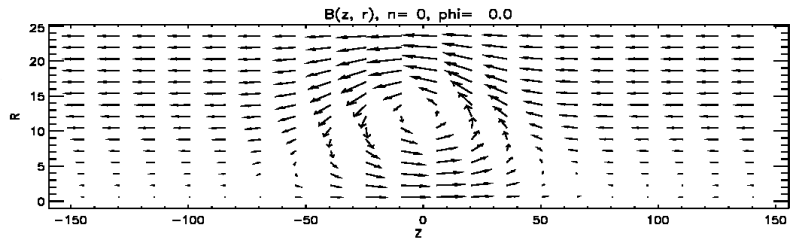
Hybrid simulations with E=4 s=2, elliptical separatrix



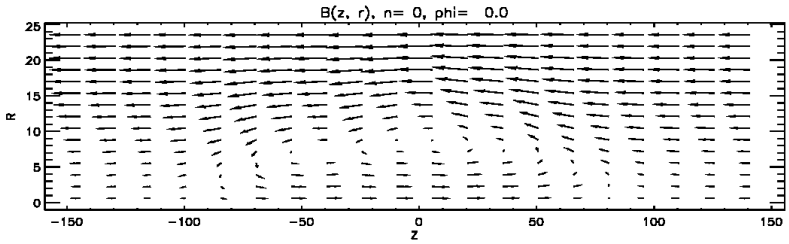
Nonlinear hybrid simulations with E=4 and s=2

Vector plots of poloidal magnetic field

t= 0

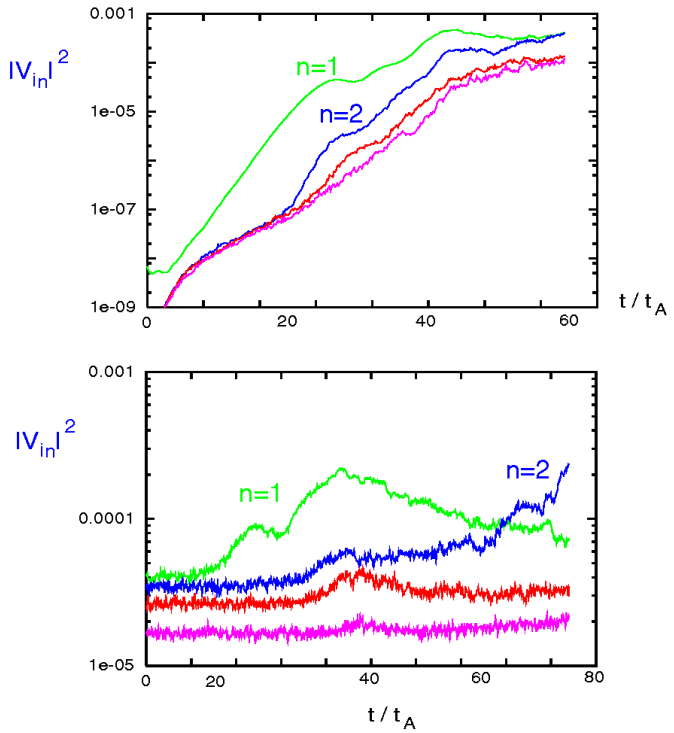


t= 62 t_A

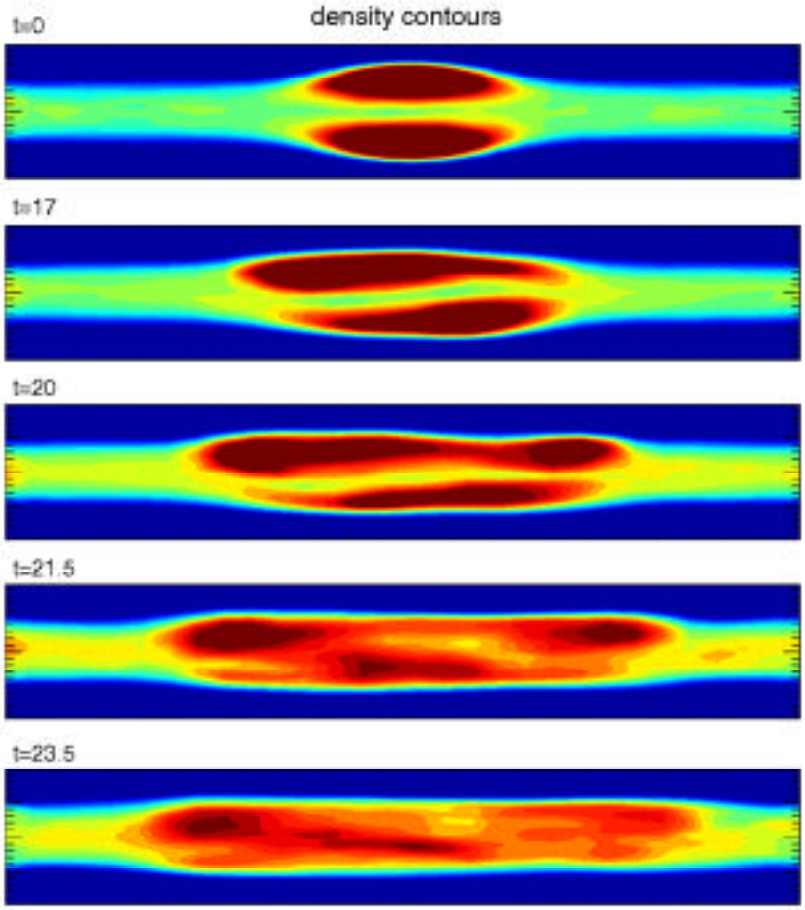


Energy plots from delta-f and full-f simulations

E=4 and s=2
np=2M



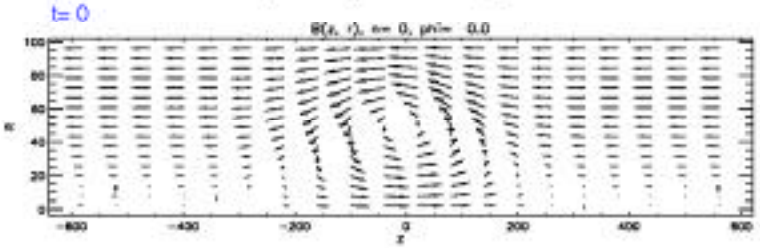
Hybrid simulations with E=4 s=7.4



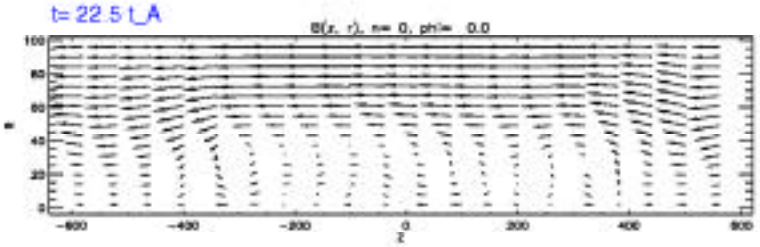
Nonlinear hybrid simulations with E=4 and s=7.4

Vector plots of poloidal magnetic field

t= 0

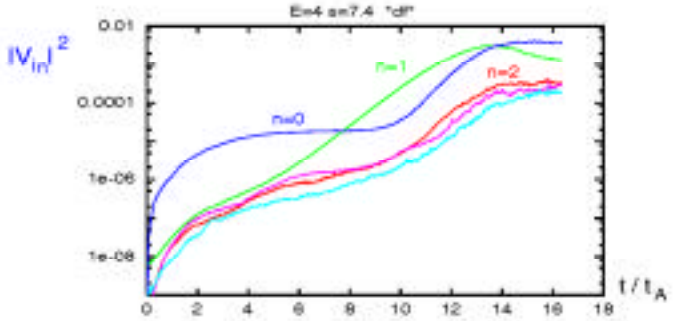


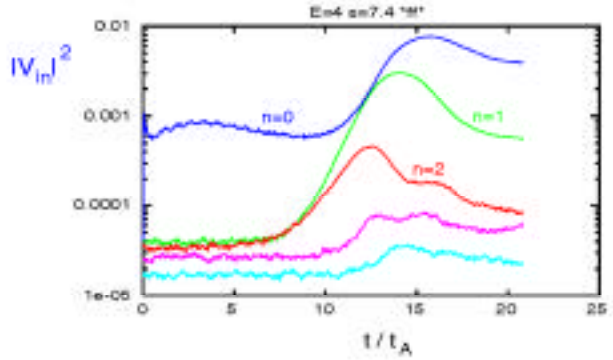
t= 22.5 t_A



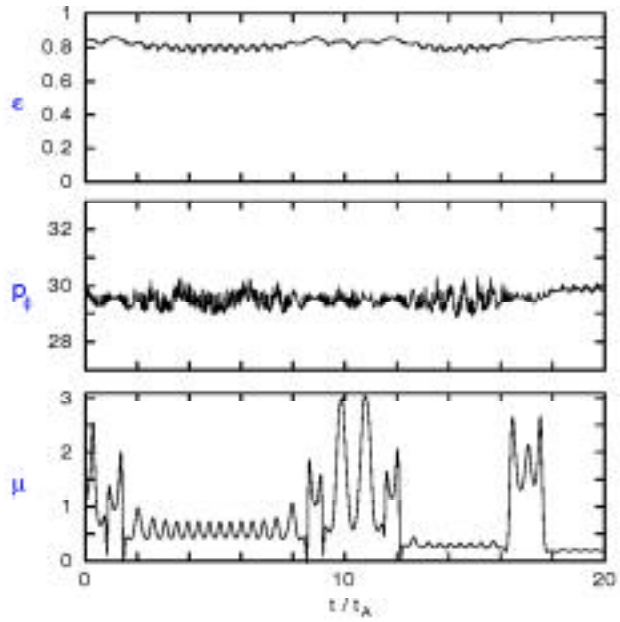
Energy plots from delta-f and full-f simulations

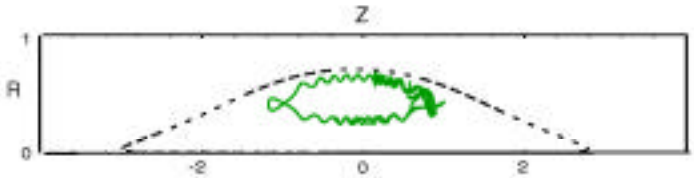
E=4 and s=7.4
np=2M





Variation of particle energy, toroidal angular momentum and magnetic moment along equilibrium orbit.



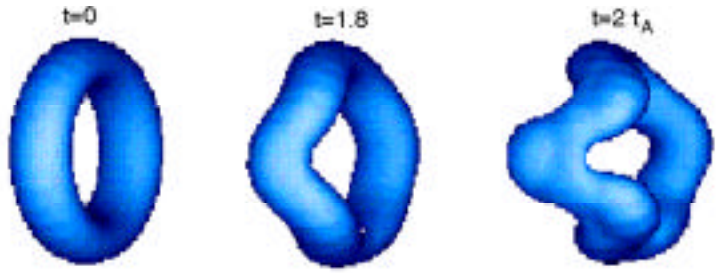


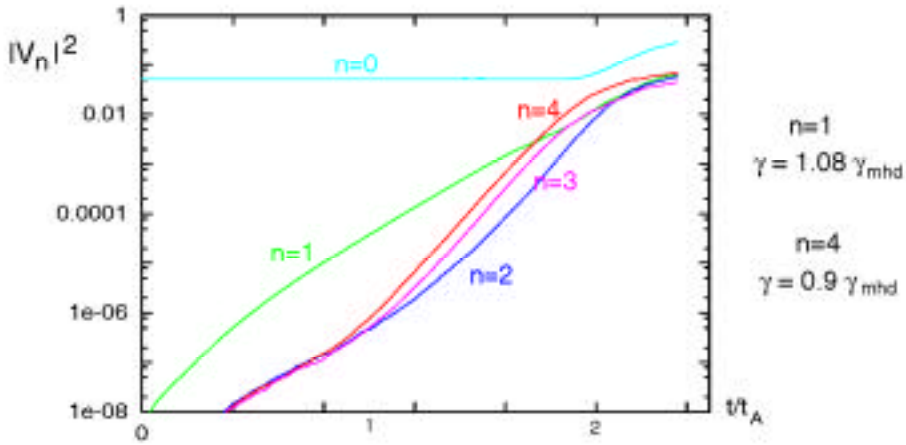
Nonlinear hybrid simulation of oblate FRC ($\bar{s}$=4, E=0.7)

80×60×32 cylindrical grid

80×51×51 particle grid; np= 0.5M

ion density isosurfaces

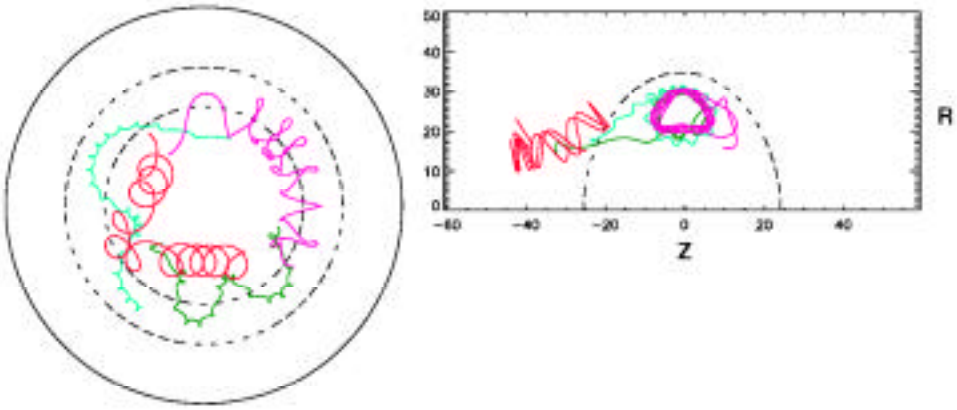




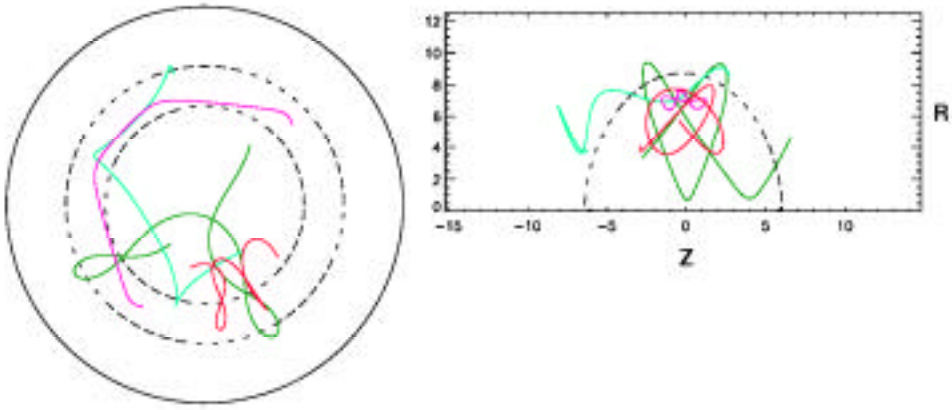
- FLR stabilization is not effective for low-n modes in oblate FRC for $\bar{s}$>1

Particle trajectories in oblate FRC (E=0.7)

Nonlinear hybrid simulation with $\bar{s}$= 4 (t=2.4t_A)



Linearized hybrid simulation with $\bar{s}$= 1 (t=1.1t_A)



Conclusions

- Linear stability E>1

n=1 tilt mode growth rate is reduced by sheared rotation and kinetic effects - no complete stabilization

linear mode structure changes at small s

- Nonlinear effects

Nonlinear saturation agrees with experiments:
elliptic separatrix
large separatrix beta
elongation E>6

Saturation mechanisms:
larger E - stability condition $\Omega > \omega_\beta$ is satisfied (?)
non-Maxwellian distribution F_i

Future work

- dependence on s, E, other shape parameters, and collisionality
- electron physics
- energetic ion beam stabilization for oblate and prolate FRCs